LAMINAR FLOW FORCED CONVECTION THROUGH ANNULAR REGULAR POLYGONAL DUCTS WITH CONSTANT HEAT FLUX PER AXIAL LENGTH

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ABSTRACT

Numerical solutions for the momentum and energy differential equations were obtained for laminar fully developed flow through annular regular polygonal ducts. The hydrodynamic results are casted in form of friction factor, incremental pressure drop and the hydrodynamic entrance length. The Nusselt numbers, under constant heat flux per unit length, are presented for three specific inner and outer circumferential boundary conditions which are, equal inner and outer wall temperatures and perfectly insulated inner or outer surfaces.

NOMENCLATURE

A	Annular duct area. (m^2)
a	Length of outer polygon side. (m)
b	Length of inner polygon side. (m)
D	Equivalent diameter (m)
f	Friction factor.
H	Spacing between nodal points in x-direction. (m)
h	Heat transfer coefficient. (W/m^2K)
K_p	Incremental pressure drop.
K_1,K_2	Spacings between nodal points in y-direction (m)
k	Thermal conductivity. (W/m k)
L_e	Hydrodynamic entrance length. (m)
N	Number of polygon sides.
Nu	Nusselt number.
P	Pressure. (N/m^2)
q	Wall heat flux per unit axial length. (W/m)
Re	Reynolds number.
S	Perimeter of solid walls. (m)
T	Temperature. (K)
u	Velocity. (m/s)
x,y	Transverse cartizian coordinates. (m)
z	Axial cartizian coordinate. (m)
α	Thermal diffusivity. (m^2/s)
ϵ	Annular ratio = b/a.
ψ	Angle of polygon symmetry = π/N.
ó	Viscosity. (kg/m.s)

SUBSCRIPTS

i	Inner wall.
m	Maximum.
o	Outer wall.

SUPERSCRIPTS

*	Dimensionless
-	Average

INTRODUCTION

The annular duct is one of the most important applications in the compact heat exchanger technology, since it is considered a simple packaged heat exchanger when two fluids pass through the inner duct and the annular cross section. The laminar flow through the annular circular duct was extensively discussed by Lunberg et al. [1]. They obtained the velocity and temperature distributions for constant and variable wall temperature. Kays [2] analysed also the problem as well as the single circular and single non-circular ducts to obtain the friction factor and Nusselt numbers. Lundgren et al. [3], using Green theory, obtained a closed form for the incremental pressure drop in the duct entrance length while Mc-Comas [4] estimated a hydrodynamic entrance length based on Lundgren [3] incremental pressure drop. The Nusselt numbers under constant heat flux of laminar flow through single regular polygonal ducts are obtained by Cheng [5]. Shah [6], using the least square matching technique, solved the momentum and energy equations in several non circular shapes. Topakoglu et al. [7] found the velocity and temperature distributions of the laminar flow through annular confocal ellipitical duct for different values of core size. They tabulated the Nusselt numbers at the inner and outer walls for equal wall temperatures and perfectly insulated inner and outer walls. Shah and London [8] gathered all hydrodynamic and heat transfer results of the laminar flow through circular and non circular, single and annular ducts.

From the review of literature in hand, the hydrodynamic and heat transfer informations about the flow through the annular non circular ducts are needed to make the heat exchangers designers able to choose the most efficient, economical and suitable geometry.

ANALYSIS

Figure (1-a) illustrates cross-section of the annular regular polygonal horizontal ducts, whose outer and inner sides lengths are "a" and "b" respectively. Figure (1-b) shows the calculation domain which represents (1/2N) of the original domain due to the two lines of symmetry. The angle "ψ" is the only parameter which represents the type of polygon $\hat{X}xd$ is equal to π/N.

GEOMETRICAL CONCEPTS

The following geometrical relations are used to calculate the area, the perimeter, and the equivalent diameter of the annular regular polygons with respect to the annular ratio (ϵ), as:

$$A = [\frac{N}{4}(1 - \epsilon^2) \cot \Psi] a^2 \quad (1\text{-a})$$

$$S = N(1 + \epsilon) a \quad (1\text{-b})$$

and

$$D = 4A/S = [(1 - \epsilon) \cot \psi] a \quad (1\text{-c})$$

The number of nodal points in "x" and "y" directions "m" and "n" as well as the nodal point spacings H, K_1 and K_2 are chosen in a way such that the nodal points are located on the inclinde symmetrical line. Therefore, the relation between i,j on the inclinde boundary is j = m + i - 1 and n = 2m - 1. Also the nodal points spacings H, K_1 and K_2 are calculated from,

$$H = [(1 - \epsilon) \cot \psi/\{2(m-1)\}]a \quad (2\text{-a})$$

$$K_1 = [\epsilon/\{2(m-1)\}]a \quad (2\text{-b})$$

and

$$K_2 = [(1 - \epsilon)/\{2(m-1)\}]a \quad (2\text{-c})$$

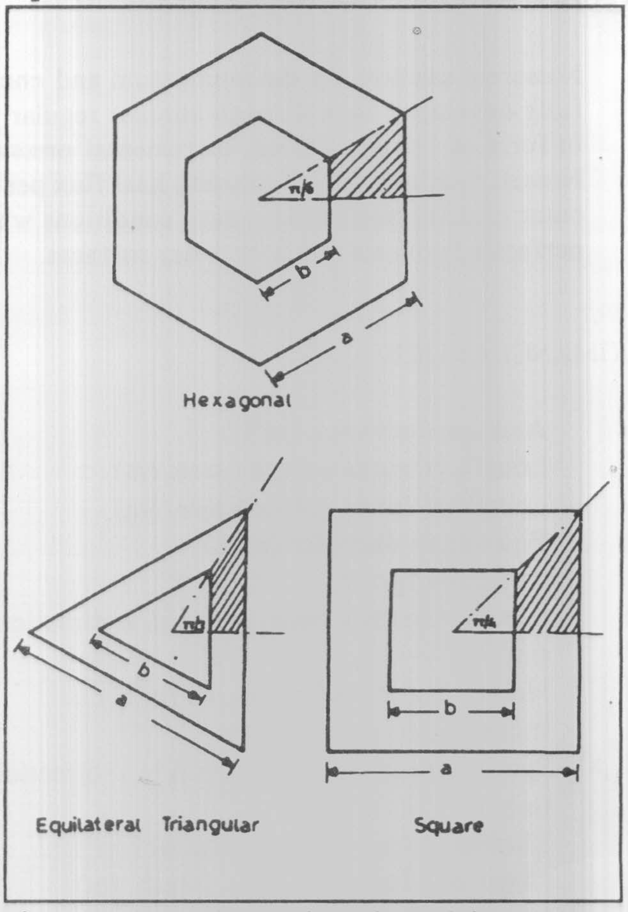

Figure 1-a. Annular regular polygonal ducts.

OMENTUM EQUATION FORMULATION

The reduced momentum differential equation governing the velocity distribution is,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = - \frac{1}{\acute{o}} \frac{dP}{dz} \quad (3)$$

In the fully developed region, the axial pressure gradient is constant while the boundary condition with no slip at the duct solid wall is u = 0 at the inner and outer surfaces. Consider the symmetrical lines, the velocity Dirchlet boundary value problem is changed to Neumann boundary value one i.e.

$\partial u/\partial y = 0$ at $y = 0$, while $(\partial u/\partial y) = (\tan \psi)\ (\partial u/\partial x)$ at the inclined symmetrical line.

The following dimensionless group are introduced in the momentum differential equation and in the boundary conditions as well as the geometrical relations,

$$x^* = x/a,\ \ y^* = y/a,\ \ A^* = A/a^2,\ \ S^* = S/a,$$
$$D^* = D/a$$

$$H^* = H/a,\ K^*_1 = K_1/a,\ K^*_2 = K_2/a$$
and $$u^* = u/(a^2/ó)(dp/dz) \tag{4}$$

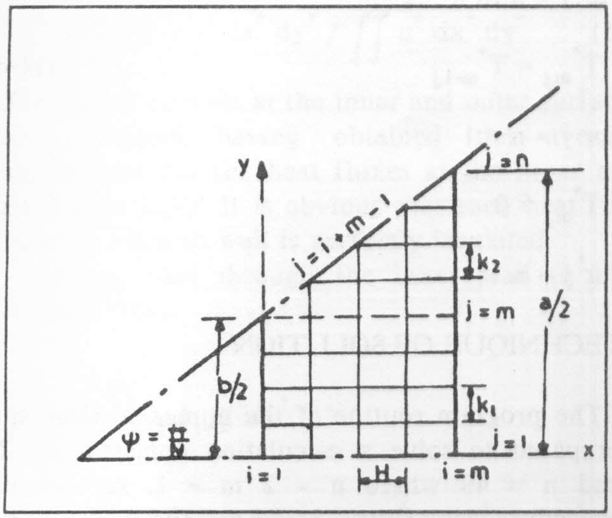

Figure 1-b. Calculation Domain.

Therefore, the dimensionless momentum differential equation becomes,

$$\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + 1 = 0 \tag{5}$$

The above equation, when it is applied on the internal nodal points, is casted in a finite difference formulation as,

$$(u^*_{i+1,j} - 2u^*_{i,j} + u^*_{i-1,j})/H^{*2} + (u^*_{i,j+1}$$
$$- 2u^*_{i,j} + u^*_{i,j-1})/K_1^{*2} + 1 = 0 \tag{6.a}$$

in the range of j = 2 to m−1

$$(u^*_{i+1,j} - 2u^*_{i,j} + u^*_{i-1,j})/H^{*2} + (u^*_{i,j+1}$$
$$- 2u^*_{i,j} + u^*_{i,j-1})/K_2^{*2} + 1 \quad = 0 \tag{6.b}$$

in the range of j = m + 1 to m + i − 2,

and

$$(u^*_{i+1,m} - 2u^*_{i,m} + u^*_{i-1,m})/H^{*2} +$$
$$2u^*_{i,m+1}/\{K^*_2(K_1^* + K^*_2)\}$$
$$+2u^*_{i,m-1}/\{K^*_1(K^*_1+K^*_2)\}$$
$$-2u^*_{i,m}/(K^*_1.K^*_2)+1=0 \tag{6.c}$$

Also the finite difference formulations for the solid boundary conditions are:

$u^*_{1,j}=0$ at i=1 and $u^*_{m,j} = 0$ at i = m

and at the lines of symmetry are:

at j = 1 $\quad \partial u^*/\partial y^* = 0$ at $y^* = 0$

or

$$- 3u^*_{i,1} + 4u^*_{i,2} - u^*_{i,3} = 0 \tag{7}$$

at j = m + i − 1 (inclined line of symmetry) i.e $\partial u^*/\partial y^* = (\tan \psi)\ \partial u^*/\partial x^*$

then,

$$\tan \Psi \left[\frac{u^*_{i+1,m+i-1} - u^*_{i,m+i-1}}{H^*} \right]$$

$$= \frac{u^*_{i,m+i-1} - u^*_{i,m+i-2}}{K_2^*}$$

Regarding the equations (2-a) to (2-c) with the dimensionless relations (4), that $\tan \psi = K_2/H$, then

$$u^*_{i,m+i-1}=[\tan^2\psi(u^*_{i+1,m+i-1})+u^*_{i,m+i-2}]/(1+\tan^2 \psi) \tag{8}$$

ENERGY EQUATION FORMULATION

The energy equation for the fully developed laminar flow under constant heat flux per unit axial length is,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{u}{\alpha}\frac{d\bar{T}}{dz} \qquad (9)$$

In that thermal situation, the average temperature gradient along the duct axis is constant. Therefore, the problem is a normal Poisson's differential equation since the term $d\bar{T}/dz$ is constant [8]. The energy equation is solved in three specific boundary conditions on the inner and outer peripheries, which are

i. Equal outer and inner surfaces temperatures.
ii. Insulated inner wall while the outside wall temperature is circumferentially constant.
iii. Insulated outer wall while the inside wall temperature is circumferentially constant.

The dimensionless temperature takes two forms for case (ii) and (iii), that

$$T^* = (T_o - T)/\left(\frac{a^4}{\text{ó}} \cdot \frac{dP}{dz} \cdot \frac{1}{\alpha} \cdot \frac{d\bar{T}}{dz}\right) \qquad (10.a)$$

in the second case

and

$$T^* = (T_i - T)/\left[\frac{a^4}{\text{ó}} \cdot \frac{dP}{dz} \cdot \frac{1}{\alpha} \cdot \frac{d\bar{T}}{dz}\right] \qquad (10\text{-}b)$$

in the third case.

The dimensionless energy differential equation becomes,

$$\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + u^* = 0 \qquad (11)$$

The finite difference formulations of the internal nodal points of the energy equation take similar forms to those of the momentum equation except that the source term is the velocity u^*_{ij} instead of unity in the momentum equation, equations (6-a), (6-b) and (6-c). The boundary conditions at the lines of symmetry are similar to the velocity boundary condition of these lines (Neumann boundary value problem).

For the inner and outer solid surfaces, the boundary conditions change according to each case of the three thermal cases:

i) For equal outer and inner surfaces temperatures case:

$$T^*_{1j} = T^*_{mj} = 0 \qquad (12)$$

ii) For insulated inner surface case:

$$T^*_{m,j} = 0 \qquad (13\text{-}a)$$

and $T^*_{1j} = (4\,T_{2j} - T^*_{3j})/3$ (13-b)

iii) For insulated outer surface case:

$$T^*_{1,j} = 0 \qquad (14\text{-}a)$$

$$T^*_{mj} = (4T^*_{m-1,j} - T^*_{m-2,j})/3 \qquad (14\text{-}b)$$

for j = 2 to n - 2

$$T^*_{mj} = T^*_{m-1,j} \qquad (14\text{-}c)$$

for j = n - 1

$$T^*_{mj} = 0 \qquad (14\text{-}d)$$

for j = n

TECHNIQUE OF SOLUTION

The program routine of the numerical solution is prepared to solve a calculation domain of m = 25 and n = 49 where n = 2 m - 1. An iterative procedure using Gauss-Seidel method is used for the momentum and energy equation respectively. The number of iteration steps increases from about 100 steps at $\epsilon = 0.9$ to 1600 steps $\epsilon = 0.05$ especially when the solid surfaces are insulated.

The velocity distribution obtained from the numerical solution of the momentum equation is integrated over the calculation domain using the modified trapezoidal method in order to calculate the average velocity as,

$$\bar{u}^* = \iint u^*\, dx^*\, dy^* / A^* \qquad (15)$$

The friction factor, the incremental pressure drop and the hydrodynamic entrance length are obtained as follows:

$$f.\ Re = -\,D^{*2}/(2.\bar{u}^*) \qquad (16)$$

while the incremental pressure drop, K_p, according to [3] is calculated from,

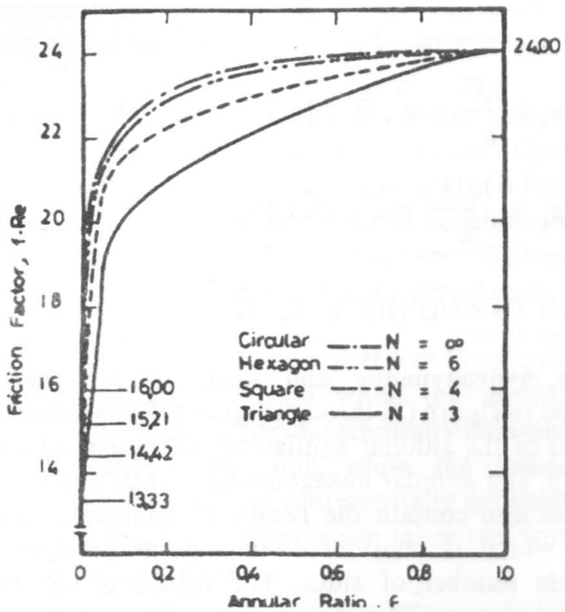

Figure 2. Friction factor versus the duct annular ratio.

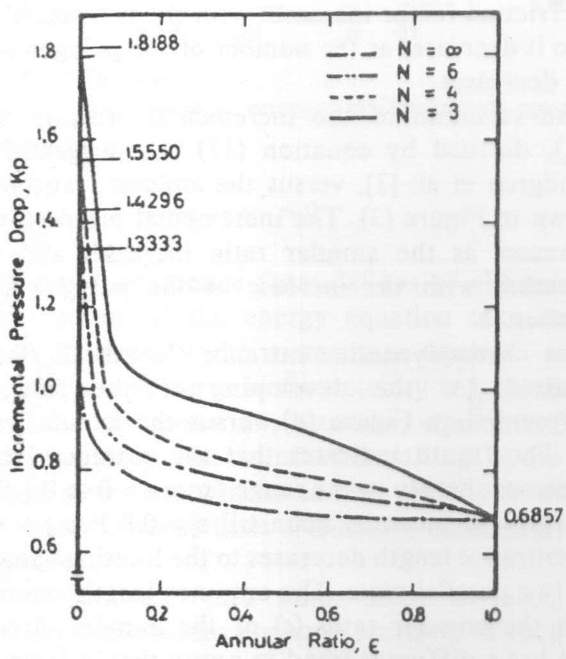

Figure 3. The incremental pressure drop versus the duct annular ratio.

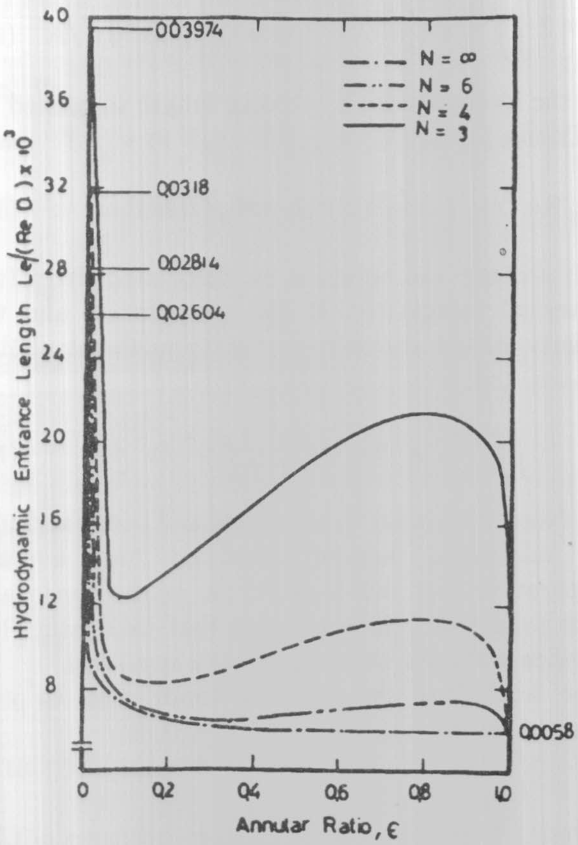

Figure 4. The hydrodynamic entrance length versus the duct annular ratio.

Generally, all the hydrodynamic functions i.e.f. Re, K_p and L_e/Re.D change with sharp rate in the range of ϵ = 0 to 0.1 since the solid boundary, whatever small is, creates a zero velocity contour inside the duct.

Figure (5) illustrates the Nusselt numbers variation at the inner surface (Nu_i) and that at the outer surface (Nu_o) in the case of equal walls temperatures. The Nusselt number at the inner surface, for all polygons, decreases from ∞ at $\epsilon \rightarrow 0$ to the parallel plate limiting value 8.235 [2,8] at ϵ = 1. The equilateral triangular has a minimum value of 7.803 at ϵ = 0.5. The Nusselt numbers of the outside surface (Nu_o) increase from the single duct values, 4.3636, 3.978, 3.605, and 3.111 for the circular, hexagonal, square, and triangular ducts, at ϵ = 0, to the parallel plate value at ϵ = 1.

In the case of perfectly insulated inner surface, the Nusselt number at the outer surface (Nu_o), Figure (6), increases with the increase of (ϵ) from the single duct values at ϵ = 0 to the limiting value of two parallel plate having one side insulated which is 5.385 [2,8]. The relation between Nu_o and ϵ seems to be straight line in the range from 0.2 to 1.0 extended to ϵ = 0.025 for the triangular one.

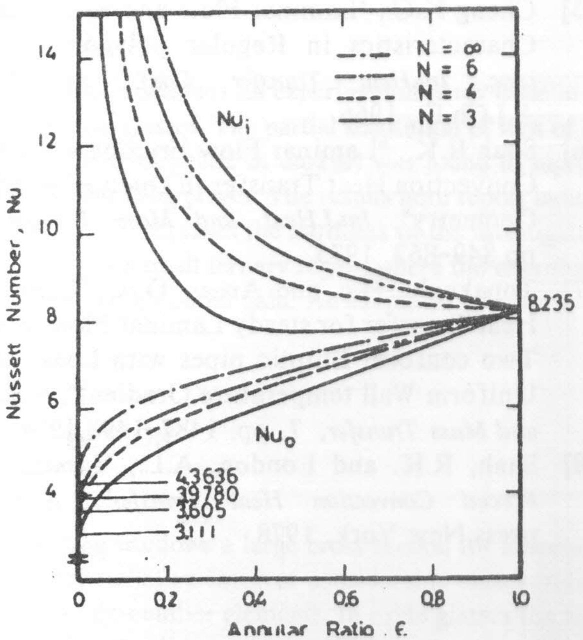

Figure 5. The Nusselt number at the inner and outer surfaces versus the duct annular ratio with equal temperatures of walls.

Figure (7) represents the Nusselt numbers of the inner surface when the outer surface is perfectly insulated. The values decrease from ∞ at ϵ = 0 to the limiting value at ϵ = 1 which is also 5.385. The annular square has a minimum value of 5.074 at ϵ = 0.06 while for the annular triangle, a minimum value of 3.133 occures at ϵ = 0.5.

Inspection of figure (5), (6), and (7), indicates that the Nusselt number values of the annualr circular duct for all annular ratios and all thermal cases are higher than those of the other polygons and the value decreases as the number of the polygon sides decreases.

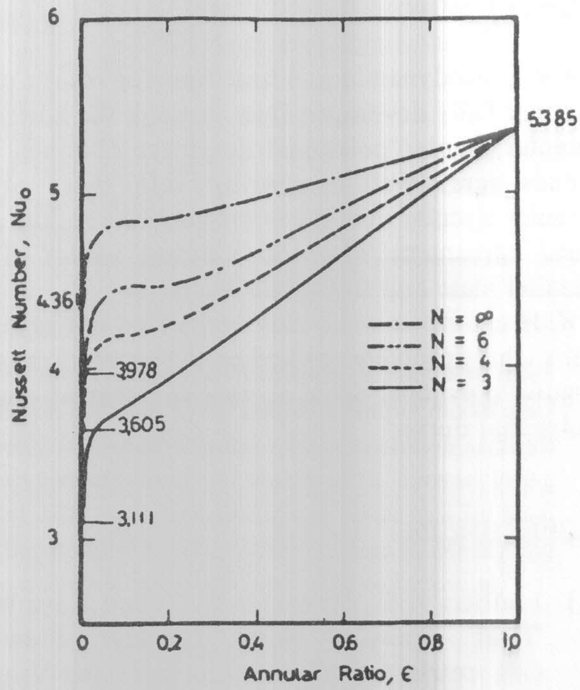

Figure 6. The Nusselt number at the outer wall versus the duct annular ratio with insulated inner wall.

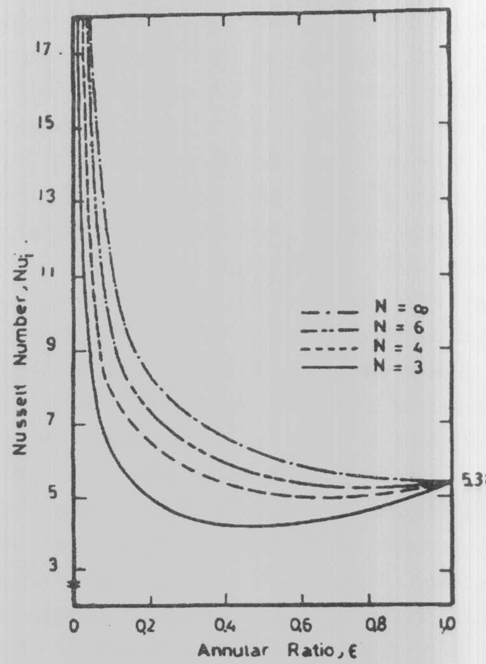

Figure 7. The Nusselt number at the inner wall versus the duct annular ratio with insulated ouuter wall.

CONCLUSION

The hydrodynamic and heat transfer results of the laminar fully developed flow through the horizontal annular regular polygonal ducts are obtained with trends agree well comparing with those of the annular circular duct ($N \rightarrow \infty$). Also the results show good agreement with the limiting values of the parallel plate and the single ducts.

Wide extensive work may be carried out in future for such a pproblem applied on the eccentric annular regular polygonal ducts as well as irregular annular polygonal ducts.

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