# A PHYSICAL ANALYSIS OF THE COUPLED AEROSOL-LIGHT BEAM SYSTEM

## Kamal M. Aly

Engineering Mathematics and Physics Department, Faculty
of Engineering, Alexandria Unversity, Egypt.

## ABSTRACT

The coupled aerosel-light beam system is studied. For water droplets the energy balance equation is solved by numerical integrations. A comparison between the present results and previous results is offered for the maximum temperature rise of the water droplets. The effect of the beam intensity on the droplet heating is studied. The mass flux and the vaporization rate are investingated in both the low and high energy case. A new parameter describing the droplet tempeature decay is defined.

## INTRODUCTION

The problem of the heating and vaporization of absorbing droplets in intense light beams has been studied by a number of investigators [1-3]. In a previous investigation [1], an approximate solution was obtained for the energy balance in a spherical fluid droplet irradiated by a pulsed light beam. The vapor kinetic energy term has been omitted from the energy conservation equation. Furthermore, the mass flux has been expanded in a power series up to the seconed order. The results for this second-order appoximation is compared with the corresponding present resuts obtained in our paper using numercial integration solution.

In many applications of practical importance, and at high irradiance, the aerosol heating, vaporization, and conductive heat losses are the dominant energy dissipation mechanisms for the propagating beam. As the pulse energy increases, the vaporization is the dominant energy draining mechanism. When the droplet radius dosen't exceed 10 $\mu$m and the incident fulx level is in the range of $10^3$ -$10^5$ W/cm$^2$, diffusive evaportion is the dominant process in the high-energy laser interaction with water aerosols. By way of contrast, for fluxes reaching $10^7$ W/cm$^2$, explosive vaporization and shock formation become important. The effects of nonuniform droplet heating and the localization of the beam at the droplet walls are significant at short times and for larger droplets whose radius exceed 10 $\mu$m.

In the present work, we consider the heating and varopization of absorbing water droplets in an intense pulsed light. The droplets absorb energy from the propagated light so that their temperature rises as a function of the droplet size beam intensity and the pulse length. Due to the rise of the droplets temperture, they vaporize and heat the ambient medium conductively. In otherwords, the amount of the energy absorbed by the fdropet, during passage of the pulse, is deposited into hheating the droplet, vaporizing the droplet, and heating the surrounding medium.

In this paper, we study the effects of the beam parmeters, as well as the droplet size, on the temperature rise of the water droplets. The energy balance equation, using the exact expression of the mass flux, is solved using the numerical integrations. The results of the present numerical solution show that the previous second-order approximation breaks down except for the cases disussed later.

For a more rigourous solution of the energy conservation equation, the vapor kinetic energy term and the droplet shrinkage term are included. The former term has an appropriate contribution to the droplet temperature in the high energy case. For the sake of completeness, we include the second discussed previously by Davies and Brock [4]. This energy term is about 30% of the droplet heating energy in the low energy case[5]; it becomes comparable with the heating energy case, but then the energies of shrinkage, heating , and conduction are small compared with the vaporization energy.

In this paper, we study the effect of the beam intensity on the droplet heating through a parameter defined as the critical beam intensity which is required to heat the water droplet up to its boiling temperature. The mass flux and the vaporization rate are studied carefully through the effects of the beam parameters and droplet size. The

temperature decay time, as a new parameter, is defined in this paper. It is investigated accurately as a function of the beam paramters and droplet radius.

*Droplet temperature rise*

The time-dependent droplet temperature T, may be obtained from the solution to an energy-conservation equation that describes the balance between absorption and dissipative processes within a droplet of radius $\gamma$. The energy-balance is [1]

$$\frac{4}{3}\pi\sigma^3\rho C\frac{\partial T}{\partial t} + 4\pi\sigma^2\ [mL+mC(T-T_o)$$

$$-K(\frac{\partial T}{\partial r})_{r=\sigma}+\frac{m^3}{2\rho'^2}\ ]= \frac{4}{3}\pi\sigma^3\alpha I, \tag{1}$$

where $\rho$, C, L, and $\alpha$ are, respectively, the droplet density specific heat, heat of evaporization, and the frequency-dependent bulk absorption cofficient; $\rho'$, $T_o$, and K are, resultively, the density, temperature, and the thermal conductivity of the surrounding medium. I is the intensity of the beam and m is the mass flux.

The term $4\pi\sigma^2\ mC(T-T_o)$ is the droplet shrinkage energy term which is included in this paper for the sake of completeness. The term $4\pi\sigma^2[mL+mC(T-To)]$ represents an effective heat of evaporization.

In the quasi-stedy approximation, we use[1]

$$m = \frac{D}{\sigma}\left[\frac{1-Y_o}{1-Y_o\exp\ (\ \frac{LM}{RT_o}-\frac{LM}{RT})}\right] \tag{2}$$

$$K(\frac{\partial T}{\partial r})_{r=\sigma} = -\frac{mC_p(T-T_o)}{\exp(mC_p\sigma/K)-1} \tag{3}$$

where D, M, and $C_p$ are the vapor difusion coefficient, molecular weight, and speciffic heat, respectively. R is ideal gas constant, and $Y_o$ is the ambient vapor mass fraction.

Inserting Eq.(3) into Eq.(1), and using the dimensionless variable $X\ (t)=[T(t)-T_o]/T_o$, we obtained the following energy balance equation for water droplets

$$\frac{dX(t)}{dt} = 8\times10^{-10}\alpha I-1.2\times10^{-9}\ \frac{m^3(t)}{\sigma\rho'^2}$$

$$5.5\times10^{-3}\ \frac{m(t)}{\sigma} - 3x10^{-3}\ \frac{m(t)X(t)}{\sigma} -$$

$$7.2\times10^{-7}\ \frac{m(t)C_pX(t)}{\sigma[\exp\{m(t)C_p//K\}-1]} \tag{4}$$

where $T_o=300\ °K$, m (t) is computed for water droplet from Eq.(2) as

$$m(t) = \frac{3x10^{-5}}{\sigma}\ \ln\ [\ \frac{0.9862}{1-0.0138\ \exp(B)}\ ], \tag{5-a}$$

and

$$B = 16.32\ \frac{X(t)}{1+X(t)} \tag{5-b}$$

Equation (4) is solved numerically using an intensity profile I (t) of thye form

$$I\ (t)\quad = I_o,\qquad 0\le\ t\le t_p, \tag{6}$$
$$=\ 0,\qquad\text{elsewhere}$$

The time step $\Delta t$ is selected not more than $10^{-7}$ sec, for pulse lengths not less than 5 $\mu$sec, to give a high accuracy for the time-dependent droplet temperature. This achieves stability in the numerical integration scheme. The pulse length $t_p$ is chosen so that $t_p \gg t_b$, where $t_b$ is the droplet heationg time: the time scale for the droplet to approach a steady state temperaturer [1].

For water droplet we estimted, at $t_o=300$ oK, the droplet heating time which is obtained as a function of dropet radius in following formula

$$\tau_b(\sigma,I) = 3.4\times10^7\sigma^2\ [3.2\ +\ \sigma^2\ \alpha I]^{-1/2},\ \text{sec} \tag{7}$$

where $\sigma^2$ is in $m^2$, and $(\sigma^2\ \alpha I)$ is in W/m.

Figure (1) illustrates the droplet temperature rise $\Delta T$ (t) versus normalized time $t/t_p$ for the selected value of droplet radius and beam intensity $\alpha I$ for a 5 $\mu$sec pulses. a frist feature of interest is that, the temperature of the larger droplet radii increases more faster with time than the temperature of the smallest ones. This occurs during the passage of the pulse up to a maximum temperature rise $\Delta T_m$ at the pulse end $(t=t_p)$.. It is clear that the droplets of samall sizes reach a steady state temperature during the heating phase. This is because, according to Eq. (7), such droplets have heating time $t_b \ll t_p$.

Table 1. shows a comparison between our present results, obtained through numerical-integration solution, and the results of Ref. [1] obtained by the second-order solution. We can find that the previous results are in good agreement with the present results for $16>\sigma^2\alpha I$ (W/m)$\ge1$, It is clear that the second-order approximation used in Ref.[1] is excellent approximation for$\sigma^2\alpha I = W/m$.
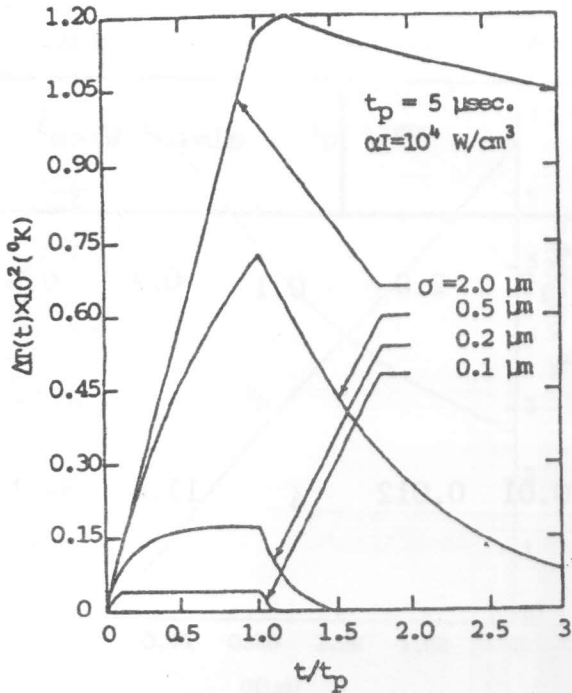
**Figure 1.** Variations of the droplet temperature rise with normalized time.

Otherwise, the numerical solution numerical values for $\Delta T_m$ less than those of Ref.[1] which obtained by the secound-order solution.

*Critical beam intensity*

Figure (2) gives the intensity dependene of the maximum temperature rise $\Delta T^m$, for selected values of the droplet radii. It is evident that for a given droplet size there is a critical beam intensity $\alpha I_c(\sigma)$ required to heat droplet up to bolong temperature $T_B$. The correlation between $\alpha I_c$ ($\sigma$) and $\sigma$ is given in Figure (3). The feature of interest is that all droplets having radii more than 2 $\mu$m need approximately the same critical beam intensity to heat up to $T_B$. For dropletswith $0.3 \leq \sigma(\mu m) \leq 1$, the critical beam intensity is inversely proportional to $\sigma$with a neagative rate $(\partial^2 \alpha I_c)/\partial\sigma^2$. All droplet less than 0.3 $\mu$m have a linear proportionality with the critical intensity.

*Steady-state vaporization rate*

For water droplets, we estimate the droplet decay time $\tau_d$ [1], as a function of the intensity, $\alpha I$ under the final following from
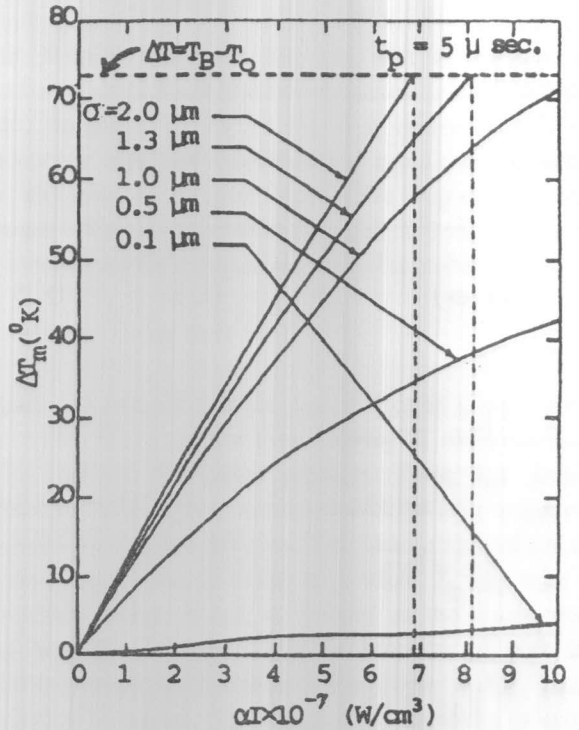


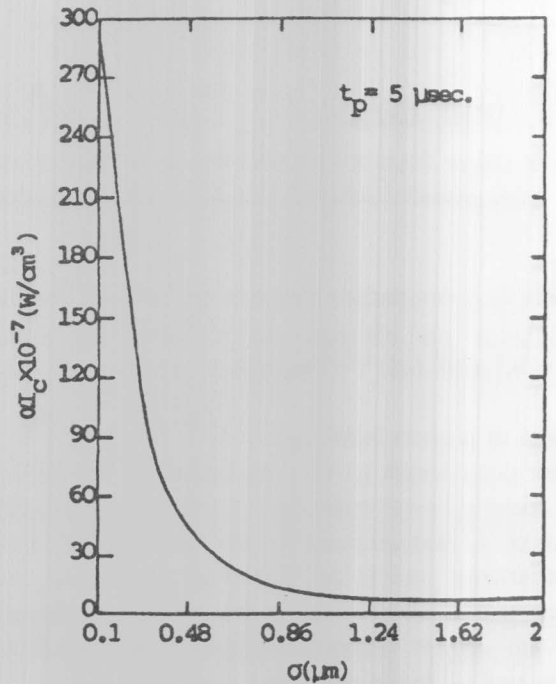**Figure 2.** Variations of the maximum droplet temperature rixe with beam intensity.



**Figure 3.** Variations of the critical beam.

| | $\alpha I = 10^4$ W/cm$^3$ | | | | $\alpha I = 10^8$ W/cm$^3$ | | |
|---|---|---|---|---|---|---|---|
| $\sigma(\mu m)$ | 0.1 | 0.5 | 1.0 | 2.0 | 0.1 | 0.2 | 0.4 |
| The present results $\Delta T_m(^\circ K)$ | $4 \times 10^{-4}$ | $7.3 \times 10^{-3}$ | 0.01 | 0.012 | 4 | 13.4 | 34.1 |
| Ref.[1] results $\Delta T_m(^\circ K)$ | $3 \times 10^{-3}$ | 0.07 | 0.10 | 0.118 | 4 | 14 | 41 |
| $\sigma^2 \alpha I$ (W/m) | $10^{-4}$ | $2.5 \times 10^{-3}$ | 0.01 | 0.040 | 1 | 4 | 16 |

Table 1. A comparison between our present results and the results of Ref. [1].

$$\tau_d = 9.5 \times 10^3 \, (\alpha I)^{-1}, \text{ sec} \qquad (8)$$

$$V_R(\sigma,I) = 4\pi\sigma^2 m(\sigma,I). \qquad (15)$$

where $\alpha I$ is given in W/cm$^3$.

For pulse length $t_p \ll \tau_d$, the radius of the froplet may be taken to be approximately constant, i.e. $\sigma(t) = \sigma(0)$. The droplet is characterized by the ambient pressure and temperature prior to the arrival of the pulse at $t=0$. For times $t>0$ the droplet energy from the beam. hence, both of the droplet temperature and vaporization rate is increased up to the pulse end where $t=t_p$.

The vaporization rate $V_R$, as a function of droplet radius $\sigma$, is given by

When the droplet radius and the beam intensity are selected so that, according to Eq.(9), $\tau_h \ll t_p$, the droplet temperature reaches to its steady state. This case leads to steady state values for both $m(\sigma,I)$ and $V_R(\sigma,I)$. Figures 4 and 5 show the variations of both the steady state vaporization rate and mass flux with the droplet radius for the low and high energy cases.

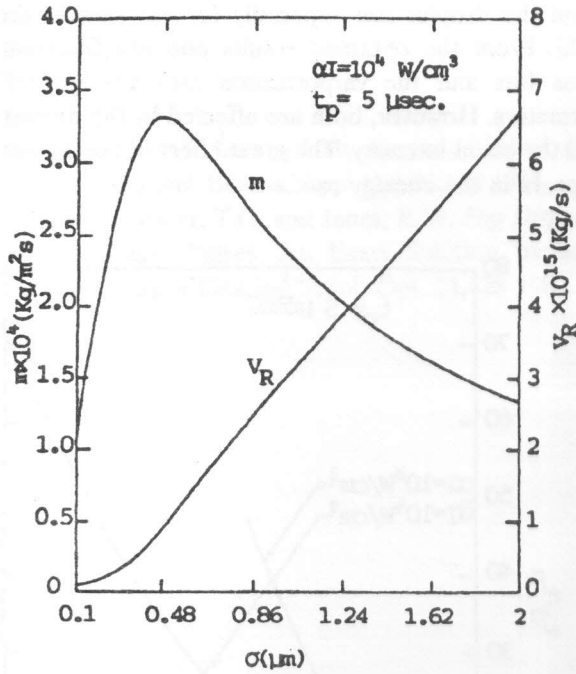In Figure (4) it is clear that, in the low energy case, the

**Figure 4.** Variations of the mass flux m and vaporization rate $V_R$ with droplet radius, $\alpha I = 10^4$ W/cm$^3$.
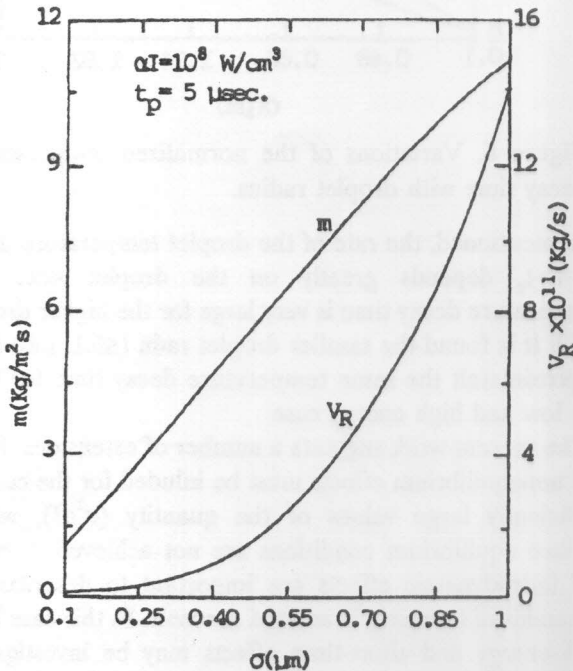


**Figure 5.** Same as in figure 4, but with $\alpha I = 10^8$ W/cm$^3$.

mass flux has a positive rapid correlation with the smallest droplet radii, and a negative slow correlation with the larger droplet radii. for droplets with $\sigma > 0.3$ $\mu$m, the vaporization rate has a linear dependence with the droplet raduis. As shown in Figure (5), it is evident that, in the high energy case, the mass flux has a positive correlation with all selected droplet sizes. The vaporization rate increases with a positive rate $(\partial^2 V_R)/(\partial \sigma^2)$ while the mass flux increases with a negative rate $(\partial^2 m)/(\partial \sigma^2)$.

*Temperature decay time*

Figure (1) reveals that the water droplet temperture is incresed for times greater then zero up to a maximu value at the end of the pulse passage. Then, the droplet temperture begins to decrease with a rate negatively correlated to the droplet size. The time, required for the droplet temperature to decay to a value $T_f$ very colse to the ambient temperature, is defined as the temperature decay time $t_d$. This paremter is function of both the maximum temperature rise and the droplet raduis. $t_f$ amy be selected to equal $AT_o$, where A is very close to unity.

Now, for small droplet sizes, $\sigma \leq 0.1$ $\mu$m, one can formulate an expression for the prameter $t_d$ . When $\sigma$ is so small that one can neglect the conducting, kinetic, and shrinkage energy terms, the energy balance equation, for $t > t_p$, can be written as

$$\frac{4}{3} \pi \sigma^3 \rho C \frac{dT}{dt} + 4\pi \sigma^2 \, mL = 0, \qquad (16)$$

where m may be approximated, for small values of the dimensionless variable X, to be in the following from

$$m = \frac{DLMY_oX}{\sigma RT_o(1-Y_o)} \qquad (17)$$

Substiute m from Eq.(17) into Eq.(16), using the dimensionless variable X instead of T, we obtain

$$\rho CT_o \frac{dX}{dt} + \frac{3DL^2MY_o}{\sigma^2 RT_o(1-Y_o)} \quad X = 0. \qquad (18)$$

For water droplet Eq.(18) can be integrated and rewritten as

$$\int_{X_m}^{X_f} \frac{dX}{X} = - \frac{4x10^{-8}}{\sigma^2} \int_{t_p}^{t_d} dt,$$

$$t_d = 2.5x10^7 \, \sigma^2 \ln \frac{X_m}{X_f} + t_p , \qquad (19)$$

Where $X_f$ is the dimensionless variable, and the maximum

dimensionless variable ( $X_m = \Delta T_m/T_o$) can be obtained by rewiting Eq.(18) for the time range $0 \le t \le t_p$. Hence, we have

$$\rho CT_o \frac{dX}{dt} + \frac{3D^2 LMY_o X}{\sigma^2 RT_o(1-Y_o)} = \alpha I. \qquad (20)$$

For water droplets, integration of Eq.(20) yields the following formula

$$\int_0^{X_m} \frac{dX}{8 \times 10^{-10} \alpha I - \frac{3 \times 10^{-7}}{\sigma^2} X )} = \int_0^{t_p} dt,$$

which gives the maximum dimensionless variable as

$$x_m = 2.7 \times 10^{-3} \sigma^2 \alpha I \, [1 - \exp(-3 \times 10^{-7} t_p/\sigma^2)]. \qquad (21)$$

The final dimensionless variable $X_f$ can be easily obtained by using $T_f = AT_o$. This leads to

$$X_f = \frac{T_f - T_o}{T_o} = A - 1 \qquad (22)$$

Equations (22),(21), and (19) provide three equations which determine the  temperature decay time $t_d$  in terms of the plotted radius, pulse length, and the beam intensity.

In Figure (6) we polled the variation of the normalized temperature decay time $(t_d/t_p)$ with the droplet raduis for the selected values of $\alpha I$. The plotted results are obtained using the numerical integration of the energy blance equation, Eq.(4), under the condition of $x_f = A-1 = 10^{-8}$. It is clear from Figure (6). that the parameter $t_d$ increases for the larger droplet sizes. The rate $(\partial t_d/\partial \sigma)$ is positively correlated with the beam intensity.

## CONCLUSION

In this paper, we have studied carefully the interaction between the water droplets and a monochromatic pulsed beam of electromagetic radiation. The couples nonlinear differential equations describing the energy and mass conservation are numerically solved.

The results of the present work reveal that the second-order solutions obtained in Ref.[1] are in agreement with the present numerical solution for the condition $16 > \sigma^2 \alpha$ $I(W/m) \le 1$. The excellent agreement occured for the condition $\sigma^2 \alpha I = 1$ W/m.

The critical beam intensity, required to heat the droplet

up to the boiling termperature, has a great dependence upon the droplet size especially for the smaller droplet radii. From the obtained results one can find that the mass flux and the varporization rate are independent parmeters. However, both are affected by the droplet size and the beam intensity. The great effect of the droplet size appears in the enerfgy case.
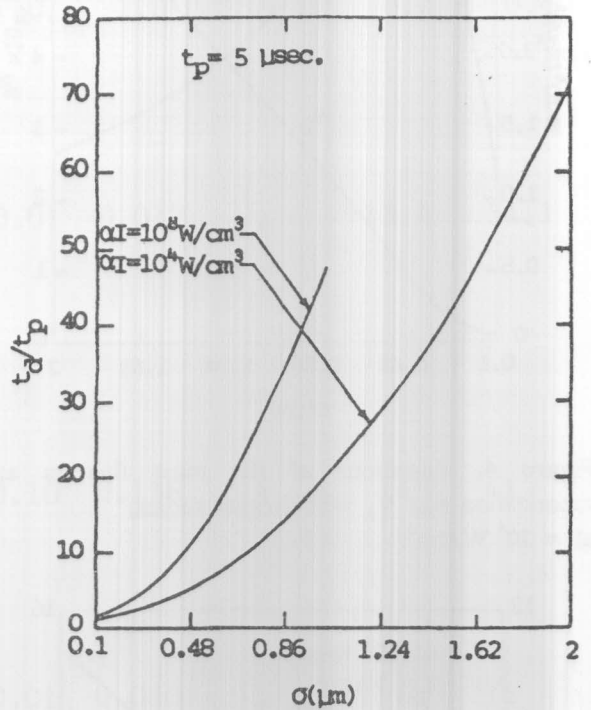


Figure 6. Variations of the normalized temperature decay time with droplet radius.

as mentioned, the rate of the droplet temperature decay at $t > t_p$ depends greatly on the droplet size. The temperature decay time is very large for the higher droplet radii. It is found the samiler droplet radii ($\le 0.1$ $\mu m$) have approximatelt the same temperature decay time for both the low and high energy case.

The present work suggests a number of extensions. First, the nonequilibrium effects must be inluded for the case of sufficiently large values of the quantity $(\sigma^2 \alpha I)$, where surface equilibrium conditions are not achieved. Second, the thdrodynamic effects are important to describe the dynamics of intensely irradiated aerosols. In this case both high-energy and short-time effects may be investigated. Finally, one can consider the more general problem of beam propagation in a poly-disperse size distribution which require knowlege of the particle concentration per unit volume, the particle-size distrbution function, and the

single-particle extinction coefficient.

## REFERENCE

[1] Armstrong, R.L. *"Aerosol* Heating and Vaporization by Pulsed Light Beams,"Appl.Opt.23,148 1984.

[2] Lee, C.T. Miller, T.G. and Janes, R.W. *Fog Dispersal by $co_2$ Laser Pulses: An Exact Solution Including Prevaporizaion Heating,"*Appl. Opt. 21,428 1982.

[3] Sutton, G.W. *"Fog Hole Boring with Pulsed High Energy Lasers: An Exact Solution Including Scattering and Absorption,"* Appl. Opt. 17., 3424 1987.

[4] Davies S.C. Brock, J.R. *"laser Evaporation of Droplets,"* Appl. Opt. 26, 786 1987.

[5] Zardecki A. and Armstrong R.L. *"Energy balance in Laser-irradiated vaporizing droplets,"* Appl.Opt.27, 3690 1988.