STIFFNESS CONSTANTS AND INTERACTION FACTORS FOR VERTICAL RESPONSE OF PILE GROUPS

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ABSTRACT

Stiffness constants and flexibility coefficients of single piles and interaction factors established for group of piles are presented to facilitate the analysis of pile groups exposed to static vertical loads. A continuous transition from friction to end-bearing piles is accounted for. A new concept of interaction factors is introduced for end bearing piles based on interaction of a groups of 5 end-baring piles. Numerical solutions for axial statically loaded single piles and pile groups are presented as a set of non-dimensional curves over a wide range of governing parameters of piles and soil. The study allows for either rigid or arbitrary flexible caps.

INTRODUCTION

Over the past many years, static response of pile foundations has been investigated using a variety of empirical, analytical and numerical techniques. Static and dynamic analyses of piles were adopted and made readily applicable by Poulos [19-21], Banerjee and Butterfield [1, 2, 4-6], Kuhlemeyer [10], Novak [12-18], Wolf [26], Nogami [11], Waas [25], Kausel [3], Radolph [22], El-Sharnouby [7-8], Blaney [3], and others. While the mentioned studies brought out considerable progress in analytical capability and understanding of single piles and pile groups behavior, most of them rely on the use of quite large and often proprietary computer programs.

The stiffness constants and flexibility coefficients of single piles are basic item of information. While the amount of data readily available is increasing much of it is limited to homogeneous soil, and completely frictional or infinitely end-bearing tip conditions. Moreover, elasticity based data are usually obtained from mathematical model of piles divided into 10 elements. The latter may yield a considerable error particular for long flexible piles. Also, most of the data are complicated with a variety of curves and charts to interpolate from, particularly, for piles resting on stiffer stratum.

In groups of closely spaced piles, the character of groups stiffness is complicated by interaction between individual piles known as pile-soil-pile interaction or groups effect. The groups effect increases group settlement, redistributes the loads on individual piles and modifies groups flexibility and thus its stiffens. Analysis of pile groups can be conducted in two ways :(a) using a computer-direct analysis of the whole group [4, 8, 11, 20, 23, 24, 25, 26] and (b) approximately using superposition of interaction factors [9, 21, 22].

The direct analysis is based on large computer programs and preferable because it is accurate within the validity of the assumptions made and provides more information. The advantages of the interaction factor approach are that it is simple and the analysis can be conducted by means of long hand calculation or very small computer program; the only conditions that a complete set of sufficiently accurate interaction factors be available and that the conditions of current case do not preclude their use.

Simple static analysis was made readily applicable by Poulos [19, 20, 21] through the concept of interaction factors. The concept has been quite useful and popular particularly for small groups but its applicability may suffer from few drawbacks [8]: (1) The evaluation of large pile group stiffness becomes tedious and additional lonmxhand calculations may be needed to determine the loading of individual piles, (2) Data published was obtained by dividing the pile into 10 elements and it was found that for long pile more elements (up to 50) are needed to produce better results and error induced due to using small number of elements is up to 30%, (3) The pile interaction effects may be over estimated, particularly for end-bearing piles under vertical loads. The latter may occur because the interaction factors to be superimposed are calculated for any two piles in the groups ignoring the stiffening effect of the other piles.

It is the purpose of this research to eliminate most of limitations and drawbacks of the traditional interaction approach and present new flexibility coefficients and new formulated interaction factors that would facilitate the analysis of vertical loaded pile groups, allowing for either rigid or arbitrary flexible caps, a broad range of pile-soil parameters and pile tip conditions. The study is limited to linear, elastic behavior of pile-soil-pile system and to vertical response of single piles and pile groups. Horizontal flexibility coefficients and interaction factors are given elsewhere [9].

METHOD OF ANALYSIS

The basic idea of the approach is to view the whole pile or group with the soil as one composite continuum whose conditions of equilibrium are specified for a number of discrete points (nodes). The conditions of equilibrium are expressed in terms of the stiffness method in which the structure stiffness of the pile is combined with the stiffness of the soil medium. The nodal soil flexibility coefficients are basically generated by the application of uniform vertical stresses upon each pile element. In actual calculations. the shear stresses are replaced by a carefully chosen equivalent discrete point loads system which yield displacements at reference point almost equal to those produced by the vertical stresses.

The piles are assumed to be vertical and of constant cross section. Each pile is divided into an equal number of elements. A total of 30 elements was generally found to yield sufficient accuracy. For long piles (L/d> 50), 50 elements were needed where L = pile length and d is the pile diameter.

CONTINUUM MODEL FOR PILE-SOIL PILE SYSTEM

The pile-soil-pile system considered in this study is shown in Figure (1):

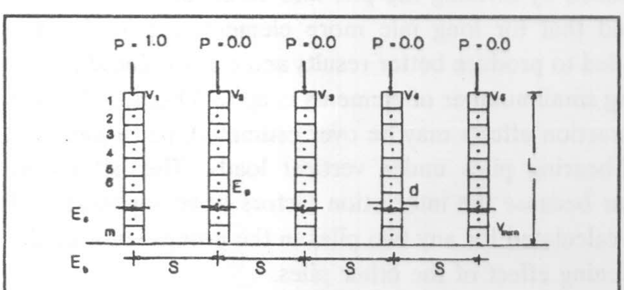

Figure 1. The continuum model for pile-soil-pile systems.

Referring to Figure (1), Young's modulus of pile surrounding soil, and bearing stratum are denoted by E_p, E_m and E_b respectively. For the analysis, each pile is divided into a number, n, of elements. Nodal points, for which pile displacements are to be specified, are located on the axis of the piles. For each pile, the first node is placed at top of the pile and the last node is located at the bottom of the lowest element to accommodate the best reaction.

the structural stiffness matrix of the i^{th} element has the standard form, e.g., for the vertical reaction

$$[K_p]_i = (E_p A_i/l_i) \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \quad (1)$$

where A_i and l_i are pile element's cross-sectional area length. The overall (global) structural stiffness matrix of the whole pile group $[K_p]$ is obtained by superimposing the individual element stiffness.

Stiffness of soil is generated by applying shear stresses along and around the pile and calculating associated displacements by means of Mindlin's solution. In the actual calculations, the continuously distributed shears are replaced by discrete point loads applied and located such that the resultant flexibility coefficients are almost the same as those obtained from continuously distributed shears [8].

Soil nonhomogenity is approximately accounted for calculating the flexibility coefficients f_{ij} with a Young's modulus equal to the average of the moduli pertinent to the stations i and j. The stiffness of the soil-pile system, [K], is obtained by the superposition of the soil and pile stiffness matrices, i.e.,

$$[K] = [K_m]+[K_p] \quad (2)$$

For the group illustrated in Figure (1), the equilibrium condition can be expressed for all pile nodes as

$$\{P\} = [K]\{v\} \quad (3)$$

in which {P} is the vector of vertical forces acting on the nodes and {v} is the vector nodal displacements.

$$\{P\} = \{1\,0\,0\,0\,0\,0\,0 \ldots\ldots 0\,0\}^T \quad (4)$$

$$\{v\} = \{v_1\; v_2\; v_3\; v_4\; v_5\; v_6 \;.\;.\;.\;.\;.\; v_{nxm}\}^T \quad (5)$$

where v_1 is the displacement of the top node of the reference loaded pile and v_2, v_3, v_4 and v_5 are the displacements of the top nodes of the rest of the unloaded piles in the model group. n and m are the number of piles and number of elements per pile respectively.

Substituting equations (4) and (5) into equation (3), the unknown displacements are obtained.

For floating single pile, the first pile only is considered in the analysis and the load vector becomes:

$$\{v\} = \{v_1 \cdot\cdot\cdot\cdot\cdot\cdot\cdot v_m\}^T \qquad (6)$$

For group of two frictional piles, the first and the second piles only are considered in the analysis and the load vector becomes:

$$\{v\} = \{v_i\ v_2 \cdot\cdot\cdot\cdot\cdot v_{2\times m}\}^T \qquad (7)$$

For piles resting on stiffer stratum, the whole group of 5 piles are considered and the dimension of the stiffness matrix is (5×m) by (5×m) while the number of elements in both load vector and displacements vector is (5×m). The number of elements per pile considered in this analysis is 50 for single pile and group of two piles and 30 for group of 5 piles.

VERTICAL STIFFNESS AND FLEXIBILITY COEFFICIENTS OF SINGLE PILES

The analysis was efficiently computerized and vertical flexibility coefficients and constants of vertical stiffness were calculated and presented in the from of charts and design curves.

The following conditions were considered: soil is homogeneous or its shear modulus diminishes upward according to a quadratic parabola or linear distribution; the piles are floating or a continuous transition from friction to end-bearing piles is allowed; slenderness ratio (L/d) varies from 25 to 100 and pile stiffness ratio, E_p/E_m, varies from 100 to 10000 where E_p and E_m are Young's modulus for pile and soil materials respectively. Piles are with cross sectional properties uniform with depth and divided in the analysis into 50 elements for maximum accuracy. Stiffness is generated by calculating the force needed to produce unit displacement at the pile head and relates the applied force, P, and the displacement, v, as

$$P = Kv \qquad (8)$$

Theoretical studies have shown that the stiffness constant, K, of single pile can be described as follows:

$$K = (E_m A_p/r)\ K' \qquad (9)$$

where K is the true vertical pile stiffness, E_m is Young's modulus of soil, r is the pile radius, A_p is the pile cross-sectional area and K' is the dimensionless stiffness constant.

Conversely, the true pile flexibility coefficient, F, can be calculated using the dimensionless flexibility coefficient, F', as

$$F = \{r/(A_p E_m)\}\ F' \qquad (10)$$

Figure (2) shows the dimensionless stiffness of single floating piles evaluated for different slenderness ratio, L/d, by Poulos, Solinero, Rajapakse and the authors. From Figure (2), it can be seen that for flexible piles (E_p/E_m = 100), calculated theoretical vertical response is quite sensitive to the number of pile elements.

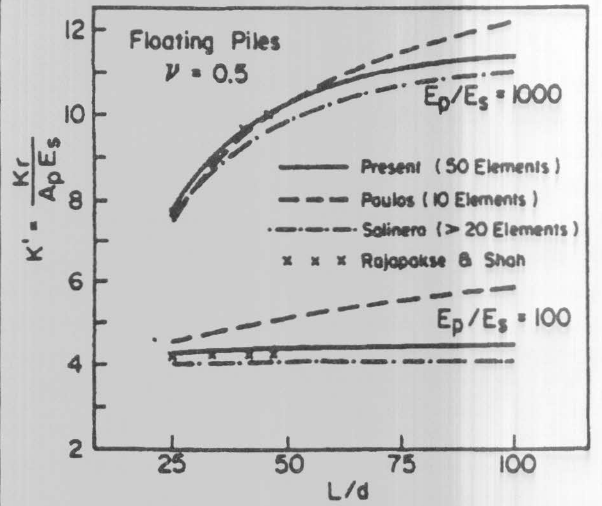

Figure 2. Dimensionless stiffness of single floating piles for homogeneous soil profile (E_p/E_m = 1000, 100).

Data based on a small number of elements (10 elements) may induce an error of an order up to 40%. For end-bearing piles, number of elements does not only affect the accuracy but also deviates the relation between the stiffness ratio, L/d. This can be seen from Figure (3-a), showing the dimensionless stiffness for end-bearing piles characterized by the ratio E_p/E_m = 100. Figures (3-

a) and (3-b) compare the results due to Poulos obtained using 10 elements per pile, Blaney using 20 elements, Solinero using more than 20 elements with the results of the present analysis obtained using 30 and 50 elements. As L/d increases approaches diverge, with Poulos approach giving higher stiffness constants. When the pile gets stiffer (E_p/E_m = 1000), the approaches converge with the present analysis giving intermediate results (Figure (3-b)).

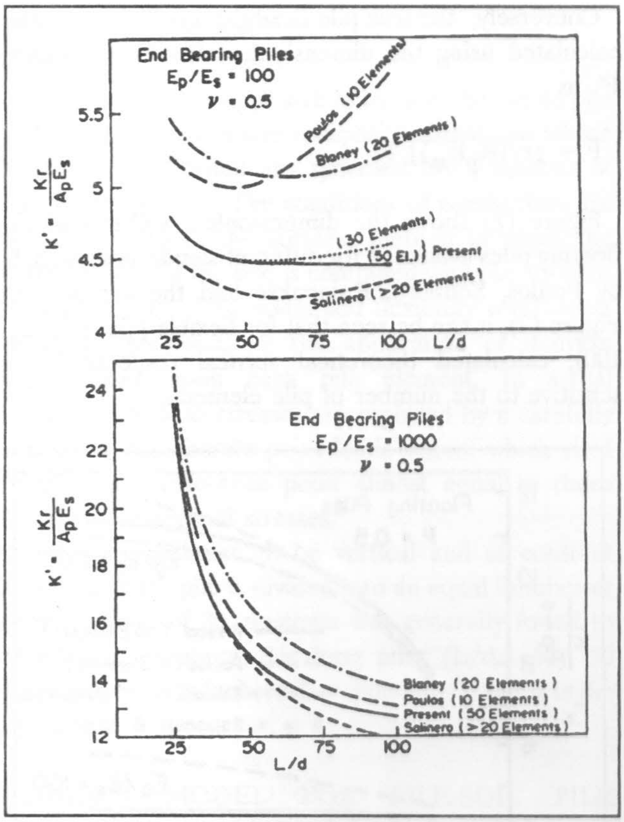

Figure 3. Dimensionless stiffness of single end-bearing piles for homogeneous soil profile (a-E_p/E_m = 100, b-E_p/E_m = 100).

The dimensionless stiffness, K′, calculated using the present approach, is plotted versus the pile stiffness-soil stiffness characteristic ratio, E_p/E_m, in Figures (4), (5), (6). The dimensionless stiffness constants are given for three soil profiles considered, i.e. homogeneous, parabolic, and linear (notice the different scale in Figures (4), (5) and (6)). The charts were calculated for fully embedded piles with L/d = 25, 50, 100, E_m = 10 MPa, variable E_p, and soil Poisson's ratio υ = 0.5, but they can be used for other pile-soil data as well since the stiffnesses are in the dimensions from. The poisson's ratio effect is not strong particularly when the data are presented in terms of the E_p/E_m ratio. For the parabolic and linear (Gibson) soil profiles, the nominal value of E_m has to be specified at a depth L where L is the pile length, i.e. 25, 50 or 100 d.

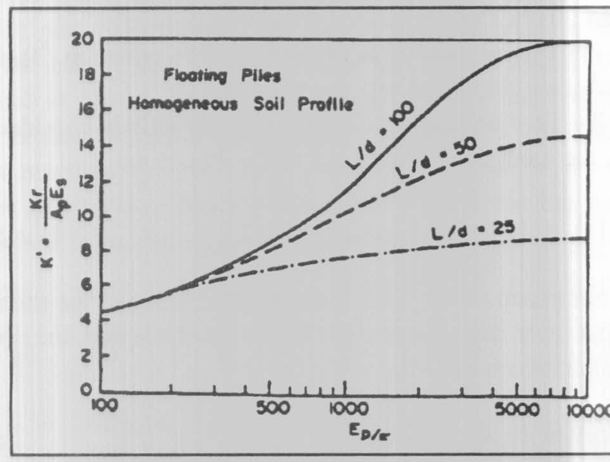

Figure 4. Dimensionless pile stiffness vs. pile-soil stiffness ratio for homogeneous soil profile.

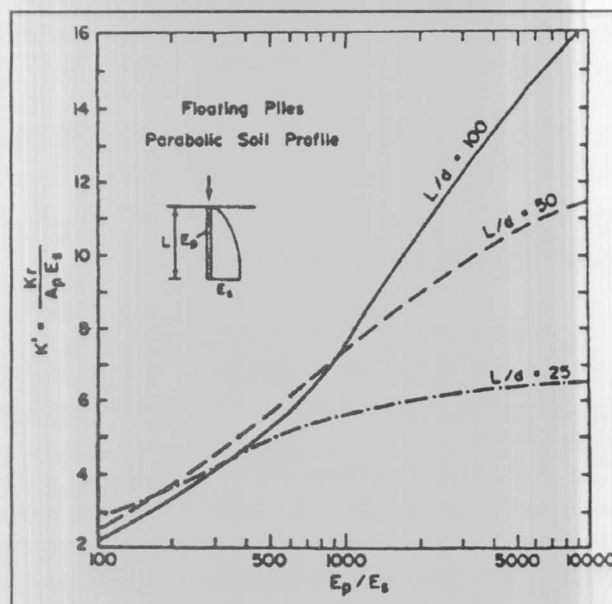

Figure 5. Dimensionless pile stiffness vs. pile-soil stiffness ratio for parabolic soil profile.

From Figure (4), it can be seen that stiffness of long pile changes more dramatically with E_p/E_m than dose stiffness of shorter piles. The figure also shows that vertical stiffness floating flexible piles are almost independent if its length (100< E_p/E_m < 500) with the change in stiffness with pile length is pronounced for stiffer piles.

Figure (5) and (6) show that for piles in parabolic or linear soil profiles the stiffness of short flexible piles is slightly greater than the stiffness of a longer one. This is because the Young's modulus, E_m, is specified at the pile tip. So, the most upper part of the long pile is adjacent to soil having very small Young's modulus, in addition to non-uniform stresses distribution along the flexible pile. The magnitude of pile stiffness decreases significantly with the changing of the soil profile from homogeneous to parabolic to linear (Figure (7)). This indicates the importance of determining the soil profile correctly and the need to account for the reduction of the soil modulus towards ground surface associated with the reduction of confining pressure.

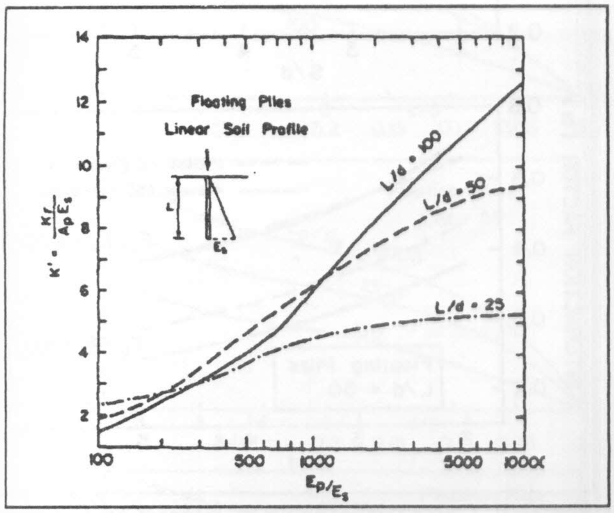

Figure 6. Dimensionless pile stiffness vs. pile-soil stiffness ratio for linear soil profile.

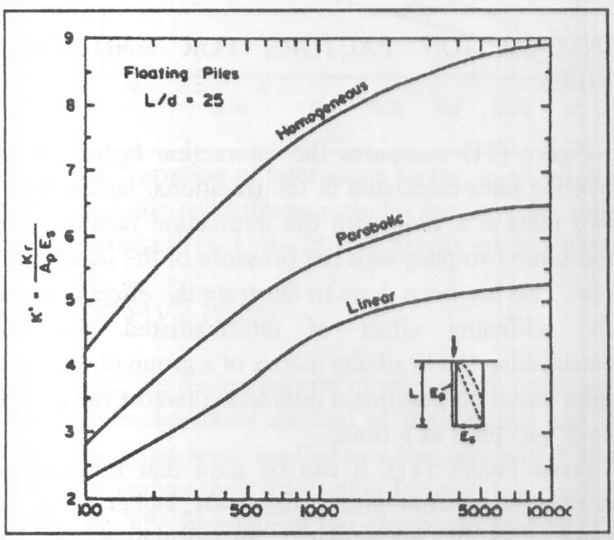

Figure 7. Dimensionless pile stiffness vs. pile-soil stiffness ratio for different soil profiles and slenderness ratio, L/d = 25.

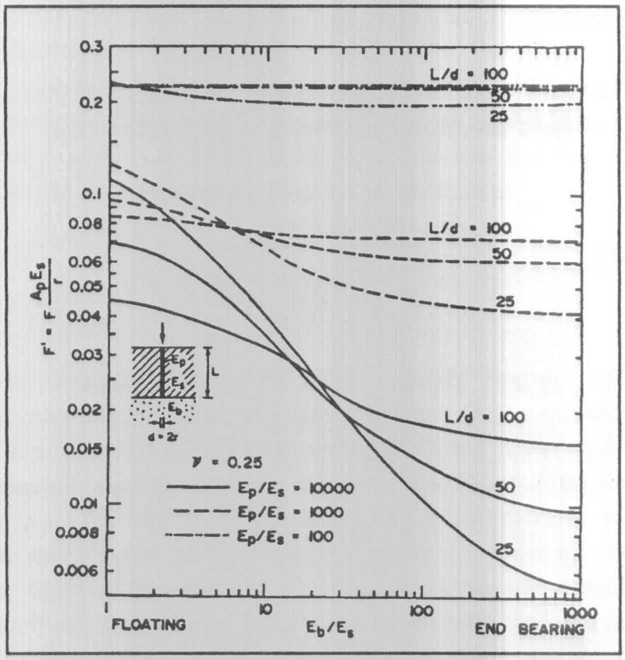

Figure 8. Dimensionless flexibility of single pile vs. stiffness of underlaying stratum for homogeneous soil profile.

Figure (8) shows the vertical dimensionless flexibility of single piles in homogeneous soil calculated for a wide range of pile-soil stiffness ratio, E_p/E_m, slenderness ratio L/d, and stiffness of underlaying stratum, E_p/E_m, where E_p is Young's modulus of the underlaying stratum. Figure (8) indicates a quite number of important features. pile-soil stiffness ratio, E_p/E_m, is a major factor affecting pile flexibility (notice the logarithmic scale for the vertical and horizontal axes). The flexibility dramatically increases with the decrease of E_p/E_m for all pile slenderness ratio, L/d, and over the whole range from floating to end-bearing piles. Flexibility of flexible piles are almost independent of tip condition particularly for long piles while the change of flexibility with tip condition is quite considerable for stiff piles.

Effect of pile length on flexibility is significantly affected by the tip condition. The longer pile poses less flexibility for relaxed tip condition and moderate stiffer underlayer stratum, but for a quite stiff base, shorter pile shows more stiff behavior and produces less flexibility than longer pile does. This latter observation is important in determining the proper pile length, diameter, and material for certain soil conditions.

INTERACTION FACTORS

The traditional interaction factors , introduced by Poulos, are defined for two equally loaded piles as the ratio

$$\alpha = \frac{\text{settlement of reference pile due to an adjacent pile load}}{\text{settlement of reference pile under its own load}} \qquad (11)$$

For larger group, these interaction factors are superimposed to yield the total settlements of the piles in group. These traditional interaction factors may be quite over estimated, particularly for end-bearing piles, because they are calculated for any two piles in the group at a time ignoring the stiffening effect of the other piles. In addition, the limited number of 10 elements per pile in that analysis may induce an error particularly for long flexible friction piles.

GROUP OF TWO PILES

The present method has been employed to verify the accuracy of the traditional interaction factors for floating piles and introduce a set of alternative factors. Figures (9-a) and (9-b) show the interaction factors plotted together with those produced by Poulos for two values of slenderness ratio, L/d, and three values of stiffness ratio, E_p/E_m where L is pile length, d is pile diameter, and E_p and E_m are Young's modulus of pile and soil respectively. The interaction factors obtained by Poulos are up to 25% greater than those from the present formulation for flexible piles, but the differences close up for stiffer piles. This may be due to the lack of accuracy of 10 elements based analysis. In the present analysis 50 elements per pile were used.

For the dimensionless ratio E_p/E_m and spacing ratio S/d, where S is pile spacing the interaction factor established are plotted in Figure (10). The charts are for L/d =25, 50 and 100, and homogeneous soil. This figure actually presents a complete set of flexibility coefficients normalized by deflections of the reference pile and pertinent to group of two piles. Using these interaction factors in addition to single pile flexibility coefficients, a complete flexibility matrix for a larger pile group can be established.

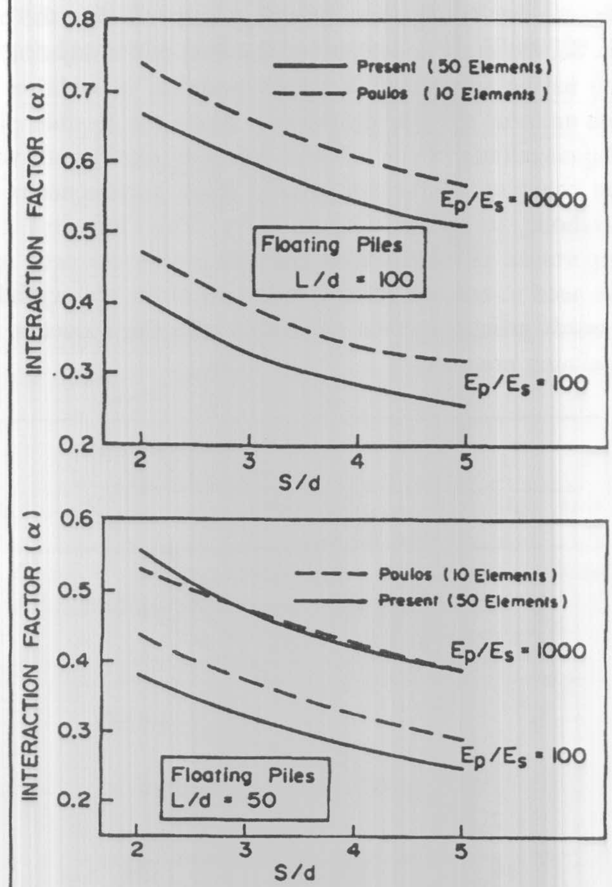

Figure 9. Comparison between interaction factors by Poulos and by the present formulation for floating piles for a- L/d = 100 and b- L/d = 50.

INTERACTION FACTORS FOR END-BEARING PILES

Figure (11) compares the interaction factors for end-bearing piles calculated in the traditional faction between two piles at a time with the interaction factors between the same two piles with the presence of the intermediated pile. This has been done to illustrate the effect of ignoring the stiffening effect of intermediated piles when establishing the flexibility matrix of a group of end bearing piles based on traditional interaction factors calculated for each two piles at a time.

From Figure (11), it can be seen that the traditional interaction factors give consistently higher value. This error may be accumulated in calculating the group settlement of end-bearing piles. This observation is valid not only for completely end-bearing piles, but also for piles resting on any stiffer stratum. The latter may be considered the dominant case in practice. Thus, there is a need to more reliable method of calculating group settlement. Although the computer programs based on direct methods of solving pile group are the best answer, they are usually large often proprietary and unavailable to all designers.

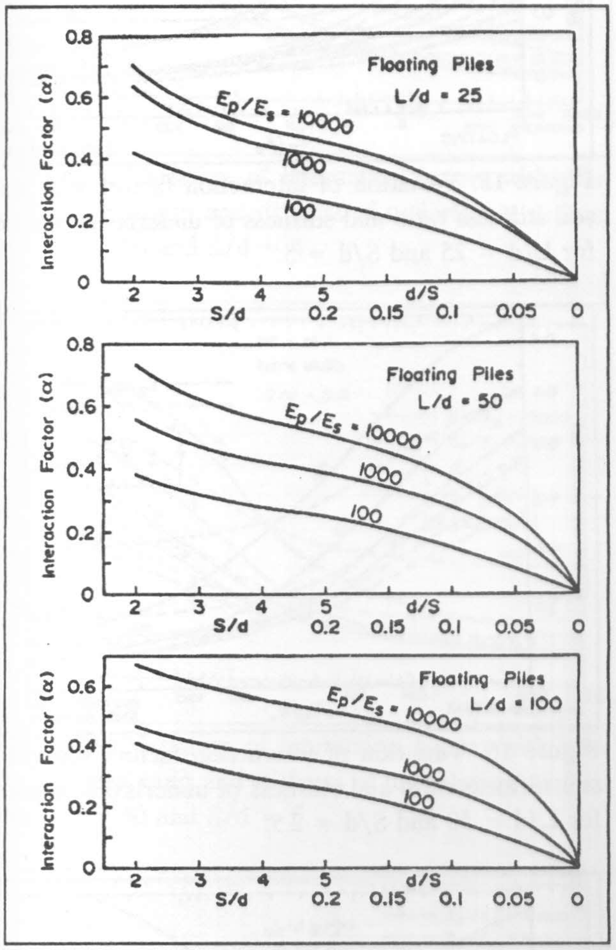

Figure 10. Variation of interaction factor, α, with pile spacing and pile-soil stiffness ratio for friction piles and homogeneous soil (a- L/d=25, b- L/d=50, c-L/d=100)

GROUP OF FIVE PILES

To facilitate the design criteria of pile groups resting on stiffer stratum the direct method of the analysis and its computer program were applied to a line group of 5 piles and used to analyze their interaction factors for a number of situations. The first pile of the group was loaded by a unit vertical load and the other piles remained unloaded. The group was solved and flexibilities of all piles were determined and the flexibilities of the unloaded piles were normalized by dividing them by the flexibility of the reference loaded pile. These normalized flexibilities present the interaction factors. This new model of a group of 5 piles is believed to be representative and may yield adequate accuracy for practical situations.

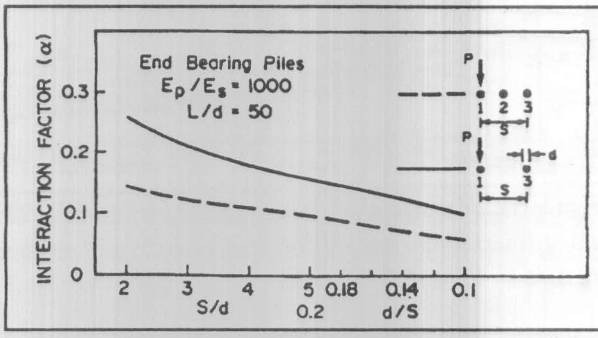

Figure 11. Comparison between the traditional interaction factors and interactions of a group of 3 piles for end-bearing piles.

The interaction factors calculated this way are defined as:

$$\alpha_{ij} = \frac{\text{Deflection of any unloaded pile due to the loaded reference pile}}{\text{Deflection of the loaded reference pile}} \qquad (12)$$

For large arbitrary or symmetric pile groups, these interaction factors can be superimposed to yield the total behavior of the group flexibility matrix. When evaluating the interaction factor, only the displacements occurring in the vertical plane parallel to the plane of load application are considered; the displacements perpendicular to that plane are neglected as insignificant, if they are present.

For the dimensionless ratios, E_p/E_m, L/d, and S/d, interaction factors are plotted in Figures (12) to (23). These charts are for homogeneous soil and a continuous transition from floating to end-bearing piles represented by the ratio E_b/E_m where E_b is Young's modulus of the underlaying stratum.

In each chart, four interaction factors are plotted for each pile-stiffness ratio. α_{31} represents the interaction factor between a pile at a distance S from the reference pile. The interaction factors α_{41},and α_{51} are the interaction factors of piles at a distance of 2S, 3S and 4S from the reference pile respectively. For very large pile groups, interaction factors of piles at larger distances can be determined by interpolations.

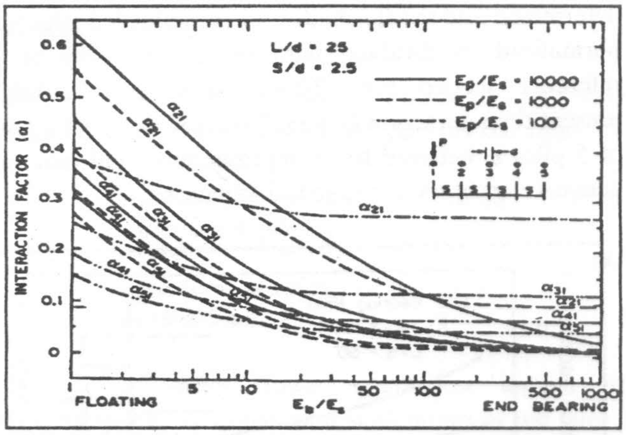

Figure 12. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 25 and S/d = 2.5.

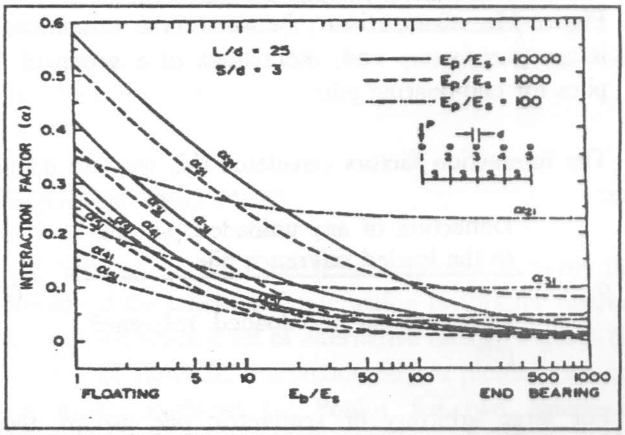

Figure 13. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 25 and S/d = 3.

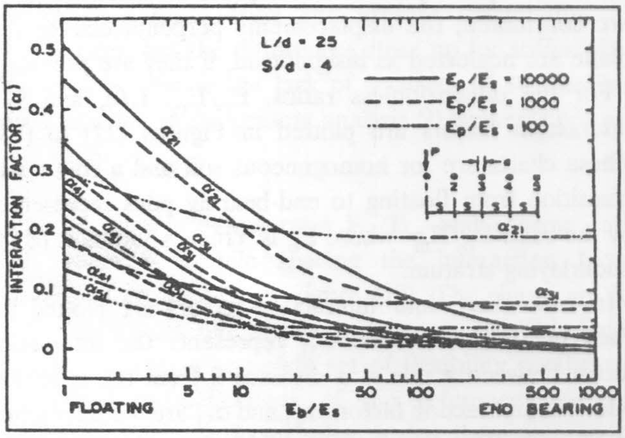

Figure 14. Variation of interaction factors with pile-soil stiffness and stiffness of underlaying stratum for L/d = 25 and S/d = 4.

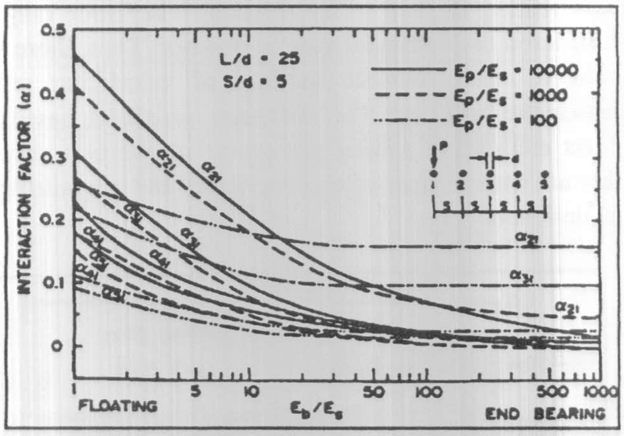

Figure 15. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 25 and S/d = 5.

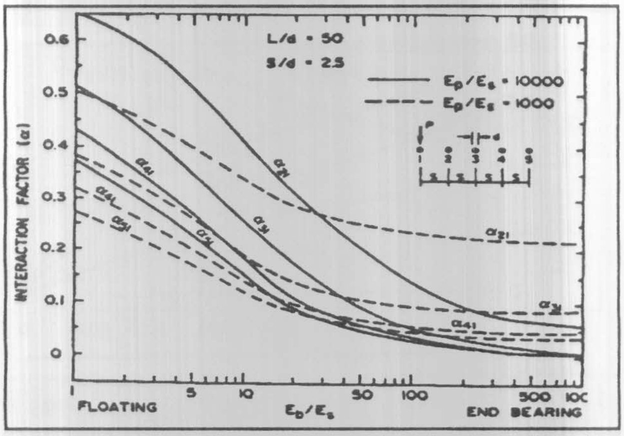

Figure 16. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 50 and S/d = 2.5.

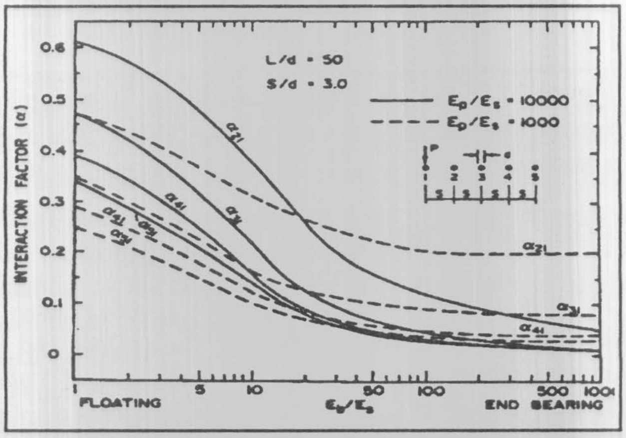

Figure 17. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 50 and S/d = 3.

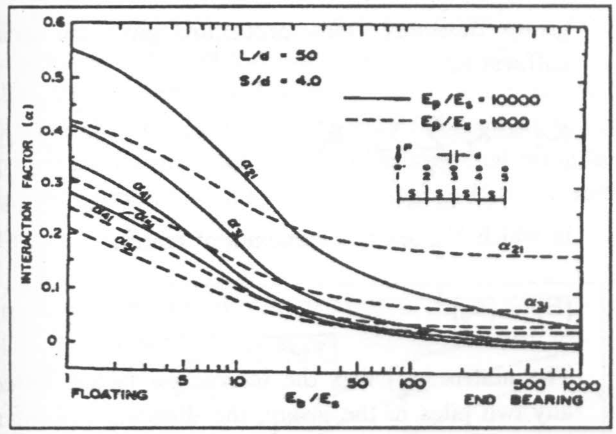

Figure 18. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 50 and S/d = 4.

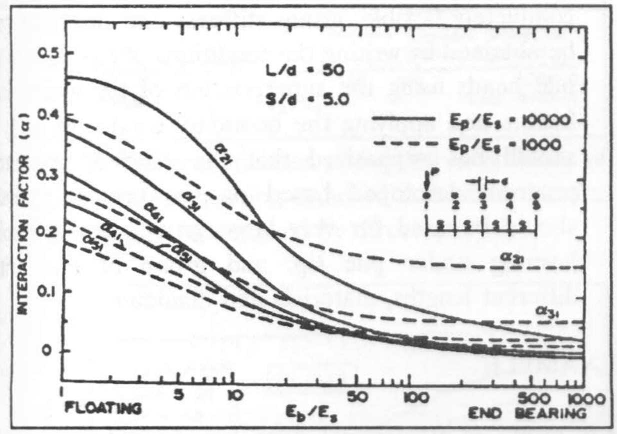

Figure 19. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 50 and S/d = 5.

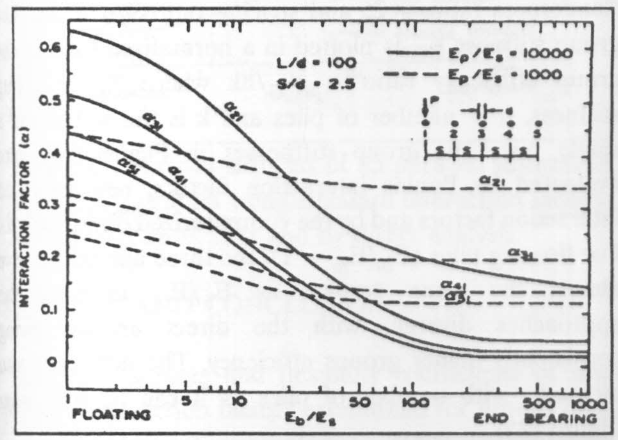

Figure 20. Variation of interaction factors with pile-soil stiffness ratio and stifness of underlaying stratum for L/d = 100 and S/d = 2.5.

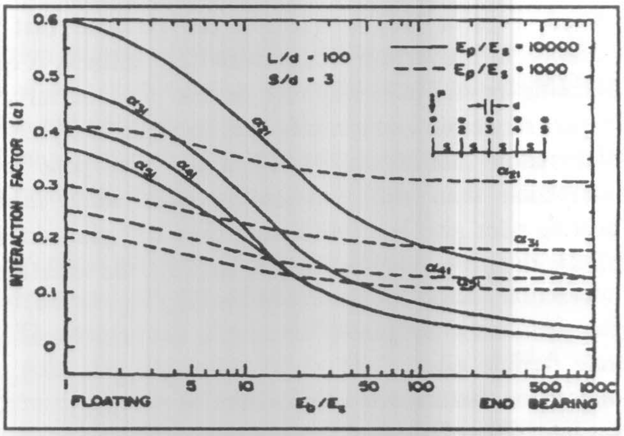

Figure 21. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 100 and S/d = 3.

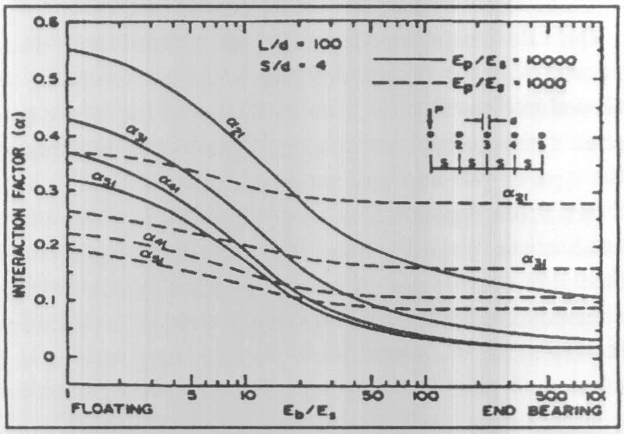

Figure 22. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 100 and S/d = 4.

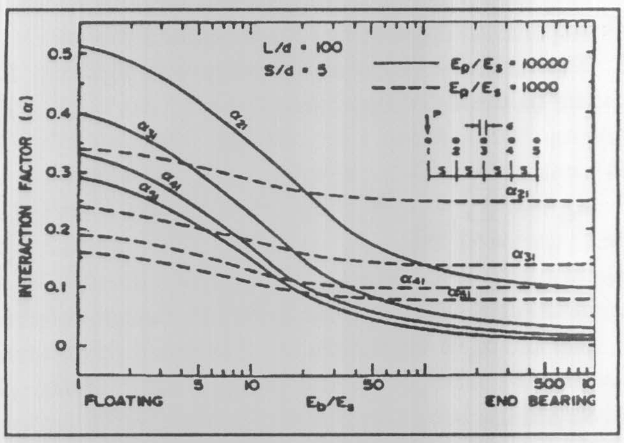

Figure 23. Variation of interaction factors with pile-soil stiffness ratio and stiffness of underlaying stratum for L/d = 100 and S/d = 5.

From Figures (12) to (15), it can be seen that the variation of interaction factors with base stiffness is quite dramatic for stiff piles while for flexible piles the variation is quite moderate and levels up quickly. This interprets the observation that flexible end-bearing piles pose more interaction than stiff end-bearing piles while flexible floating piles give less interaction than stiff piles do. For piles with $L/d \geq 50$ (Figures (16) to (23)), the variation of interaction factors of flexible piles ($E_p/E_m = 100$) with base stiffness is not plotted because it has been found that long flexible piles, ($L/d \geq 50$ and $E_p/E_m = 100$), the interaction factors are independent of the tip condition. So, interaction factors of floating piles, (Figure (3)) can be used for this particular situation.

PILE GROUP STIFFNESS AND FLEXIBILITY

The flexibility coefficients and interaction factors presented in the companion paper and above can be used to evaluate the flexibility and stiffness of the whole group piles. This can be done in a number of ways depending on the type of pile cap and accuracy required.

First, the group flexibility matrix $[F_G]$ should be established; the diagonal elements are equal to the flexibility coefficients of individual piles (F_i) and the off-diagonal elements are expressed using the interaction factors as α_{ji} F_1 where $i,j = 1, 2, 3, .., n$, and n is the number of piles in the group. Then:

1. For flexible caps, offshore structures and interactive superstructures, the group stiffness matrix $[K_G] = [F_G]^{-1}$ can be combined with the stiffness matrix of the flexible cap or superstructure and the analysis of the response to any external loads can be proceed.
2. The vertical stiffness of the group with rigid cap, K_G, can be evaluated approximately as

$$K_G = \sum_{i=1}^{n} K_r / \sum_{i=1}^{n} \alpha_{ri} \tag{13}$$

in which $K_r = 1/F_r$ is the stiffness of single reference pile and α_{ri} = interaction factor between the reference pile and any pile in the group. The reference pile should not be at the extremity or right in the center of the group.
3. For rigid caps, a more rigorous formula can be derived by imposing identical displacements on all pile heads and using, again, the interaction factors to describe group flexibility. This procedure gives the vertical stiffness as

$$K_G = K_r \sum_{i} \sum_{j} E_{ij} \tag{14}$$

in which E_{ij}, are the elements of matrix

$$[E] = [\alpha_{ij}]^{-1} \tag{15}$$

The matrix $[\alpha_{ij}]$ lists the interaction factors between any two piles in the group, the diagonal elements α_{ii} are equal to unity and the matrix is symmetrical with dimensions nxn. The double sum indicates the summation of all elements of the matrix [E].
4. For small groups whose caps are either rigid or completely flexible, group stiffness and flexibility can be obtained by writing the conditions of equilibrium at pile heads using the superposition of the interaction factors and applying the boundary conditions [21]. It should be emphasized that the efficient computer program developed based on the present analysis should be used for very large group, stratified soil, layering under pile tip, and group of piles with different lengths, materials and diameters.

EXAMPLE

A group of 9 identical piles has a rigid cap and is embedded in a homogeneous deposit was analyzed and the group stiffness vs. Underlayer stiffness is plotted in Figure (24). The piles are of pile-soil stiffness ratio = 100 and slenderness ratio = 25 and spacing ratio $S/d = 2.5$. The group stiffness K_G is plotted in a normalized form as the group efficiency ratio = K_G/nk where K_G = group stiffness, n = number of piles and k is the stiffness of a single pile. The group stiffnesses in Figure (24) were evaluated by Poulos interaction factors, new improved interaction factors and by the computerized direct analysis. For floating piles ($E_b/E_m = 1$), the three approaches give almost the same results; as E_b/E_m increases the approaches diverge, with the direct analysis giving consistently higher groups efficiency. The difference also increases with number of piles as it can be seen from Figure (25).

Figures (24) and (25) suggest that the superposition of traditional interaction factors may exaggerate pile-soil-pile interaction effect by up to 50%. The use of new introduced interaction factors may significantly improve the accuracy of the interaction factors approach.

It should be mentioned that the group stiffness of 9 and 25 piles in Figures (24) and (25) were obtained using the approximate formula (13) and it is believed that using the rigorous formula (14) may produce slightly better results [16].

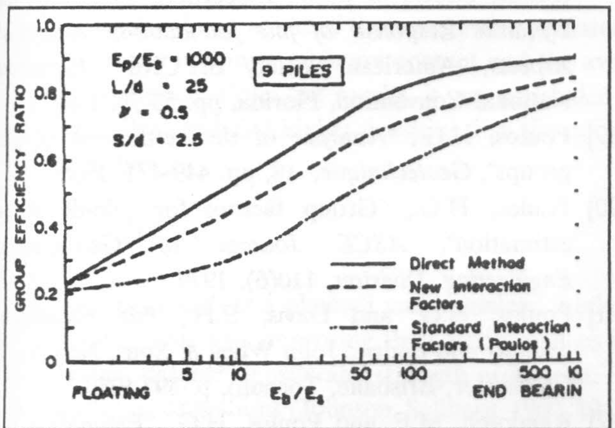

Figure 24. Vertical stiffness of 9 piles vs. stiffness of undrelaying stratum using standard interaction factors, new interaction factors, and by direct analysis.

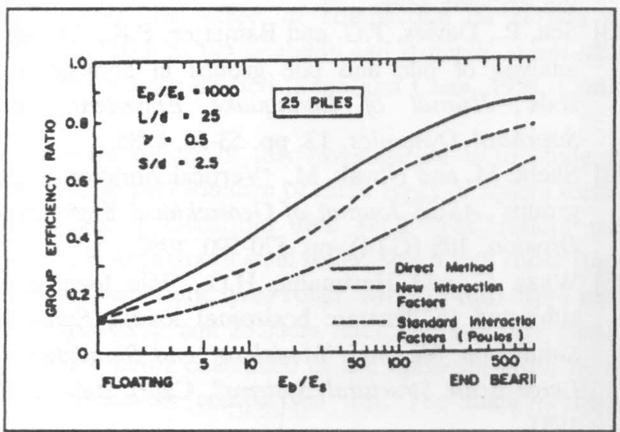

Figure 25. Vertival stiffness of 25 piles vs. stiffness of underlaying stratum using standard interaction factors, new interaction factors and by direct analysis.

SUMMARY AND CONCLUSIONS

Stiffness constants and flexibility coefficients of single piles and interaction factors established for groups of two and five piles are presented to facilitate analysis of arbitrary pile groups exposed to static vertical loads. The established constants and coefficients, and interaction factors are plotted in the form of dimensionless curves for the following conditions: a continuous transition from friction to end-bearing piles, homogeneous, parabolic or linear soil profile and a wide range of pile-soil stiffness ratio and pile slenderness ratio.

Comparison of data presented suggests a number of observations:

1. Vertical stiffness of floating flexible piles are almost independent of its length.
2. The magnitude of pile stiffness decreases significantly with the changing of the soil profile from homogeneous to parabolic to linear.
3. Pile-soil stiffness ratio is a major factor affecting pile flexibility for pile resting on stiffer underlaying stratum.
4. Stiffness of flexible piles is almost independent of its pile tip conditions.
5. Effect of pile length on the magnitude of flexibility is significantly affected by the tip condition
6. Numerical examples showed that the new formulated interaction factors alleviate the difference between the computer based direct approach and the superposition of interaction factors.
7. The group flexibility matrix can be easily employed to solve groups with rigid, and infinitely or arbitrary flexible caps
8. Further research should include end-bearing pile in non-homogeneous soil, non-linearity, flexible caps and field experiments.

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