SCREW CONVEYOR EQUIVALENT ROTATIONAL SURFACE AND ITS OPTIMUM FORMING FOR MINIMUM PLASTIC DEFORMATION

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ABSTRACT

The problem of manufacturing screw shaped conveyors is discussed. A screw-equivalent rotational surface with the same dimensional characteristics is found. The parameters of a cone shape are optimized to get minimum plastic strain energy in forming the screw-equivalent rotational surface.

INTRODUCTION

Screw conveyors are widely used in material handling. The problem of forming the screw shape from a metal sheet has practically very high interest. The forming methods used in practice cause high strain energy for the material besides the lack of precision. These methods [1], [2] depend mainly on forming an incomplete annulus shape which has an inside and outside circumference equaling the screw inside and outside helical length respectively, in the same time the radial distance in the screw is equal to the difference between the inside and outside radius of the annulus. Experience shows that these methods have several disadvantages, which are obvious from simple calculation of the areas of the helix surface and annulus. The area of helix surface is smaller than that of annulus, (specially when the difference between the inside and outside diameter is large), which means that the annulus plate should be deformed (compressed) in its plane to obtain the helix shape, such deformation is practically impossible.

In the following we find the exact rotational surface to form one pitch of the helix, and then determine the optimum cone shape which gives minimum strain energy due to plastic deformation from conical to the exact rotational surface.

SCREW-EQUIVALENT ROTATIONAL SURFACE

This work will be taken on two steps; the first concerns with finding the rotational surface which is exactly equivalent in dimensions (lengths, area, angles, ...) to the screw surface and consequently gives the exact screw shape by simple bending. The second step is to find the parameters of conical shape which minimize the plastic strain necessary for getting the rotational surface.

The length and radius of curvature of the helix shown in Figure (1) are calculated using parametric equation of helix [3]; and given by,

$$\mathcal{L} = 2\pi\sqrt{r^2+b^2} \;\;\&\;\; \rho = r+b^2/r \;\;\text{for one turn}$$

where $\mathcal{L}$ is the length of turn of helix of radius r, ρ is the radius of curvature of helix form, and b = pitch/2π.

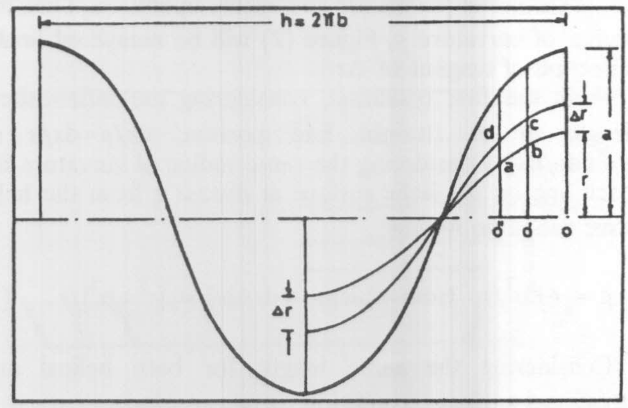

Figure 1.

To get the function which describes the equivalent rotation surface, Figure (2), the following conditions are considered:

i- The distant Δr between any two points 1,2 on the generatrix; (p the curve generating the rotation surface by rotation about axis lying in the same plane as the curve); is equal to the radial distance between two similar points on the helical surface.

ii- The circumference of the circle generated by the rotation of any point on the generatrix is equal to the length of the helix generated by the motion of a similar point on the helix surface at a constant radial distance for one pitch of the helix.

iii- According to the two previous conditions, the similar areas abcd on the two surfaces must be equals.

iv- The radius of curvature for the developed boundary curves of any strip Δr on the helix must be equal to that of the similar strip on the rotational surface.

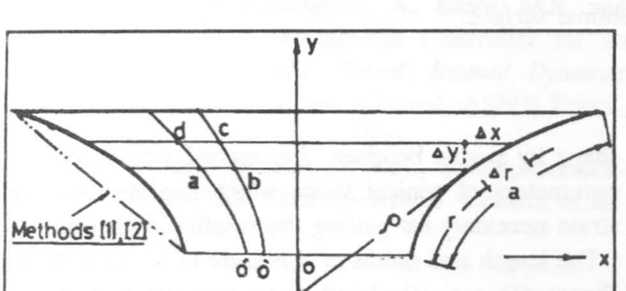

Figure 2.

For this last condition it must be noted that the forming of a plate by simple bending perpendicular to its surface is an easy operation, but forming a plate in the direction of its surface is practically impossible operation. Thus the radius of curvature ρ, Figure (2) will be measured in the direction of tangent of Δr.

From the first condition, considering the infinitesimal length dr as straight line portion; $dr/\rho = dx/x$ or $\rho = x[dr/dx]$. Considering the same radius of curvature for each line on the helix surface at distant r from the helix axis; condition iv thus;

$$\rho = r + b^2/r; \text{ from which; } x[dr/dx] = [r^2+b^2]/r \qquad (1)$$

Considering the same length for both helical and rotational surface; (condition ii); we conclude;

$$2\pi x = \mathcal{L} = 2\pi\sqrt{r^2+b^2} \text{ or } x^2 = r^2+b^2 \qquad (2)$$

From equations (1) and (2), we get;

$$dr/dx = x/\sqrt{x^2-b^2} \qquad (3)$$

The relation between dx, dy, dr, is given by; $dy/dx = [(dr/dx)^2-1]^{1/2}$; and equation (3) may thus take the form $dy/dx = [x^2/(x^2-b^2)-1]^{1/2}$; from which by direct integration, $y = b\ln[x+\sqrt{x^2-b^2}\ ] + C$, where C is an integration constant obtained by substitution of boundary conditions; at $y=0$ $(r=0)$, $x=\pm b$; thus $C=-b(\ln b)$; and the equation describes the generatrix of the rotational surface will be;

$$y = b\ln[(x/b)+\sqrt{(x/b)^2-1}\ ] \qquad (4)$$

The generatrix of the surface of rotation obtained from equation (4) has the same arc length and radius of curvature of the developed boundary curve at any point; measured in the direction of tangent of the surface exactly as those of the helix.

OPTIMUM CONE DIMENSIONS TO OBTAIN THE ROTATIONAL SURFACE

To obtain rotational surface in practice from a plate, it is obvious that the nearest shape to begin with is a cone. Our problem is to find the optimum dimensions of such cone to minimize the plastic deformation in forming the rotational surface equivalent to the helix.

According to the perpendicular strain, the length between two points (0,1) on the generatrix of the cone before deformation, Figure (4), must differ from the length of the rotational surface between the points (0,1) after deformation Figure (3).

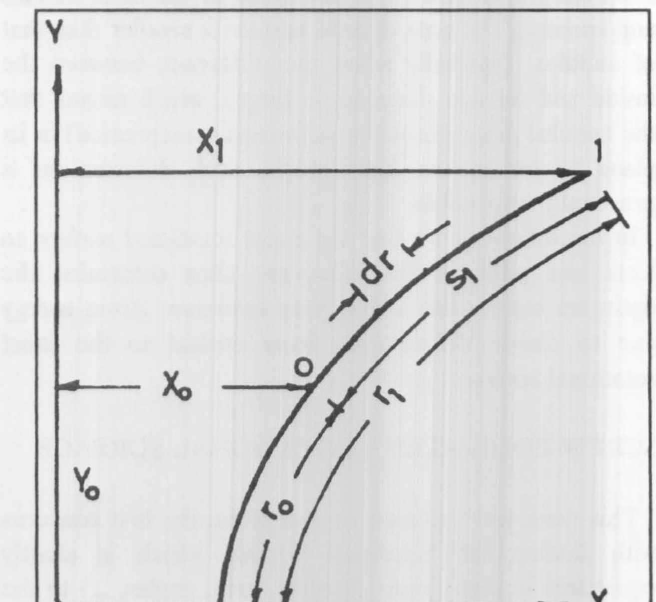

Figure 3.

The radius of rotational surface at point 1 will be;

$$x_1^2 = r_1^2 + b^2 \tag{5}$$

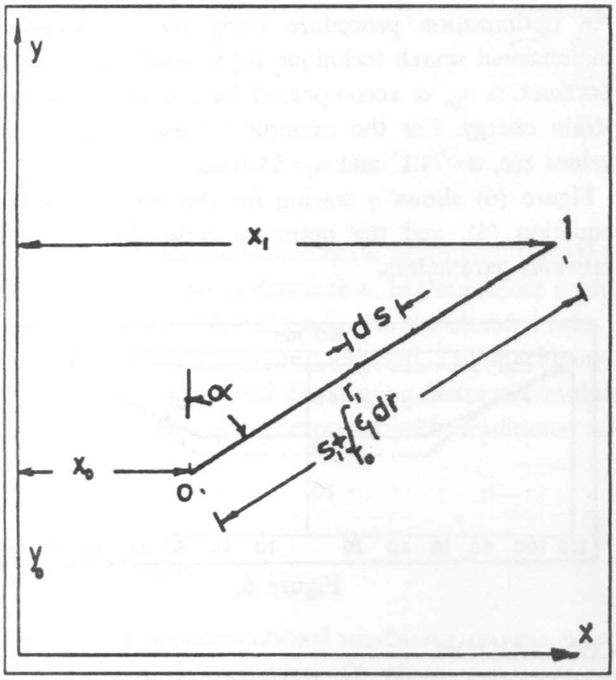

Figure 4.

The relation between an infinitesimal length ds on the generatrix of the cone shape and the deformed length dr on the generatrix of the rotational surface could be written as [4];

$$\epsilon_{\xi r} = \ln[dr/ds] \tag{6}$$

where $\epsilon_{\xi r}$ is the longitudinal strain in r direction.

Assuming that the material will behave as an ideal plastic material, and thus obeys Levy-Mises relation, and considering that our case as a unidirectional stress state (all surface at x_0, X_0, x_1, X_1 are free surfaces), then we can write, [5];

$$\epsilon_{\xi r} = -\epsilon_{xr}/2 = -(1/2)\ln[X/x]$$

where ϵ_{xr} is the strain in x direction, and equation (6) will be;

$$\epsilon_{\xi r} = -(1/2)\epsilon_{xr} = -(1/2)\ln[X/x] = \ln[dr/ds]$$

from which $ds = dr\sqrt{X/x}$. Substituting $ds = dx/\sin\alpha$ and separation of variables gives;

$$\int_{x_0}^{x_1} \sqrt{x}\,dx = \sin\alpha \int_{X_0}^{X_1} \sqrt{X}\,dr$$

Substitution by equation (3) in the right hand side and direct integration gives;

$$x_1 = \left\{[x_0]^{3/2} + (3/2)\sin\alpha\left\{\int_{X_0}^{X_1} [X]^{3/2}/\sqrt{X^2-b^2}\,dX\right\}\right\}^{2/3} \tag{7}$$

Equation (7) gives the radius x_1 of the cone at point (1) before deformation.

To minimize the plastic strain; it will be reasonable to find the cone parameters such that the plastic strain energy necessary to deform the conical surface to the specified helix-equivalent rotational surface will be minimum.

PLASTIC STRAIN ENERGY CALCULATIONS

The strain energy necessary to deform an elementary annular shape of thickness t and length dr_1 from cone radius x_1 to rotational surface radius X_1 will be; Figure (5);

$$\Delta E = \int_{x_1}^{X_1} 2\pi F'_1 dX'_1 = \int_{x_1}^{X_1} 2\pi\sigma_x t dX'_1 dr$$

where F'_1 is the circumferential force in the annulus at a position X'_1.

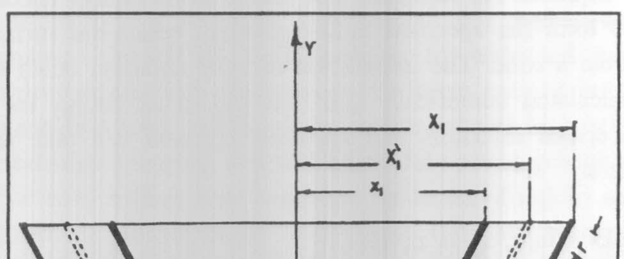

Figure 5.

For the unidirectional stress state of the Levy-Mises ideal plastic material, we have;

$$d\epsilon_x = [d\bar{\epsilon}/\sigma]\sigma_x \quad \text{or} \quad \sigma_x = \bar{\sigma}[d\epsilon_x/d\epsilon]$$

where $d\bar{\epsilon}$ is the incremental strain invariant, $\bar{\sigma}$ the stress invariant, and $d\epsilon_x$, σ_x are the incremental strain and the stress in direction x respectively. Also;

$$d\bar{\epsilon} = [(2/3)(d\epsilon_x^2 + d\epsilon_y^2 + d\epsilon_z^2)]^{\frac{1}{2}} = d\epsilon_x$$

And considering that the stress-strain relation of the material will take the form $\bar{\sigma} = k[\bar{\epsilon}]^n$, where k and are experimental constants.

Thus the strain energy equation for the infinitesimal annulus will take the form;

$$\Delta E = \int_{x_1}^{X_1} 2\pi t dX_1' . k[\bar{\epsilon}]^n dr = \int_{x_1}^{X_1} 2\pi k t\{\ln[X/x]\}^n dX_1 dr,$$

and the total plastic strain energy necessary to form the rotational surface form the cone will be;

$$E = \int_{r_0}^{R} \int_{x_1}^{X} 2\pi k t\{\ln[X/x]\}^2 dX dr \qquad (8)$$

where r_0 and R are the inner and outer radii of the helix respectively.

The variation of thickness t is small compared with the variations in the plane direction, thus t could be considered constant in the above integration. Also the parameter x which represent the cone radius at a point is constant in the integration with respect to X in equation (8).

Equation (8) represent the plastic strain energy needed to form the specified helix-equivalent rotational surface from a cone. The integration of such equation could be calculated numerically and thus the strain energy for a specified thickness and specified material will take the form;

$$E = fn[\alpha, x_0, b, r_0, R]$$

Hence, for a specified helix, (i.e. known b, r_0, R) the optimum values of the two parameters α, x_0 defining the cone which minimize the plastic strain energy could be obtained.

NUMERICAL EXAMPLE

For a specified screw conveyor has a helix with inner radius r_0=45mm and outer radius R=114mm and b=36.3mm (the pitch is equal to the outer diameter) the plastic strain energy, equation (8), is calculated numerically; (using the numerical calculations of equations (4) and (7) as a function of the cone parameters x_0, α.

An optimization procedure using the alternating one dimensional search technique [6] is conducted to get the parameters x_0, α accompanied by the minimum plastic strain energy. For the example mentioned above these values are, α=74.1° and x_0=53.4mm.

Figure (6) shows a tracing for the rotational surface; equation (4), and the optimum cone obtained by the previous parameters.

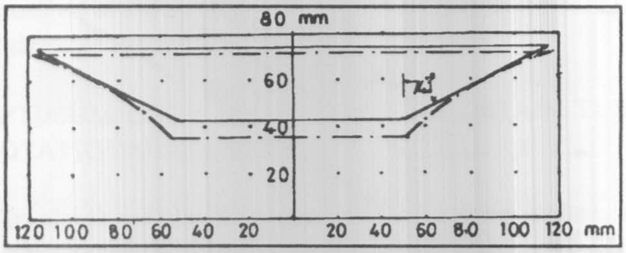

Figure 6.

CONCLUSION

1- An screw-equivalent rotational surface with the same radial distances, areas radius of curvature at a point is obtained.
2- The optimum cone dimensions which minimize the strain energy required to form the rotational surface are calculated.

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