

OPTIMAL LOAD SHEDDING SOLUTION IN POWER SYSTEMS  
WITH SPECIAL CONSIDERATION TO  
STABILITY PROBLEM

**M. Y. ABDELFATTAH**

Department of Electrical Engineering

Faculty of Engineering

Alexandria University

Alexandria, EGYPT

Abstract

In this paper a novel technique for handling the stability problem associated with load shedding strategy is studied. The solution of the problem is performed in two steps :

1. A steady state solution for the load shedding strategy is obtained using an optimization technique.
2. The transient stability associated with the load shedding is then studied knowing the amount of load to be shed from step 1. To study the transient stability each synchronous machine is represented with its dynamic equations.

This novel technique is applied to a 5-busbars and 12-busbars systems. Study has revealed that beside the load shedding strategy a control over the machines exciter has to be applied to keep the system stable throughout the period of the load shedding process.

## 1. Introduction

With present time interconnected power systems, conditions often arise which would result in the isolation of a part of the system with an excess of load on it. A declining frequency situation will, therefore, develop in that part. Load shedding provides the means of arresting such frequency decline after all available generation reserves are used up. Certain percentages of the loads are dropped in succession according to the declining frequency, until conditions become favourable and system frequency starts to return back to normal<sup>1,2</sup>. If such process of load shedding is successful a steady state, at normal frequency, will finally ensue with decreased loads.

The problem of optimal load shedding is essentially the problem of obtaining such steady state which guarantees a minimum of load dropped. At the same time, that steady state must be feasible in the sense that no operational constraints are violated. This ensures the practicability of the load schedule obtained. The problem is, therefore, a constrained power system optimization problem to which numerous mathematical optimization techniques may be applied.

Many papers have been directed to the optimal load shedding problem<sup>3-9</sup>. However, all these papers have studied the problem at steady state without consideration to the arising problem of transient stability.

In this paper the transient stability problem associated with the load shedding strategy is studied. The algorithm is applied to a 5-bus system and 12-bus system, and the results obtained are reported and discussed.

2. Problem Formulation

2.1 Optimal Load Shedding Problem

Consider a system of  $N$  busses with  $N_L$  pure load busses, and  $N_G = N - N_L$  pure generator busses. Then the problem is :

Minimize the scalar function

$$F(\underline{V}, \underline{\delta}) = \sum_{i \in S} [ C_i^* - C_i(\underline{V}, \underline{\delta}) ]^2 / 2k_i C_i^* \tag{1}$$

Subject to

$$D_i(\underline{V}, \underline{\delta}) - \gamma_i C_i(\underline{V}, \underline{\delta}) = 0 \quad i=1, 2, \dots, N_L \tag{2}$$

$$P_i(\underline{V}, \underline{\delta}) - P_i = 0 \quad i=N_L+1, \dots, N \tag{3}$$

$$C_i^m \leq C_i(\underline{V}, \underline{\delta}) \leq C_i^* \quad i=1, 2, \dots, N_L \tag{4}$$

$$D_i^m \leq D_i(\underline{V}, \underline{\delta}) \leq D_i^* \quad i=1, 2, \dots, N_L \tag{5}$$

$$Q_i^m \leq Q_i(\underline{V}, \underline{\delta}) \leq Q_i^M \quad i=N_L+1, \dots, N \tag{6}$$

$$V_i^m \leq V_i \leq V_i^M \quad i=1, 2, \dots, N \tag{7}$$

where,

$S$  is the set of load busses

$k_i$  is the priority factor assigned to that load

$$C_i(\underline{V}, \underline{\delta}) = -V_i \sum_{j=1}^N V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$D_i(\underline{V}, \underline{\delta}) = -V_i \sum_{j=1}^N V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij})$$

$$P_i(\underline{V}, \underline{\delta}) = V_i \sum_{j=1}^N V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$V_i e^{j\delta_i}$  is the voltage at bus  $i$

$[ Y_{ij} e^{j\theta_{ij}} ]$  is the bus admittance matrix of the post emergency system

$\gamma_i = \tan(-\phi_i)$ , where  $\phi_i$  is the power factor angle at bus  $i$

$C_i^*$  &  $D_i^*$  are active & reactive demands at load bus  $i$  prior to emergency respectively

$C_i^m$  &  $D_i^m$  are the minimum real & reactive generations at load bus  $i$  respectively

$Q_i^m$  &  $Q_i^M$  are the minimum & maximum reactive generations at generator bus  $i$  respectively

$V_i^m$  &  $V_i^M$  are the minimum & maximum limits of the voltage magnitude at bus  $i$  load or generator, respectively.

The solution of this problem has been described in detail in reference (5).

## 2.2 Stability Consideration

In transient stability studies, a synchronous machine may be represented by a voltage source in back of transient reactance<sup>10</sup>. With this representation for a synchronous machine, two first order differential equations are solved to obtain the change in the internal voltage angle  $\delta$  and machine speed  $\omega$ . Thus for a system with  $N_G$  generator busses :

$$d\delta_i/dt = \omega_i - 2\pi f \quad i=N_L+1, \dots, N \quad (8)$$

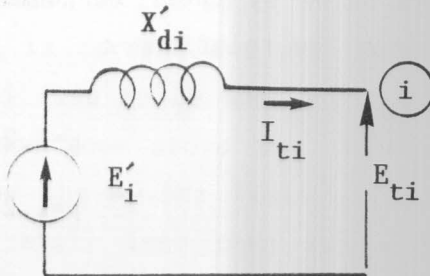
$$d\omega_i/dt = \frac{\pi f}{H_i} (P_{mi} - P_{ei}) \quad i=N_L+1, \dots, N \quad (9)$$

where  $f$  is the system frequency,  $H_i$  inertia constant,  $P_{mi}$  mechanical input power and  $P_{ei}$  is the electrical air gap power for machine at bus  $i$ .

To calculate  $P_{ei}$ , from the representation shown in figure(1), for the synchronous machine we find

$$I_{ti} = (E'_i - E_{ti})/jX'_{di} \quad (10)$$

$$P_{ei} - jQ_{ei} = I_{ti} (E'_i)^* \quad (11)$$



Figure(1): Simplified representation of a synchronous machine for transient analysis

The solution of the load shedding problem proceeds as follows :

1. Load flow analysis is used to obtain system conditions prior the disturbance. From this analysis the initial machines internal voltages are then calculated from :

$$I_{ti} = P_i - jQ_i / E_{ti}^* \quad (12)$$

$$E'_i = E_{ti} + jX'_{di} I_{ti} \quad (13)$$

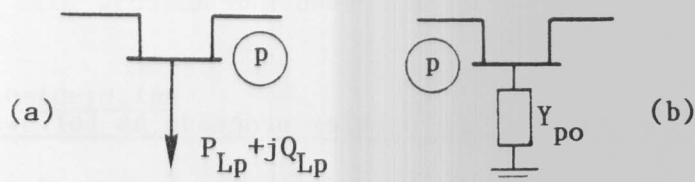
where  $P_i$  &  $Q_i$  are the real and reactive terminal powers at bus  $i$ .

2. Simulate the system disturbance. In our case, the input mechanical power at the faulty generator busses are reduced while there is no reserve at the remaining busses. Then applying the optimal load shedding technique described in section 2.1 to get the new steady state loads distribution at the different load busses, and voltage magnitudes and phase angles for all busses.
3. Calculate the post disturbance internal voltages of machines from the steady state conditions obtained from step 2. To get this state, excitation control should be applied as described in next section.
4. The different loads are represented by their respective admittances to ground as shown in figure(2) from the relations :

$$I_{po} = P_{Lp} - jQ_{Lp} / E_p^* \quad p=1,2,\dots,N_L \quad (14)$$

$$Y_{po} = I_{po} / E_p \quad p=1,2,\dots,N_L \quad (15)$$

where  $P_{Lp}$  &  $Q_{Lp}$  are the scheduled bus loads and  $E_p$  is the calculated bus voltage.



Figure(2): Representation of load

a: Constant Power representation

b: Admittance to ground representation

Now, our strategy is to completely shed the excess loads which are determined from step 2 in steps with a constraint that the drop in any machine frequency does not exceed a predetermined percent e.g. 1% i.e. at any step of shedding. To know the frequency of each machine at each step equations (8) & (9) have to be applied to calculate the derivatives. Then modified Euler technique for numerical integration is applied with a time increment  $\Delta t$ . During this application load flow solution has to be carried out for the modified system which contains the machine transient reactances and the loads represented with their respective admittances to ground.

5. At the end of each numerical integration step a check is performed to all machines speeds. If the speed at any bus drops below say 49.5Hz then a load shedding is carried out with a predetermined constant ratio say 20% of the total load to be shed known from step 2. This shedding will stop if the speed goes above the 49.5Hz and will resume again if the speed drop below the 49.5Hz limit. If after dropping all loads to be shed any speed still less than say 49.0Hz then this means that the system is unstable.

### 3. Exciter Effect

It is a well known fact that exciter control system is vital for preserving the stability of synchronous machines when subjected to sudden fault. Its action is to control the excitation emf in order to reduce the output power to reach the power balance between the output and the power consumed by the system during the fault. One can look to the load shedding problem in a similar manner. After each load shedding process, the power consumed by the system reduces. This can be regarded as electrically equivalent to a fault on the system. Hence, the importance of control action during the

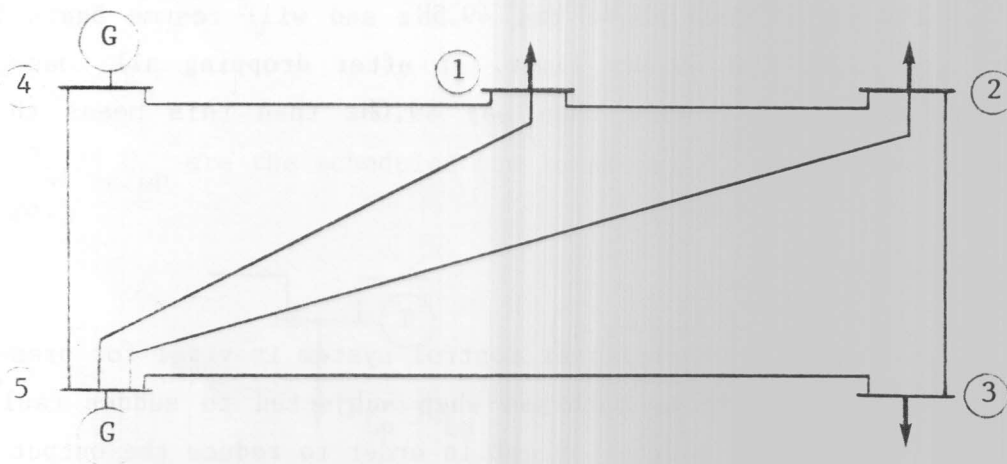
load shedding process arises. In the following application exciter control system is assumed to act on each machine having the following performance: After each load shedding step the internal emf of each machine is reduced by a percentage equal to the percentage of the load shed.

#### 4. Applications

The method described above has been applied to two test systems :

##### A: 5-Bus System

The power system shown in figure(3) is used as a test system. The transmission line impedances in per unit on a 100MVA base are given in Table(1). The scheduled generation and loads and assumed bus voltages in per unit are given in Table(2). The inertia constants and direct-axis transient reactances at busses 4 & 5 in per unit are given in Table(3).



Figure(3): 5-Bus Sample System



Table(1): Transmission Line Impedances

Bus Code p-q	Impedance
1-2	0.01 + j0.03
1-4	0.08 + j0.24
1-5	0.06 + j0.18
2-3	0.08 + j0.24
2-5	0.06 + j0.18
3-5	0.04 + j0.12
4-5	0.02 + j0.06

Table(2): Scheduled Generation And Loads And Assumed Bus Voltages

Bus Code p	Assumed Bus Voltage	Generation		Load	
		P	Q	C	D
1	1.0 + j0.0	0.0	0.0	0.5	0.1
2	1.0 + j0.0	0.0	0.0	0.6	0.1
3	1.0 + j0.0	0.0	0.0	0.8	0.2
4	1.06+ j0.0	1.0	?	0.0	0.0
5	1.05+ j0.0	?	?	0.0	0.0

Table(3): Inertia Constants And Direct-Axis Transient Reactances

Bus Code p	Inertia Constant H	Direct-Axis Transient
		Reactance $X'_d$
4	50.0	0.25
5	1.0	1.50

Table(4) shows the state prior the emergency. Also shown are the upper and lower limits of the different inequalities. A normal load flow is used to obtain that state, with generator bus 5 acting as the slack generator. This is followed by a rotation of axes to make the voltage phase angle of load bus 3, the would be reference, equal to zero.

Table(4): System Pre-Emergency State

Bus Code p	V <sub>min</sub>	V	V <sub>max</sub>	$\delta$	P <sub>min</sub>	P	P <sub>max</sub>	Q <sub>min</sub>	Q	Q <sub>max</sub>
1	0.9	1.01159	1.05	1.232	0.0	-0.5	-0.5	0.0	-0.1	-0.1
2	0.9	1.00786	1.05	0.805	0.0	-0.6	-0.6	0.0	-0.1	-0.1
3	0.9	0.99635	1.05	0.0	0.0	-0.8	-0.8	0.0	-0.2	-0.2
4	1.0	1.06	1.1	6.237	0.25	1.0	1.0	-0.4	0.0833	0.8
5	1.0	1.05	1.1	4.437	0.25	0.961	1.0	-0.4	0.5002	0.8

Emergency is simulated by a reduction in the available real generation at bus 5 from 1.0 p.u. to 0.3 p.u. Optimal load shedding is obtained while priorities are equal for all loads. Table(5) shows the solution for this case.

Table(5): Optimal Solution,  $k_1=k_2=k_3=1$ 

Bus Code p	V	$\delta$	C <sub>min</sub>	C	C*	D <sub>min</sub>	D	D*	P	Q
1	1.01674	0.989	0.0	0.3306	0.5	0.0	0.0661	0.1		
2	1.01396	0.658	0.0	0.4015	0.6	0.0	0.0669	0.1		
3	1.00567	0.0	0.0	0.5346	0.8	0.0	0.1337	0.2		
4	1.05289	4.989							1.0	0.0727
5	1.04010	2.902							0.3	0.2939

To study the transient response associated with the load shedding problem, the following load shedding parameters should be incorporated :

1. Relay set point, i.e. the frequency at which shedding takes place. In our case will be chosen as 49.5Hz.
2. Step size, i.e. the amount of load to be shed at each step as a percentage of the total amount to be shed known from the solution of the optimal load shedding problem described in section 2.1. In our case the total amount to be shed is 0.6333 p.u. as given from Table(5) and the step size will be chosen as 50%.

3. Number of steps. In our case will be chosen to be 2 steps.
4. Time delay between each shedding step. In our case will be chosen to be 0.02 second.

Table(6) shows shedding steps and their respective times in conjunction with the amount of load to be shed at each step.

Table(6): Shedding Process For The 5-Bus System

Step Number	Respective Time For Shedding	Amount Of Load Shed
1	0.04 sec	0.31665 p.u.
2	0.06 sec	0.31665 p.u.

Figure(4) shows the transient response for electric power, internal emf phase angle and the speed deviation from nominal system frequency(50Hz) for machines connected to busses 4 & 5.

**Note:** Speed Deviation =  $\frac{\text{Speed} - \text{Rated Speed}}{\text{Rated Speed}}$

#### B: 12-Bus System

The power system shown in figure(5) is used as another test system. Tables 7, 8 & 9 give the required data in per unit on a 100MVA base.

Table(7): Transmission Line Impedances And Line Charging

Bus Code p-q	Impedance	Line Charging $y'_{pq}/2$
1-2	0.06701+j0.17103	0.0173
1-11	0.04699+j0.19797	0.0219
2-3	0.01335+j0.04211	0.0064
2-4*	0.00000+j0.55618	0.0
2-11	0.05811+j0.17632	0.0187
3-10*	0.00000+j0.25202	0.0
3-11	0.05695+j0.17388	0.0170

.....

Continue Table(7).....

3-12	0.05403+j0.22304	0.0246
4-5	0.03181+j0.08450	0.0
4-7	0.12711+j0.27038	0.0
5-6	0.08205+j0.19207	0.0
6-10	0.09498+j0.19890	0.0
7-8	0.17093+j0.34802	0.0
8-9	0.22092+j0.19988	0.0
8-10	0.06615+j0.13027	0.0
9-10	0.12291+j0.25581	0.0
11-12	0.01938+j0.05917	0.0264

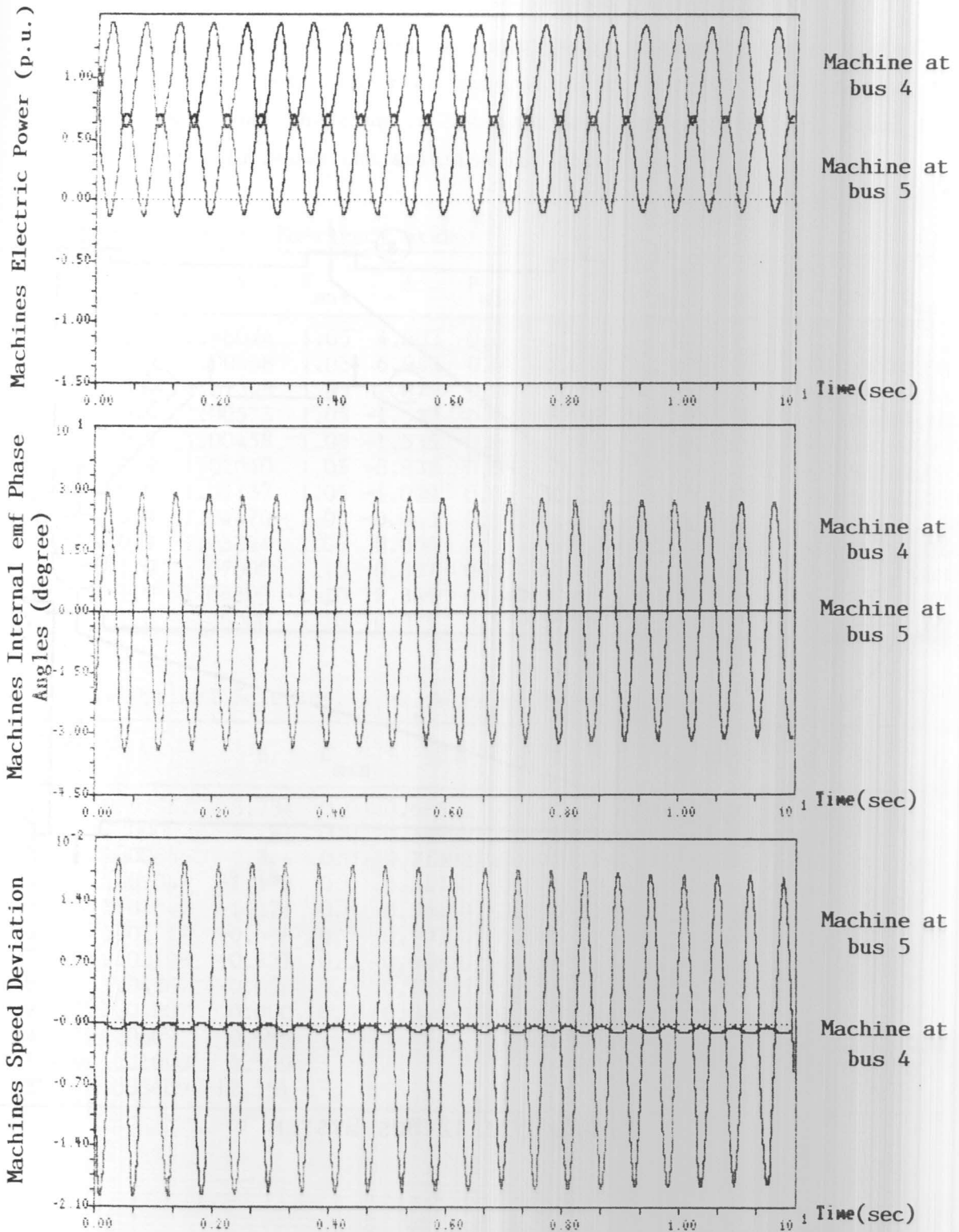
\* Impedance of a transformer

Table(8): Scheduled Generation And Loads And Assumed Bus Voltages

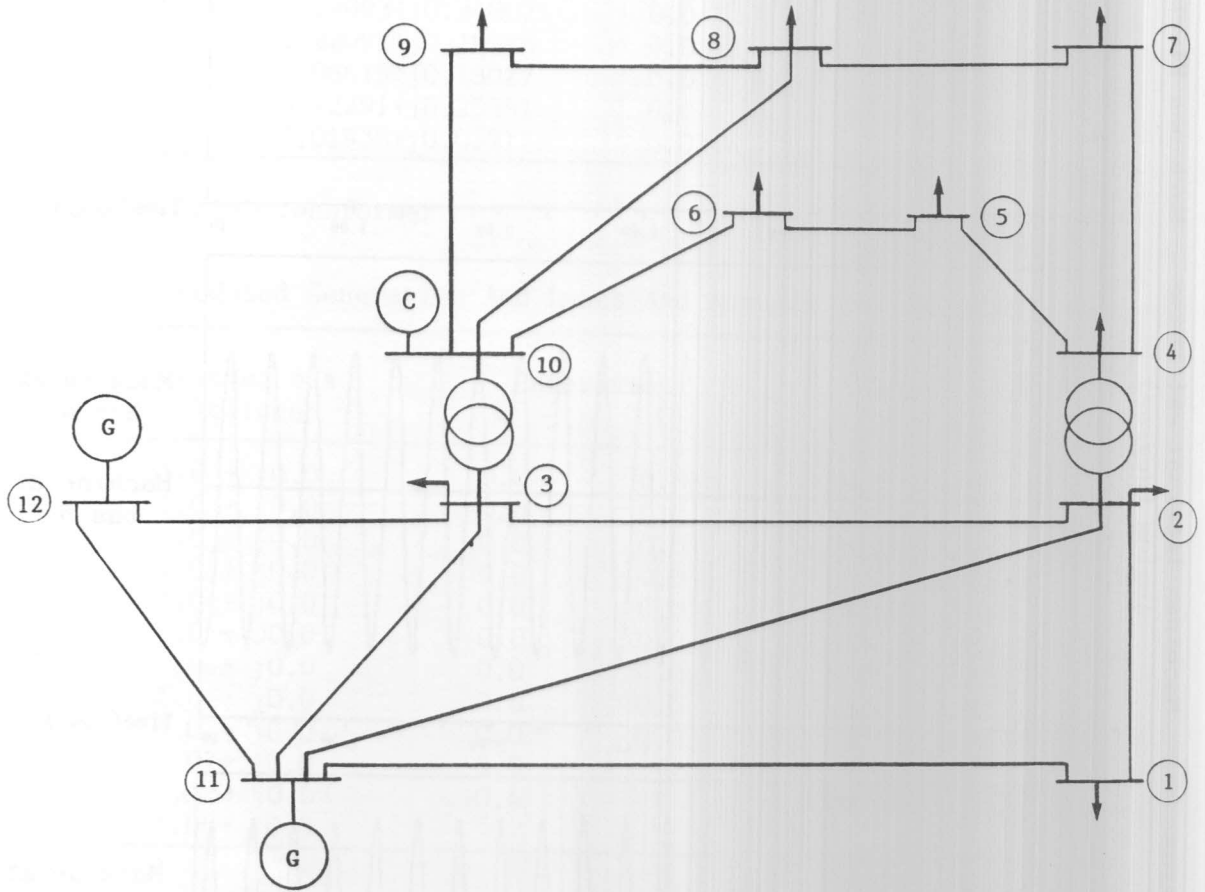
Bus Code p	Assumed Bus Voltage	Generation		Load	
		P	Q	C	D
1	1.0 + j0.0	0.0	0.0	0.942	0.190
2	1.0 + j0.0	0.0	0.0	0.478	0.039
3	1.0 + j0.0	0.0	0.0	0.293	0.143
4	1.0 + j0.0	0.0	0.0	0.295	0.166
5	1.0 + j0.0	0.0	0.0	0.090	0.058
6	1.0 + j0.0	0.0	0.0	0.147	0.093
7	1.0 + j0.0	0.0	0.0	0.050	0.050
8	1.0 + j0.0	0.0	0.0	0.135	0.058
9	1.0 + j0.0	0.0	0.0	0.061	0.016
10	1.07+ j0.0	0.0	?	0.0	0.0
11	1.045+ j0.0	0.4	?	0.0	0.0
12	1.06+ j0.0	?	?	0.0	0.0

Table(9): Inertia Constants And Direct-Axis Transient Reactances

Bus Code p	Inertia Constant H	Direct-Axis Transient Reactance $X_d$
10	0.25	1.0
11	2.00	0.5
12	20.00	0.1



Figure(4): Transient Response For 5-Bus System



FIGURE(5): 12-BUS SYSTEM

Table(10) shows the state prior the emergency. Emergency is simulated by a reduction in the available real generation at bus 12 from 2.4 p.u. to 1.5 p.u. Optimal load shedding is obtained while priorities are equal for all loads. Table(11) shows the solution for this case.

Table(10): System Pre-Emergency State

Bus Code p	V <sub>min</sub>	V	V <sub>max</sub>	δ	P <sub>min</sub>	P	P <sub>max</sub>	Q <sub>min</sub>	Q	Q <sub>max</sub>
1	0.9	0.98074	1.05	4.803	0.0	-0.942	-0.942	0.0	-0.190	-0.190
2	0.9	1.00568	1.05	6.943	0.0	-0.478	-0.478	0.0	-0.039	-0.039
3	0.9	1.01398	1.05	7.879	0.0	-0.293	-0.293	0.0	-0.143	-0.143
4	0.9	1.00575	1.05	-1.533	0.0	-0.295	-0.295	0.0	-0.166	-0.166
5	0.9	1.00458	1.05	-1.535	0.0	-0.090	-0.090	0.0	-0.058	-0.058
6	0.9	1.02040	1.05	-0.832	0.0	-0.147	-0.147	0.0	-0.093	-0.093
7	0.9	1.01137	1.05	-1.090	0.0	-0.050	-0.050	0.0	-0.050	-0.050
8	0.9	1.04470	1.05	-0.063	0.0	-0.135	-0.135	0.0	-0.058	-0.058
9	0.9	1.05244	1.05	0.000	0.0	-0.061	-0.061	0.0	-0.016	-0.016
10	1.0	1.07000	1.10	0.897	0.0	0.000	0.000	-0.4	0.566	0.800
11	1.0	1.04500	1.10	12.385	0.1	0.400	0.400	-0.4	0.487	0.800
12	1.0	1.06000	1.10	16.975	0.5	2.234	2.400	-0.4	-0.114	0.800

Table(11): Optimal Solution,  $k_1=k_2=k_3=k_4=k_5=k_6=k_7=k_8=k_9=1$

Bus Code p	V	δ	C <sub>min</sub>	C	C*	D <sub>min</sub>	D	D*	P	Q
1	0.97962	3.734	0.0	0.68504	0.942	0.0	0.13817	0.190		
2	1.00002	5.191	0.0	0.34997	0.478	0.0	0.02855	0.039		
3	1.00656	5.864	0.0	0.21302	0.293	0.0	0.10397	0.143		
4	1.02048	-1.178	0.0	0.21780	0.295	0.0	0.12256	0.166		
5	1.01843	-1.171	0.0	0.06639	0.090	0.0	0.04279	0.058		
6	1.02717	-0.635	0.0	0.10963	0.147	0.0	0.06936	0.093		
7	1.02198	-0.837	0.0	0.03678	0.050	0.0	0.03678	0.050		
8	1.04287	-0.056	0.0	0.10205	0.135	0.0	0.04385	0.058		
9	1.04801	0.000	0.0	0.04663	0.061	0.0	0.01223	0.016		
10	1.06071	0.686							0.0	0.437
11	1.01990	9.509							0.4	0.030
12	1.03609	12.551							1.5	-0.021



The load shedding parameters associated with the 12-bus system for the transient response are as follows :

1. Relay set point = 49.5Hz
2. Step size = 5%
3. Number of steps = 20
4. Time delay between shedding steps = 0.02 second

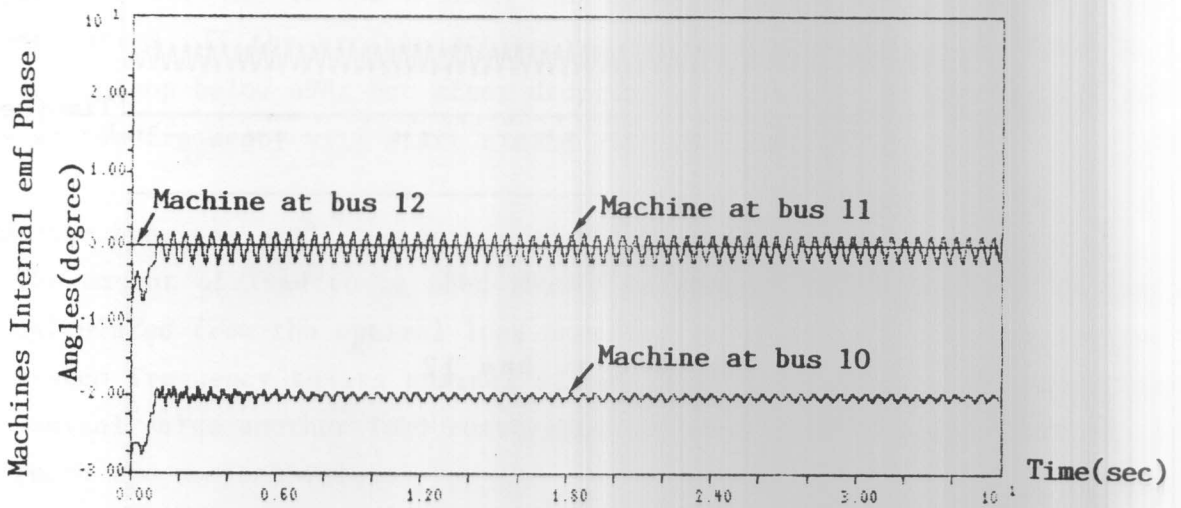
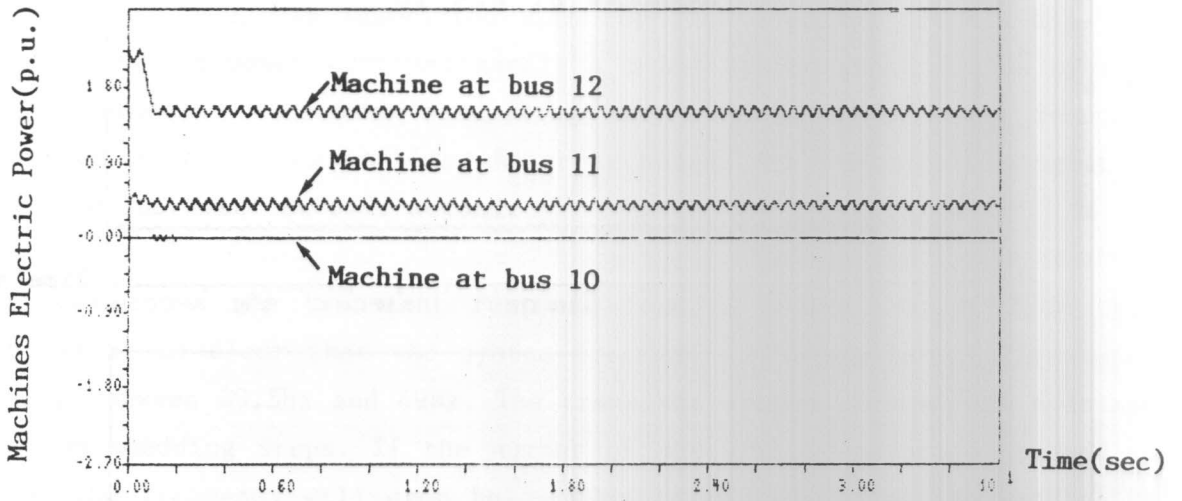
Table(12) shows shedding steps and their respective times in conjunction with the amount of load to be shed at each step for the 12-bus system.

Table(12): Shedding Process For The 12-Bus System

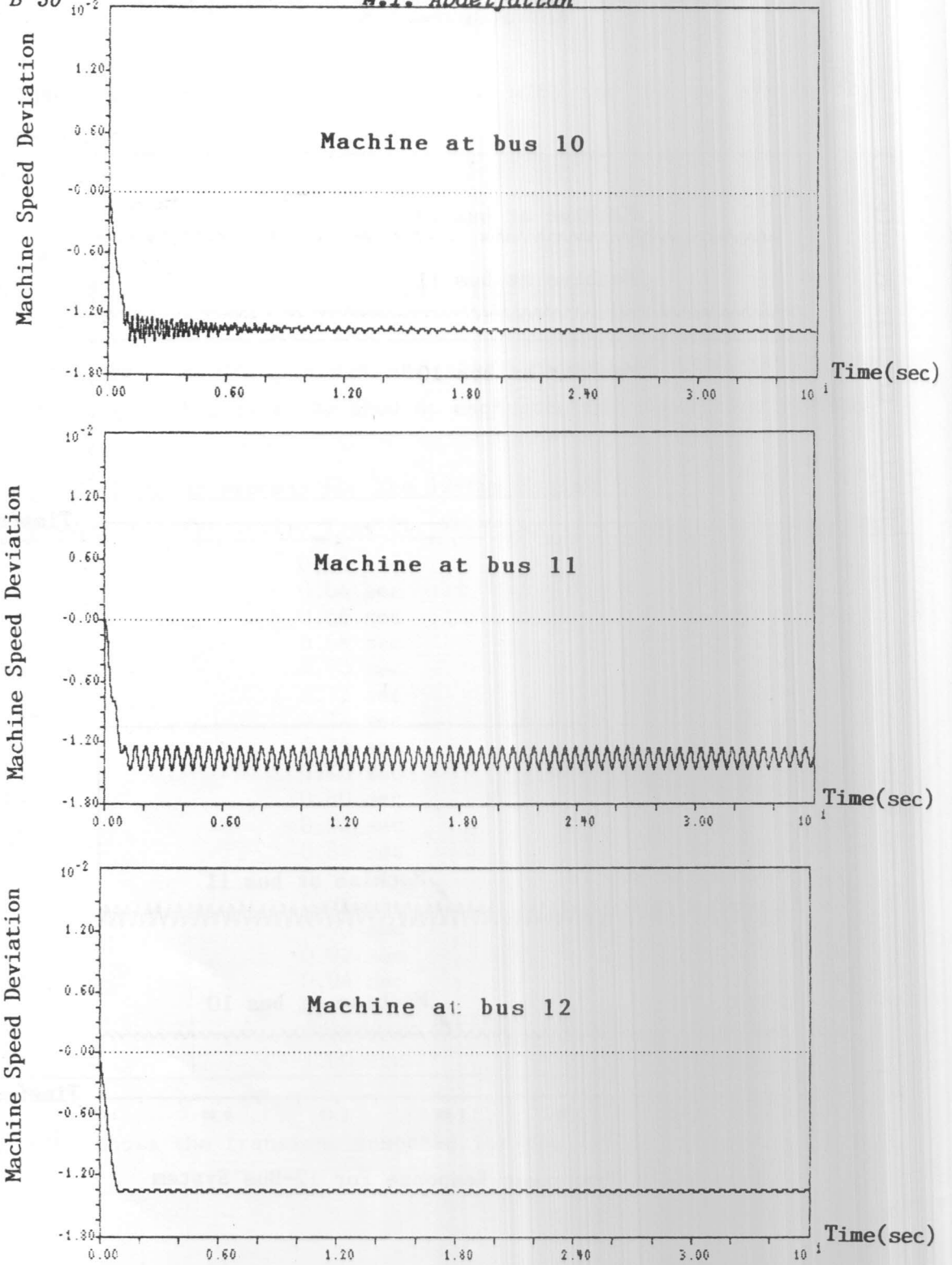
Step Number	Respective Time For Shedding	Amount Of Load Shed
1	0.62 sec	0.033184 p.u.
2	0.64 sec	0.033184 p.u.
3	0.66 sec	0.033184 p.u.
4	0.68 sec	0.033184 p.u.
5	0.70 sec	0.033184 p.u.
6	0.72 sec	0.033184 p.u.
7	0.74 sec	0.033184 p.u.
8	0.76 sec	0.033184 p.u.
9	0.78 sec	0.033184 p.u.
10	0.80 sec	0.033184 p.u.
11	0.82 sec	0.033184 p.u.
12	0.84 sec	0.033184 p.u.
13	0.86 sec	0.033184 p.u.
14	0.88 sec	0.033184 p.u.
15	0.90 sec	0.033184 p.u.
16	0.92 sec	0.033184 p.u.
17	0.94 sec	0.033184 p.u.
18	0.96 sec	0.033184 p.u.
19	0.98 sec	0.033184 p.u.
20	1.00 sec	0.033184 p.u.

Figure(6) shows the transient response for the 12-bus system.





Figure(6): Transient Response For 12-Bus System



Figure(6): Transient Response For 12-Bus System  
( continue)

## 5. Conclusions

The transient response associated with the load shedding process has been studied for two power systems; namely a 5-bus system and a 12-bus system. The differential equations describing the synchronous machines response have been solved using modified Euler technique for numerical integration with time increment  $\Delta t=0.02$  second.

Figure(4) shows the transient response for the 5-bus system. From this response it is clear that the system frequency will settle at a frequency ranging between 49.5Hz and 49Hz. The transient response shown was obtained with two shedding steps. If the number of shedding steps are increased to three, the frequency will drop below 49Hz. Also, if the exciter action is not included the system frequency will drop below 49Hz even if shedding is carried out in one step.

Figure(6), for the 12-bus system, was obtained with shedding steps equal twenty steps. If the exciter action was not included the system frequency will not drop below 49Hz but after dropping all loads to be shed at 1.0 sec the system frequency will start rising with no limit.

From the previous study two points should be emphasized :

1. The amount of load to be shed should be greater than the amount of load calculated from the optimal load shedding solution in order to raise the system frequency to its nominal value. After the system rises above the nominal value another load restoration process should be incorporated to reach the nominal value.
2. In order to keep the system stable another action should be considered. This is the exciter action. The internal emf's for synchronous machines should be reduced to achieve a stable operation.

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