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Reflection Coefficient of Neutrons from Double-Layered Media

by

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Abstract

The concept of the neutron reflection coefficient (Albedo) is recently used in the analysis of various practical applications in the fields of radiation shielding, radiation protection and neutron therapy. The importance of the Albedo concept lies in the fact that it can be used to greatly simplify neutron transport calculations or to aid in the design of the nuclear systems by considering the neutron reflecting properties of various materials. From the present study we can conclude that: *(i)* The depth dependence of the Albedo value in both the first and the second layers of a double-layered slab is strongly affected by the order of arranging the two layers; *(ii)* The difference in the Albedo values for single- and double-layered slabs is much greater for thin depths than that for deep depths in the slab; and *(iii)* The effect of the anisotropy on the Albedo value is the decrease of its rate by which the Albedo of the double-layered slab reaches the saturation value of the first layer.

Introduction

In many practical applications in the fields of radiation shielding, radiation protection and neutron therapy, multi-region systems are encountered quite often. In the calculations concerning these practical applications, treating the multi-layered system by the S_n- or the Monte Carlo-methods with the given interface and boundary conditions, is a tedious and a complicated problem.

An alternative method, which is much simpler and at the same time possesses high degree of accuracy, for treating these calculations is carried out using the Albedo concept. Information about the reflected neutrons from the multi-layered slabs are essential in many calculations of the above mentioned fields. Considering the characteristics of these reflected neutrons, the calculations of the neutron transport can be greatly simplified. The physical design of nuclear systems is also benefitted from knowing the reflected properties of neutrons from different media as used for the first wall and blanket of fusion devices, The treatment of the neutron transmission problems of complicated geometries could be simplified using the concept of Albedo. This could be accomplished in two steps. In the first step, calculation of the angular distributions and energy spectra of the reflected neutrons when the monoenergetic beam of neutrons is incident on the multi-layered medium at various angles of incidence and various neutron energies. In the second step, The original problem with its complicated geometry, in which the multi-layered medium would be replaced by a source having the previously determined reflecting properties, can be easily treated. Information or measurements on Albedos of double-layered slabs are scarce [1,2] compared to that of single-layered slabs [3-19]. The object of this study is to give numerical values for the Albedos for a large number of different double-layered media that are most important in the practical applications. In this study, the S_n-method is selected for solving the neutron transport in the double-layered medium.

Computational Methods

The Albedo is defined as the ratio of the radiation current reflected from a surface to the current incident upon that surface, mathematically,

$$R = \int_{-1}^{0} q(0,u)\, u\, du \tag{1}$$

where $q(0,u)$ = the angular flux at the surface.
u = the cosine of the scattering angle.

The range $-1<u<0$ corresponds to a backscattering of particles.

In realistic energy dependent problems, the scattering is invariably anisotropic. In order to obtain accurate solutions to these problems using one speed transport methods, the effect of the anisotropy must be introduced to get a comparatively accurate result. So, the one-speed transport equation takes the form,

$$u\, dq(x,u)/dz + q(x,u) = c \int_{0}^{2\pi} dt \int_{-1}^{+1} f(\hat{w} - w)\, q(r,\hat{u})\, d\hat{u} + Q(x,u) \tag{2}$$

where t is the azimuthal angle. The angular flux $q(x,u)$ and the source $Q(x,u)$ are assumed to be independent of the angle t, $f(\hat{w} - w)$ is a function of $w.\hat{w} = u_o$ only, where $\hat{w}$ and w are the neutron directions before and after scattering respectively. Consequently, $f(\hat{w} - w)$ may be expressed as the sum of Legendre polynomials, *i.e.*,

$$f(\hat{w} - w) = f(u_o) = \sum_{l=0}^{\infty} \{(2l+1)/4\pi\}\quad f_l\, P_l(u_o) \tag{3}$$

By the orthogonality of these polynomials, we get

$$f_l = 2\pi \int_{-1}^{+1} f(u_o)\, P_l(u_o)\, du_o,$$

with the normalization condition

$$f_o = 2\pi \int_{-1}^{+1} f(u_o)\, du_o,$$

For anisotropic scattering $l=0$, $P_o(u) = P_o(u_o) = 1$, and $f_o=1$. Considering the media in which the neutrons with a linear anisotropic scattering, then $l=1$, $P_l(u) = u$, $P_l(u_o) = u_o$ and $f_l = u_o$, where u_o is the average cosine of the scattering angle.

Isotropic Scattering

We consider the case where the neutron scattering in the media is isotropic. This case can represent a large number of materials. The main two parameters affecting the Albedo values are given below.

i) <u>The thickness of each layer</u>: In figure (1), the ordinate is the Albedo value and the abscissa is the thickness of the layers. For comparison, the variation of the Albedo value with the thickness in case of a single-layered medium is also drawn for the medium of the first layer and that for the medium of the second layer. It is observed that the Albedo of the double-layered medium increases with the increase of the thickness of the first layer and then reaches the saturation value of the first medium, and the rate of the increase of the Albedo decreases with the increase in the second layer thickness. This is because the back-scattering of the neutrons is higher for media of high c-value than of lower values.

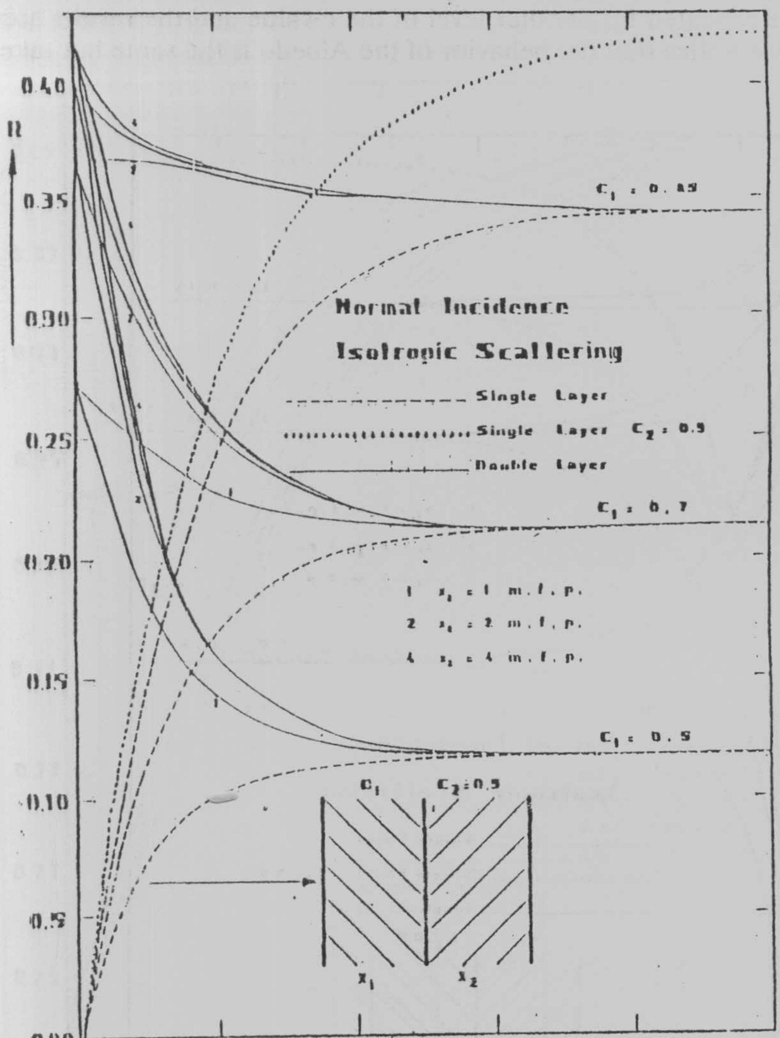

Fig. (1) Number Albedo calculated for: x-m.f.p. thick layer of c=0.9, c=0.85, c=0.7 or c=0.5 and a double-layered slab of "x-m.f.p. first layer of different c-values + x-m.f.p. second layer of c=0.9". Curve parameters are the thickness of the second layer and the c-value of the first layer.

ii) <u>The order of arrangement of the layers</u>: The effect of the order of arranging the two layers in the slab is considered. The results are shown in figures (1) and (2). The three families of curves for the case where the first layer has c-value of 0.5, 0.7 or 0.85 are all drawn in figure (1). Curve parameter in each family is the thickness of the second layer. It is observed that the Albedo value decreases with the increase of the thickness of the first layer until its value reaches the saturation value of the first layer. This can be explained due to the absorbing power of the first layer. It can be also noticed that the rate at which it reaches this saturation value decreases as the thickness of the second layer decreases. This rate also increases as the c-value of the layer decreases due to the increased absorption as the c-value decreases. The above calculations are repeated for another level of the c-value and the results are plotted in figure (2) and we notice that the behavior of the Albedo is the same but takes different values.

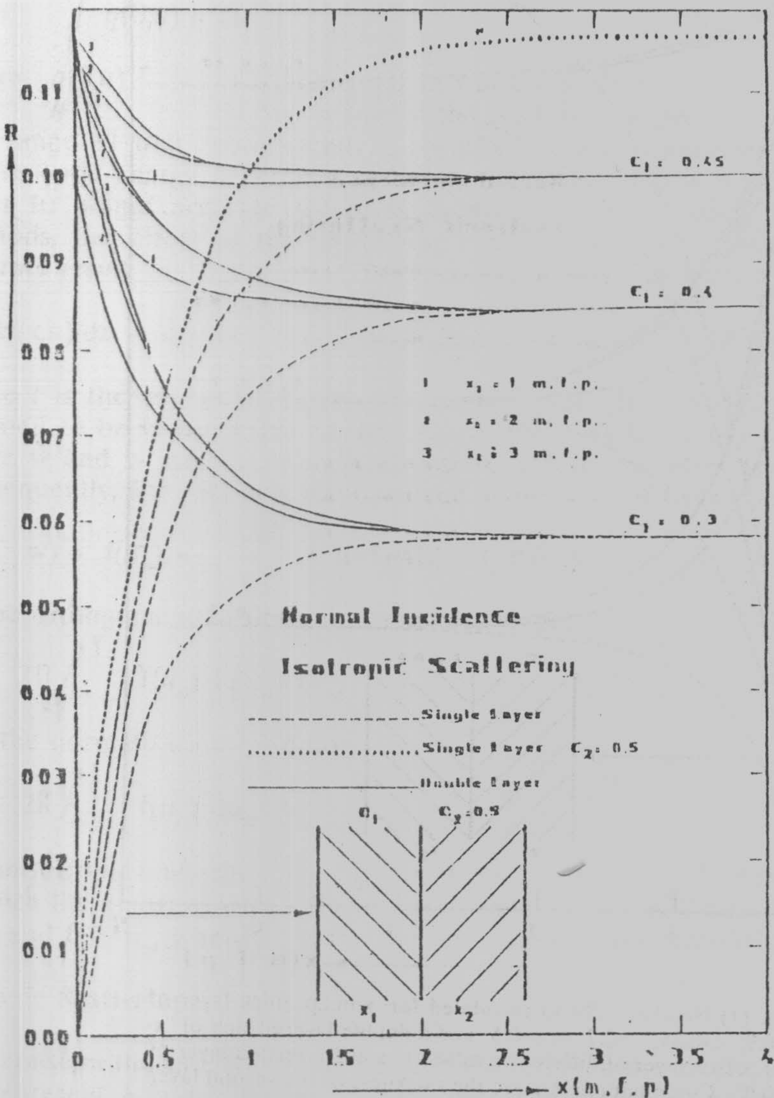

Fig. (2) Number Albedo calculated for: x-m.f.p. thick layer of c=0.5, c=0.45, c=0.4 or c=0.3 and a double-layered slab of "x-m.f.p. first layer of different c-values + x m.f.p. second layer of c=0.5". Curve parameters are the thickness of the second layer and the c-value of the first layer.

Anisotropic Scattering

The case of double-layered slab with a first layer of high c-value followed by another of relatively low c-value is considered. The Albedo values are calculated and given in figure (3). In this figure, the case where the first layer is made of medium characterized by a backward scattering (z=-0.3) followed by a second medium of forward scattering (z=+0.3) is represented. For comparison, the case where one of the two layers is characterized by isotropic scattering is plotted. Also, the case where the scattering is isotropic in both layers is plotted in the same figure. Two examples are considered; where the thickness of the first layer is relatively thin (x=1) and the second where the thickness is relatively thick (x=4).

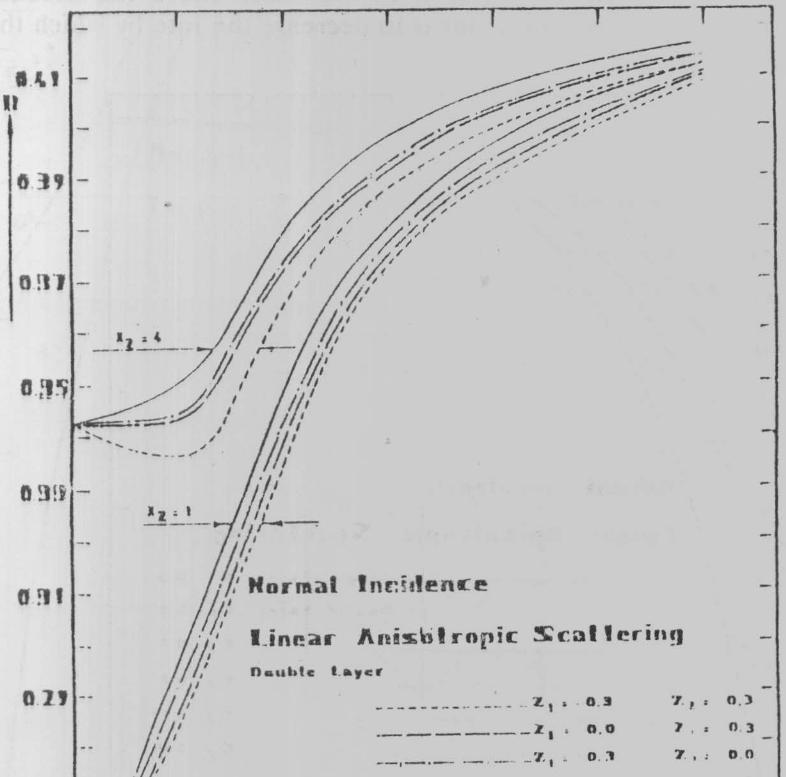

Fig. (3) Number Albedo calculated for a double-layered slab of "x-m.f.p. first layer of c=0.9 + x-m.f.p. second layer of c=0.85". The first layer is of a backward scattering medium of z=-0.3 and the second layer is of a forward scattering medium of z=+0.3. Curve parameter is the thickness of the second layer.

In the case, where the second layer is relatively thick (4 m.f.p.), it is observed that the Albedo starts from the value of the pure second layer then decreases reaching a minimum value then increases again monotonically until it reaches the saturation value of the first layer. The main contribution to Albedo in this case is due to those neutrons coming from the second layer and escaped collisions in the first layer. As the thickness of the first layer (which is a good back-scatterer) increases further, the beam of neutrons diverges within it and some will be backwardly scattered and hence contribute to the Albedo, and so the contribution to the Albedo at larger thicknesses of the first layer is not only due to those neutrons coming from the second layer and escaped collisions in the first but also due to those neutrons colliding within the first layer and being backwardly scattered. It should be noted that the depression is less when the scattering inside one of them is isotropic. In the case, where the second layer is relatively thin, the effect of the anisotropy is to decrease the rate by which the Albedo reaches its saturation value.

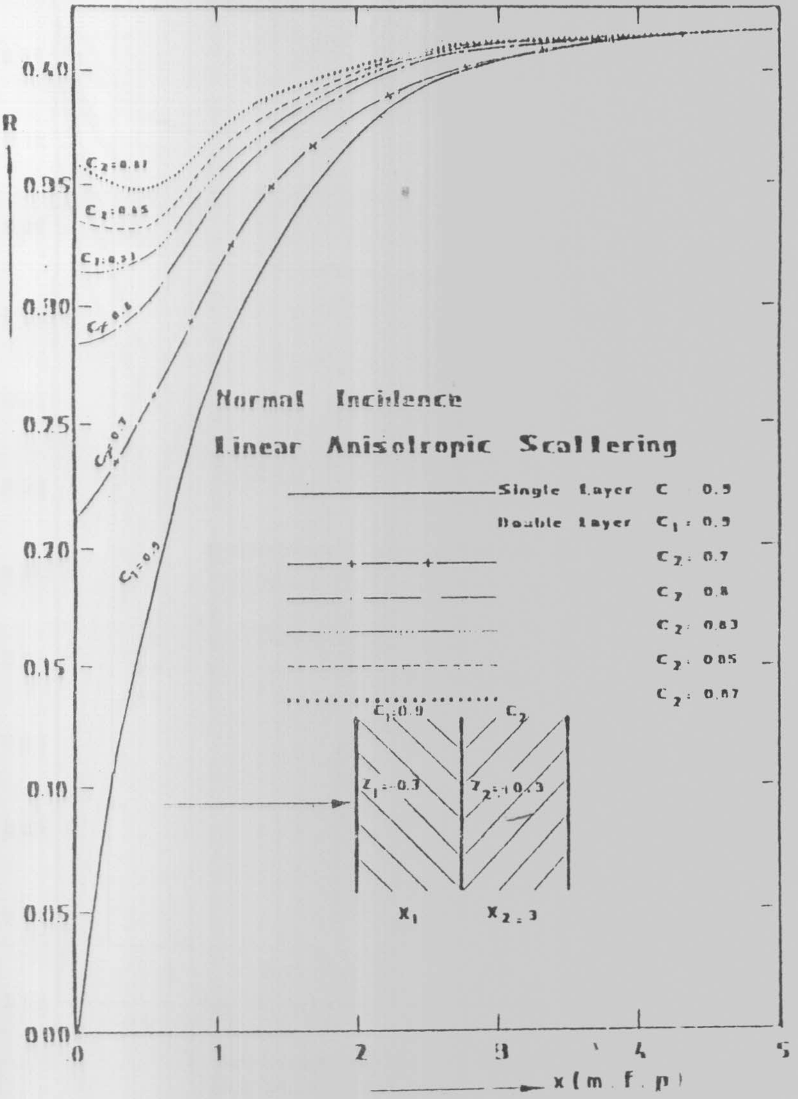

Fig. (4) Number Albedo calculated for a double-layered slab of "x-m.f.p. first layer of c=0.9 + 3-m.f.p. second layer of different c's". The first layer is of a backward scattering medium of z=-0.3 and the second layer is of a forward scattering medium of z=+0.3. Curve parameter is the thickness of the second layer.

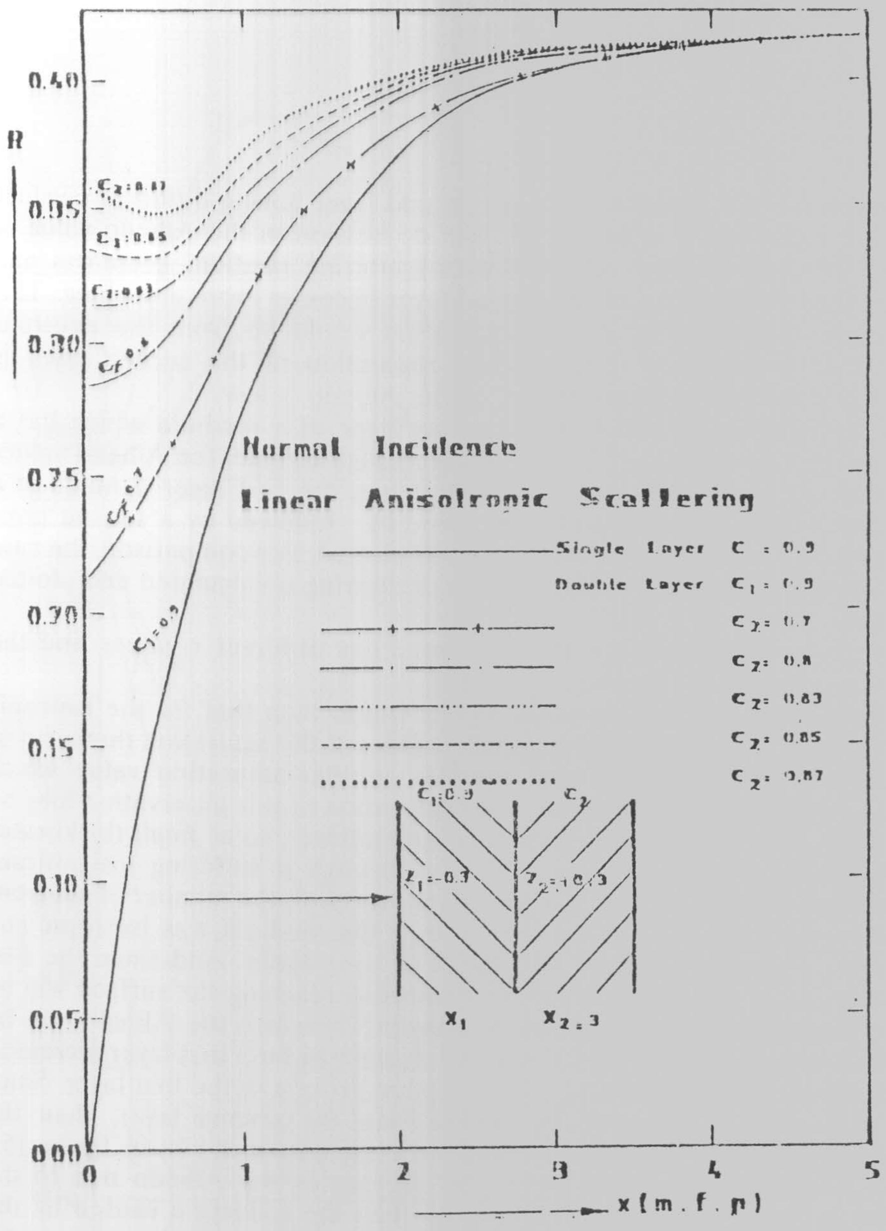

Fig. (4) Number Albedo calculated for a double-layered slab of "x-m.f.p. first layer of c=0.9 + 3-m.f.p. second layer of different c's". The first layer is of a backward scattering medium of z=-0.3 and the second layer is of a forward scattering medium of z=+0.3. Curve parameter is the thickness of the second layer.

The above calculations are repeated with the second layer having different c-values and the results are represented in figure (4). The depression in the Albedo value at small thicknesses of the first layer of good back-scattering medium decreases and begins to disappear as the medium of the second layer becomes more absorbing. This is expected since in this case the main contribution would be from the neutrons backwardly scattered from the first layer. The absorption of the second layer is becoming higher and the contribution from it to the Albedo is low.

In the case of a double-layered slab with its first layer of a medium which has a relatively low c-value followed by another of relatively high c-value, the Albedo values are calculated and are given in figure (5). In this figure, the first layer is made of a medium characterized by a forward scattering (z=+1.0) followed by a second layer which is of a backward scattering medium (z=-1.0) is plotted. For comparison, the case where the two layers are characterized by isotropic scattering is calculated and plotted in the same figure.

The calculations are repeated when the first layer has different c-values and the results are also represented in the same figure (5).

It can be concluded that the Albedo takes higher values than that for the isotropic case until the thickness of the first layer reaches or exceeds the saturation thickness of the isotropic case, then it decreases and assumes another saturation value which characterizes the medium of the first layer. The explanation of this observation may be put as follows. Since the first layer is a good forward scatterer, so at small thicknesses the normally incident beam of neutrons will pass through it suffering insignificant divergence. And since the second layer is a good back-scatterer, the number of neutrons backwardly scattered are greater than it would be if the medium was isotropic and hence more neutrons will be reflected into the first layer again. And since the first layer is a good forward scatterer the number of neutrons reaching the surface will be greater that it would be if the medium was isotropic. Therefore the Albedo will be higher than if the medium was isotropic. But as the thickness of the first layer increases, another process comes into play; namely the absorption process in the first layer. Since the first layer is higher in its absorbing power than the second layer, then the competition between the two processes accounts for the peaks shown in figure (5). Below a certain thickness of the first layer, the increase in the Albedo due to the anisotropy of the medium is dominant and higher than the reduction caused by the absorption process in the first layer. Above this thickness the reverse is true and the Albedo begins to decrease till it assumes the saturation value of the first medium. It is noted that the peak in the Albedo curve increases as the c-value of the first layer increases. This is expected since by increasing the c-value, the absorption inside the first layer becomes weaker, and the number of neutrons passing through it to the surface will be larger.

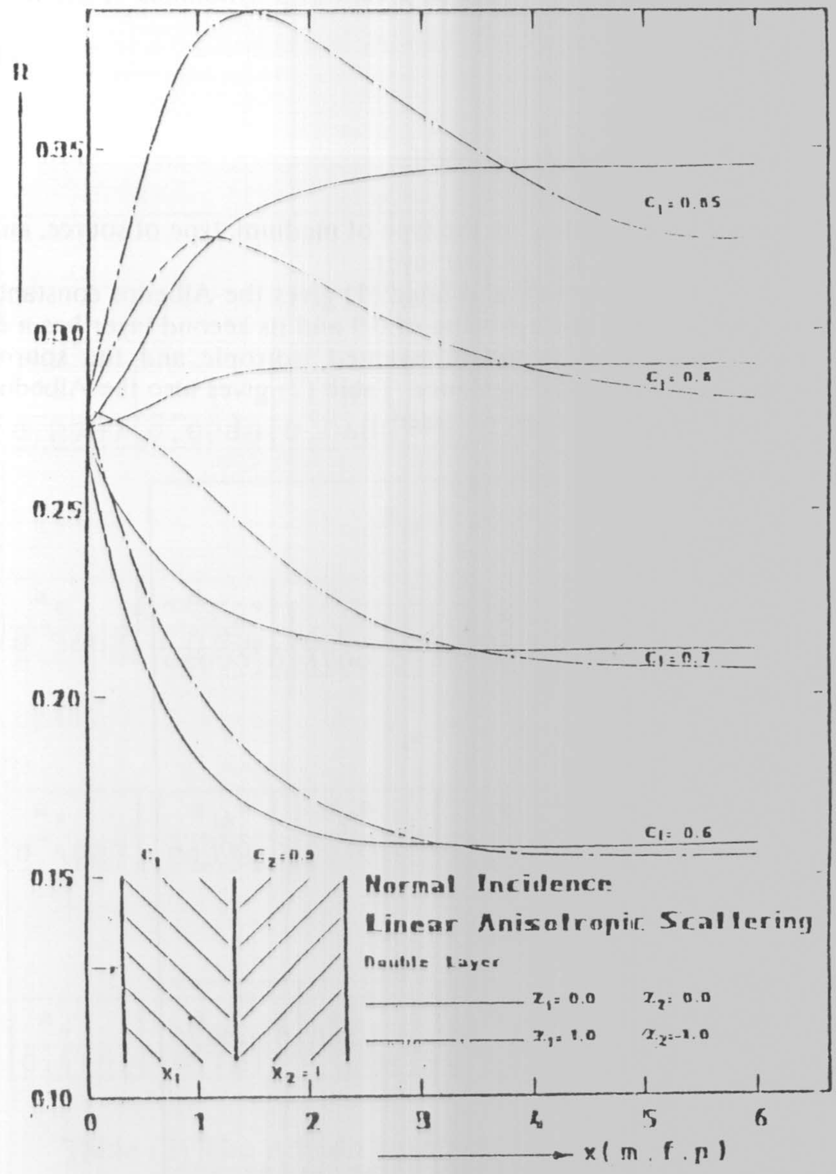

Fig. (5) Number Albedo calculated for a double-layered slab of "x-m.f.p. first layer of different c's with z=+1 +1 m.f.p. second layer c=0.9 with z=-1". For comparison the case of isotropic scattering in both media is also plotted. Curve parameter is the c-value of the first layer.

Analytical Formula for the Albedo

It is convenient to express the Albedo's values as a sum of polynomials of the form

$$R(x) = \sum_{n=0}^{N} a_n x^n \tag{4}$$

where $R(x)$ = the Albedo value.
x = the thickness of the first layer.

a_n = a constant depending on the type of medium, type of source, and the thickness of the second layer.

The summation has being truncated at N=6 and table (1) gives the Albedos constants of a double-layered slab its first layer has a c-value of 0.9 and its second layer has a c-value of 0.85. The scattering in both layers is assumed isotropic and the source configuration is isotropic planar of normal incidence. Table (2) gives also the Albedos constants, when the above arrangements are reversed.

$x_2 = 1$ m.f.p						
a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.2420	0.0846	0.0943	-0.1086	0.0429	-0.0076	0.00050
$x_2 = 2$ m.f.p						
a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.3145	-0.017	0.1817	-0.1567	0.0581	-0.0100	0.00066
$x_2 = 3$ m.f.p						
a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.3354	-0.048	0.2088	-0.1717	0.0628	-0.0108	0.00070
$x_2 = 4$ m.f.p						
a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.3428	-0.057	0.2167	-0.1759	0.0649	-0.0110	0.00072

Table (1) The Albedo constants for a double-layered slab of "x-m.f.p, first layer of c=0.9 +1, 2, 3 or 4 m.f.p. second layer of c=0.85". The source is anisotropic planar source of normal incidence and the scattering inside both layers is isotropic.

x_2 = 1 m.f.p

a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.2727	0.0288	0.0680	-0.0732	0.0291	-0.0052	0.00034

x_2 = 2 m.f.p

a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.3682	0.0128	-0.044	0.0314	-0.011	0.0017	-0.0001

x_2 = 3 m.f.p

a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.4027	-0.110	0.1338	-0.0901	0.0304	-0.0053	0.00035

x_2 = 4 m.f.p

a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.4151	-0.151	0.1934	-0.1311	0.0456	-0.0077	0.00050

Table (2) The Albedo constants for a double-layered s ab of "x-m.f.p. first layer of c=0.85 +1, 2, 3 or 4 m.f.p. second layer of c=0.9". The source is anisotropic planar source of normal incidence and the scattering inside both layers is isotropic.

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