Alexandria Eng., J., Alex. Univ., Volume 28 No.4, pp 543- 553 (1989)

ON THE VALUE AND STABILITY OF SOME EXACT STEADY STATE SOLUTION OF SOME PROBLEMS IN THE TRANSIENT MICROWAVE HEATING OF IONIZED MEDIA

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Abstract

The value and stability of the exact steady state solution of the average electron energy balance equation are investigated for three different kinds of homogenuous plasmas heated by a microwave field. These are, a weakly ionized plasma, a strongly ionized plasma and a hydrogen plasma in which the collision cross section is approximated by an analytical expression which fits experimental data. The effect of the field frequency and field amplitude on the value and stability of the steady state average electron energy as well as its limiting values are investigated for the three cases considered.

1. Introduction

A plasma is a medium subject to different kinds of instabilities. Terminology and nomenclature of plasma instabilities are numerous and confusing. In this paper, we investigate the stability and the value of the steady state average electron energy of 3 different kinds of homogenuous plasmas heated by a microwave field. These are, a weakly ionized plasma, a strongly ionized plasma and an atomic hydrogen plasma in which the collision cross section of electrons is approximated by an analytical expression which fits experimental data. We assumed that all electron generation and loss processes are neglected and that electron collisions are elastic. The fraction of electron energy lost per collision is considered to be constant. For the power absorbed by an average electron from the field, we used an average value. These assumptions were considered by several investigators to study different phenomena either exactly or numerically. Ginzburg [1] studied the steady-state characteristics of the interaction of a homogenuous plasma and a strong uniform electric field. Numerical solutions of the non linear model of this interaction for the build-up of the electron energy and electron density when a step sinusoidal high power RF field is applied to a weakly ionized medium was investigated by several investigators [2], [3], [4]. Also the same assumptions were used to study the third harmonic generation of electromagnetic waves in weakly and strongly ionized homogenuous collisional plasmas [5]. Also, in the study of the number of the steady state values of the average electron energy of a homogenuous strongly ionized plasmas heated by a microwave field, the same assumptions had been considered [6]. In this paper we have investigated the effect of the field frequency and field amplitude on the value and stability of the steady state average electron energy of the 3 different kinds of homogenuous plasmas mentioned previously. The effect of the field amplitude is found to be the same on the 3 kinds of plasmas studied. However, the field frequency had proved to have a different effect on the different kinds of plasmas studied. Meanwhile, we have found that the 3 kinds of plasmas had behaved in the same way when the field frequency is increased without limit.

In section 2, we have presented the general basic equations. In sections 3, 4 and 5 we had presented the equations as well as the results of the investigation of the first, second and third kind of plasma studied respectively. Discussion and comparison of the results of sections 3, 4 and 5 are given in section 6.

2. Basic Equations

We assumed that a step sinusoidal microwave field $E(t) = E_o \sin wt$ is applied to an ionized medium at the instant t=0. The magnitude of the amplitude of the field E_o is large enough to cause appreciable heating of the electrons but is not so large to make changes in the electron density of the medium. Also all processes of electron losses are negelcted. Under these simplified assumptions, the response of the electrons to this field is governed by the equations of motion and temperature [1], [5].

$$\frac{dv}{dt} = \frac{-eE}{m} - \nu_c v \qquad (1)$$

$$\frac{3}{2}\frac{dT_e}{dt} = - eEv - \delta\nu_c \frac{3}{2}(T_e - T_o) \qquad (2)$$

where T_o is the temperature of the heavy particle .

assume to be constant, T_e is the average temperature of the electrons at time t, at t = 0, T_e is assumed to be equal to T_o, e, m, v, and ν_c are the electron charge, mass, velocity and collision frequency respectively, δ is the fraction of electron energy lost in a collision with the heavy particle. For an elastic collision, δ is contant and equal to $\frac{2m}{M}$ where M is the mass of the heavy particle [1], [5], [7]

The first term on right hand side of equation (2) is the power absorbed by an average electron from the field $P_a(t)$. By assuming ν_c to be constant over the period of the microwave field, $P_a(t)$ has been found to have the value [4].

$$P_a(t) = \frac{e^2 E_o^2}{2m \nu_c \left[1 + \frac{w^2}{\nu_c^2} \right]} \qquad (3)$$

This is the same as one uses the classical effective field definition and considers ν_c to be slowly varying function of time. Using this value for $P_a(t)$, equation (2) becomes autonomous and can be solved either analytically or numerically once the dependence of ν_c on T_e is known.

3. Values and Stability of the Steady State Average Electron Temperature T_e of a homogenuous weakly ionized plasma

In a weakly ionized plasma, electron collision is mainly with atoms assuming a maxwellian distribution for the electron velocity, the dependence of ν_c on T_e is given by [1], [5], [7]

$$\nu_c = \nu_{co} \cdot \left(\frac{T_e}{T_o} \right)^{\frac{1}{2}} \qquad (4)$$

Substituting from (4) and (3) in (2) one obtains

$$\frac{d\theta}{d\tau} = \frac{\eta^2 (1+c^2)^{\frac{1}{2}} \theta}{(\theta + c^2)} - (\theta^{3/2} - \theta^{1/2}) \qquad (5)$$

where θ is the normalized electron Temperature $\frac{T_e}{T_o}$, τ is the normalized time = $\delta \nu_{co} t$, c is the normalized frequency = w / ν_{co} and η is the normalized field amplitude = E_o/E_p where E_p is Ginzburg's plasma field given by

$$E_p = [3m \delta \nu_{co}^2 T_o (1+c^2)/e^2]^{\frac{1}{2}}$$

The steady state value (θ_{ss}) of θ is easily found to be [7], [8].

$$\theta_{ss} = \tfrac{1}{2} (1-c^2) + \tfrac{1}{2} [(1-c^2)^2 + 4(\eta^2 + c^2 (1+\eta^2))]^{\frac{1}{2}}$$

To find the range of values of c and η for which (θ_{ss}) is stable, we made use of the linearlized theory for analysis of equilibria [9]. This consists in rewriting equation (5) in the form

$$\dot{\theta} = f(\theta) \qquad (7)$$

where $\dot{\theta} = d\theta/d\tau$ and $f(\theta)$ = right hand side of equation (5) and then we have to find the zeros of the characteristic equation [9].

$$(\lambda - f'(\theta))^2 = 0 \tag{8}$$

from which

$$\lambda = f'(\theta)\Big|_{\theta=\theta_{ss}} \tag{9}$$

Since λ is real, hence θ_{ss} is non oscillatory. Also θ_{ss} is a stable node if $\lambda < 0$ and an unstable node of $\lambda > 0$ [10]. The sign of λ is easily seen to be the sign of the quantity $1-(c^2 + 2\theta_{ss})$ since θ_{ss} is > 1 hence λ is always -ve which means that θ_{ss} is stable for all values of c and η. Fig. (1) shows the variation of θ_{ss} with η for three different values of c. It is obvious that θ_{ss} increases with the increase of c and η. No heating occurs for values of $\eta < 1$. Fig. (2) shows the variation of θ_{ss} with c for different values of η. It is easily noticed that the variation of θ_{ss} with c is negligible when c is < 1 and also when c is >>1 i.e θ_{ss} varies with c in a limited range. Also it is obvious that the length of this range increase with the increase of η.

4. Values and stability of the steady state average electron temperature T_e of a homogenuous strongly ionized plasma

For a strongly ionized plasma, electron collision is mainly with ions, the dependence of the frequency of collision of electrons with the ions ν_c on the electron temperature T_e is given by [11]

$$\nu_c = \nu_{co} \left(\frac{T_e}{T_o}\right)^{-3/2} \tag{10}$$

Substituting from (3) and (10) in (2) one obtains

$$\frac{d\theta}{d\tau} = \frac{\left[c^2\theta^4 - (\Gamma^2 + c^2)\theta^3 + \theta - 1\right]}{\theta^{3/2}(1+c^2\theta^3)} \qquad (11)$$

where $\Gamma = \eta(1+c^2)^{\frac{1}{2}}$

The exact solution of equation (11) has been found by El-Khoga and Negm [12] and the variation of θ with τ has been calculated for diffeɛ ən values of c and η. Also, the number N of the steady state values of θ as well as the steady state values of θ have been investigated in [6] and [13] respectively.

As the case of a weakly ionized plasma, the zeros of the characteristic equation that determine the stability criterion are obtained from equation (8) but here f(θ) is the right hand side of equation (11). λ is found to have the value

$$\lambda = \frac{[3(\Gamma^2 + c^2)\theta_{ss}^2 - 4c^2\theta_{ss}^3 - 1]}{\theta_{ss}^{3/2}(1+c^2\theta_{ss}^3)} \qquad (12)$$

It is easily seen that the sign of λ is the same as the sign of the numerator of this equation.

When N = 1 [6], λ is -ve and hence θ_{ss} is stable. Fig. (3) shows the variation of θ_{ss} with η at different values of c. In contrary with the behavior of a weakly ionized plasma, θ_{ss} decreases with the increase of c. from Fig. (4) we see that this decrease occurs for low values of c. From Fig. (4), we can notice also that there is a value c_{cri} of c above which the increase of c has no effect on θ_{ss}, we can also notice that the value of c_{cri} increases with the increase of η. Negm and El-Khoga [6] found that when c and η are both < 1, N may be equal to 3 and we had found that not all the three values of θ_{ss} are stable. We had found that if θ_1, θ_2 and θ_3 are the three steady state values of θ such that $\theta_1 < \theta_2 < \theta_3$ then both θ_1 and θ_3 are stable but θ_2 is unstable. Fig. (5) shows the variation of the two stable steady state average normalized electron temperature θ_1 and θ_3 and the unstable one θ_2 with η when c = 0.01. The results presented in Figs (6) and (7) are the same as those of Fig. (5) but for c = 0.1 and 0.2 respectively. From these Figures, both θ_1 and θ_3 increase with the increase of η and the decrease of c, meanwhile θ_2 has a different behaviour, it decreases with the increase of η and the decrease of c.

5. Values and stability of the steady state average electron temperature of a hydrogen plasma collision cross section is approximated from experimental results

However when the plasma is neither weakly nor strongly ionized, the theoritical calculation of ν_c is very complicated.

Usually, for this case ν_c is estimated from experimental measurements of the value of the collision cross section σ_c. The experimental results of Brackman and Fite [16] for the dependence of σ_c on the electron energy in an atomic hydrogen plasma can be approximated by a simple relation in which σ is inversely proportional the energy of electron (16). The Corresponding dependence of ν_c on T_e wes than found to be [16]

$$\nu_c = \nu_{co} \left(\frac{T_e}{T_o}\right)^{-\frac{1}{2}} \qquad (14)$$

For this case we have to use the equations (14), (3) and (2) to get the energy balance equation. This has the form

$$\frac{d\theta}{d\tau} = - \frac{[c^2 \theta^{3/2} + (1-\Gamma^2 - c^2)\theta^{\frac{1}{2}} \theta^{-\frac{1}{2}}]}{(1 + c^2\theta)} \qquad (15)$$

For this case N = 1 and θ_{ss} has the value

$$\theta_{ss} = \frac{1}{2c^2} \{(\Gamma^2 + c^2 - 1) + [(\Gamma^2 + c^2 - 1)^2 + 4c^2]^{\frac{1}{2}}\} \qquad (16)$$

Fig. (8) shows the variation of θ_{ss} with η at different values of c. We have found that the differences between the two curves of c=1 and c=4 are too small. Thus the variation of the frequency has a smaller effect on the heating process of this plasma so long as c is > 1. When c is < 1, θ_{ss} decreases with the increase of c. This result is presented in Figure (9).

6. Discussion and Conclusion

Comparing figures (2), (4) and (9) we can conclude the increase of c above a critical value has no effect on the value of θ_{ss} for the three kinds of plasmas studied here. This critical value of c for any of the three kinds of plasmas depends on the value of . Also, it is simply found from equations (6), (13) and (16) that for the three kinds of plasmas studied

$$\lim_{c \to \infty} \theta_{ss} = 1 + \frac{2}{\eta} \qquad (17)$$

Equation (17) agrees with Ginzburg's equation for high frequency heating of a fully ionized medium. Thus we proved that this equation is to be satisfied also for high frequency heating of a weakly ionized medium and a medium in which the dependence of σ_c on the electron enery fits experimental data. Also we can conclude that the stability of the heating process by a microwave field is ensured for the first and third kind of plasma studied here. The stability of the heating process of the strongly ionized plasma is satisfied when N = 1. The case in which N $\neq$ 1 may occur only when c is < 1, thus the neccessary and suffecient condition for N to be equal to 1 when c is < 1 found by Negm and El-Khoga [6] can be considered also as a necessary and suffecient condition for the stability of θ_{ss} when c < 1. This condition has the value [6]

$$\eta < \eta_{cri} = \frac{c^{2/3}(1-c^{2/3})^{\frac{1}{2}}}{(1+c^{2})^{\frac{1}{2}}} \qquad (18)$$

Also when c is < 1, the necessary bu not suffecient condition for N to be equal to 3 found by the same authors and given by

$$\eta > \eta_{cri} \qquad (19)$$

can be considered as a necessary but not suffecient condition for the instability of the steady state electron temperature of a strongly ionized homogenuous plasma heated by a microwave field.

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