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APPLICATION OF PHASE CG ORDINATES REFERENCE FRAME IN FAULTY LEVEL CALCULATION
PART 2 NUMERICAL EXAMPLES

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Abstract

The fault analysis of polyphase power networks by means of phase co ordinates method has been presented with employed in success. The technique is also applied to solve practical examples for simultaneous faults.

1. Introduction

The fault analysis of power system polyphase networks under fault conditions is made in terms of phase co-ordinates representation. Using the system representation in the phase frame of reference given [1,2,11], the generalized analysis of polyphase networks under fault conditions can be widely used.

An example previously presented in [2] is solved again in this paper, considering the vector group of power transforeer.

Analysis of some unbalanced faults single phase to earth, simultaneous fault and other types on a part of Alexandria Electric Power Network is carried out in this paper using the elegant phase co ordinates method.

Example 1

The following simple example was illustrated in [2]

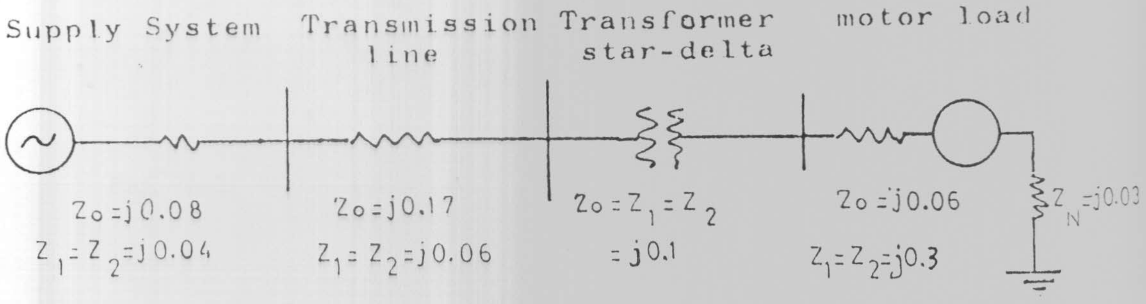

Fig. (1) Power System Example

The system shown schematically in Fig (1) constitutes star induction motor load, star-delta transformer, transmission line, and supply system. The impedances are expressed in terms of the symmetrical components zero, positive and negative sequence per unit values.

The sample system shown in Fig. (1) is drawn schematically as shown in Fig. (2) to illustrate the node-numbering sequence used.

The admittance matrices [1] of all elements are also plotted.

From the sequence impedances given, the 3-phase admittance matrices can be found from the relationship.

$$Y_{phase} = 1/3\ TY_{012}T^{*}$$

Where

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \partial^2 & \partial \\ 1 & \partial & \partial \end{bmatrix} \qquad \begin{aligned} \partial &= 1 \angle 120 \\ \partial^2 &= 1 \angle 240 \end{aligned}$$

Thus for the supply system

$$\begin{bmatrix} Y_s \end{bmatrix} = \begin{array}{c|ccc} & 7 & 8 & 9 \\ \hline 7 & -j20.833 & j4.166 & j4.166 \\ 8 & j4.166 & -j20.833 & j4.166 \\ 9 & j4.166 & j4.166 & -j20.833 \end{array}$$

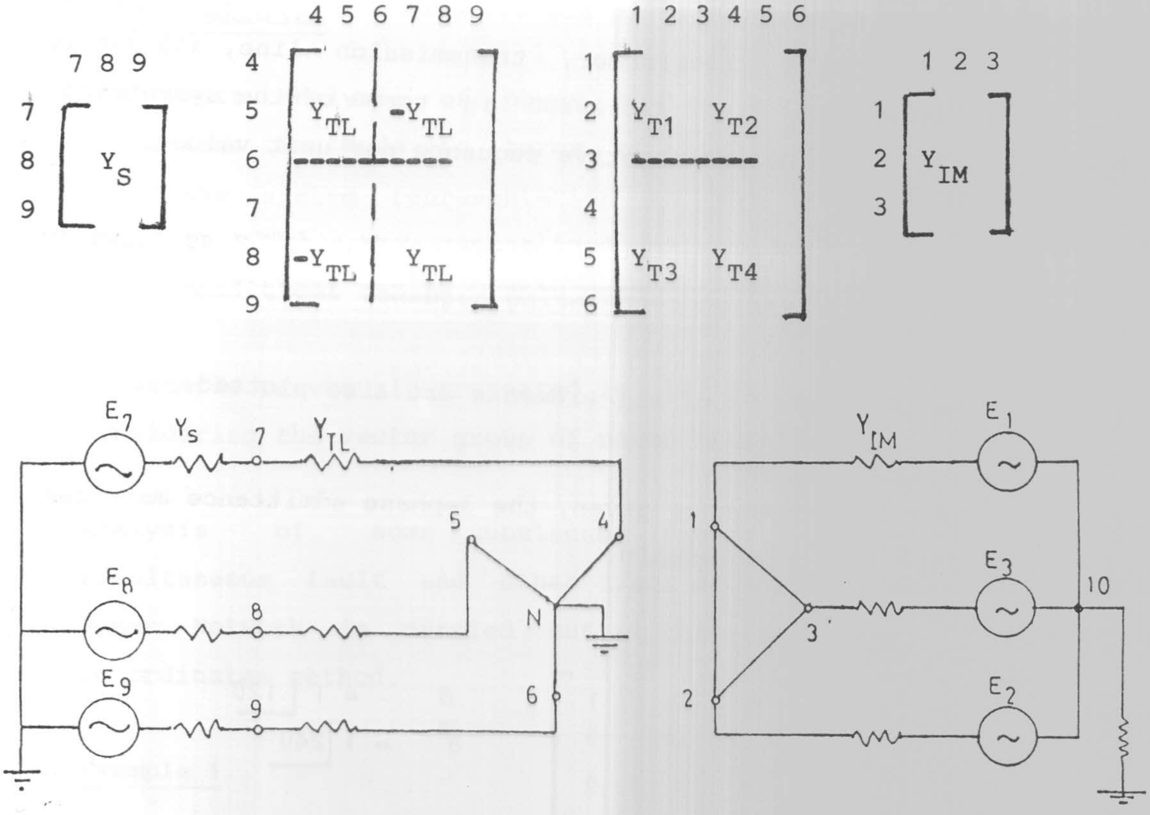

Fig. (2) Schematic representation of sample system showing phase busbar numbering sequence and admittance matrices of all elements.

and

$$\left[Y_{TL} \right] = \begin{array}{c|ccc|} & 4 & 5 & 6 \\ 4 & -j13.071 & j3.595 & j3.595 \\ 5 & j3.595 & -j13.071 & j3.595 \\ 6 & j3.595 & j3.595 & -j13.071 \end{array}$$

$$
\begin{bmatrix} Y_{IM} \end{bmatrix} = \begin{array}{c|ccc|}
 & 1 & 2 & 3 \\
1 & -j7.777 & -j4.444 & -j4.444 \\
2 & -j4.444 & -j7.777 & -j4.444 \\
3 & -j4.444 & -j4.444 & -j7.777
\end{array}
$$

From Fig. (2) the admittance matrix Y of the system is given by equation:

$$
\begin{bmatrix} Y \end{bmatrix} = \begin{array}{c|ccc|ccc|ccc|c|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & & & & & & & & & & -y_o(IM) \\
2 & & Y_{IM} + Y_{T1} & & & Y_{T2} & & & & & -y_o(IM) \\
3 & & & & & & & & & & -y_o(IM) \\
\hline
4 & & & & & & & & & & \\
5 & & & & & & & & & & \\
6 & & Y_{T3} & & & Y_{TL} + Y_{T4} & & & -Y_{TL} & & \\
\hline
7 & & & & & & & & & & \\
8 & & & & & -Y_{T1} & & & Y_s + Y_{TL} & & \\
9 & & & & & & & & & & \\
\hline
10 & -y_o(IM) & -y_o(IM) & -y_o(IM) & & & & & & & 3y_o(IM) + y_{10,0}
\end{array}
$$

where

$y_o(IM)$ is the zero sequence admittance of the induction motor,

$y_{10,10}$ is the admittance through carthed neutral of induction motor,

Y_{T1}, Y_{T2}, Y_{T3}, Y_{T4} are the submatrices of the transformer admittance matrix, Y_{TR}, depend on the type of vector group of transformer. Matrices of Y_{TR} are illustrated in table (1), [11].

Solutions are obtained for the above example considering three types of star-delta transformers: Yd1, Yd3, Yd9.

In Appendix 1 the admittance and impedance matrices of these type of star-delta transformers are plotted.

The following results are obtained:

1. All diagonal elements are the same, i.e for all types of vector groups the short circuit current for single-phase, two-phase or three-phase short circuit are the same.
2. The elements of submatrices

$$\begin{array}{cc} & \begin{array}{ccc}4&5&6\end{array} \\ \begin{array}{c}1\\2\\3\end{array} & \left[\begin{array}{ccc} & & \\ & Y & \\ & & \end{array}\right]\end{array}, \quad \begin{array}{cc} & \begin{array}{ccc}1&2&3\end{array} \\ \begin{array}{c}4\\5\\6\end{array} & \left[\begin{array}{ccc} & & \\ & Y & \\ & & \end{array}\right]\end{array} \quad \begin{array}{cc} & \begin{array}{ccc}4&5&6\end{array} \\ \begin{array}{c}1\\2\\3\end{array} & \left[\begin{array}{ccc} & & \\ & Z & \\ & & \end{array}\right]\end{array}, \quad \begin{array}{cc} & \begin{array}{ccc}1&2&3\end{array} \\ \begin{array}{c}4\\5\\6\end{array} & \left[\begin{array}{ccc} & & \\ & Z & \\ & & \end{array}\right]\end{array}, \quad \begin{array}{cc} & \begin{array}{ccc}1&2&3\end{array} \\ \begin{array}{c}7\\8\\9\end{array} & \left[\begin{array}{ccc} & & \\ & Z & \\ & & \end{array}\right]\end{array}$$

and $\begin{array}{cc} & \begin{array}{ccc}7&8&9\end{array} \\ \begin{array}{c}1\\2\\3\end{array} & \left[\begin{array}{ccc} & & \\ & Z & \\ & & \end{array}\right]\end{array}$ are differents, then for simultaneous

Faults (i.e. 3-phase short circuit between 1,2,3 and sangle-phase at 6) the short circuit currents in the elements are different.

Fig. (3), Fig. (4) and Fig. (5) are star-delta transformers under simultaneous fault conditions for Yd3, Yd9 and Yd1.

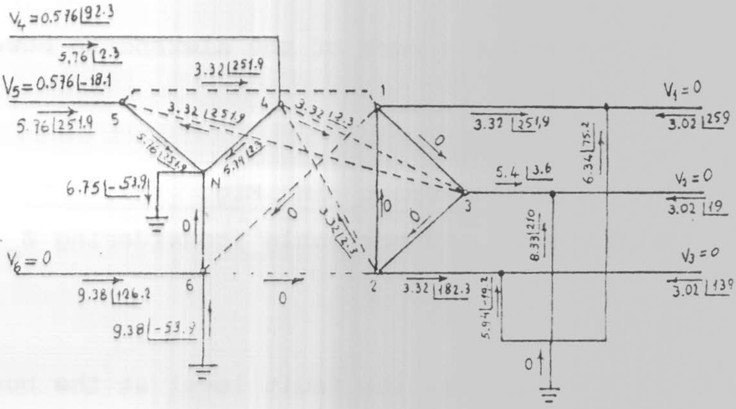

Fig.(3) Yd3 Transformer under simultaneous fault condition.

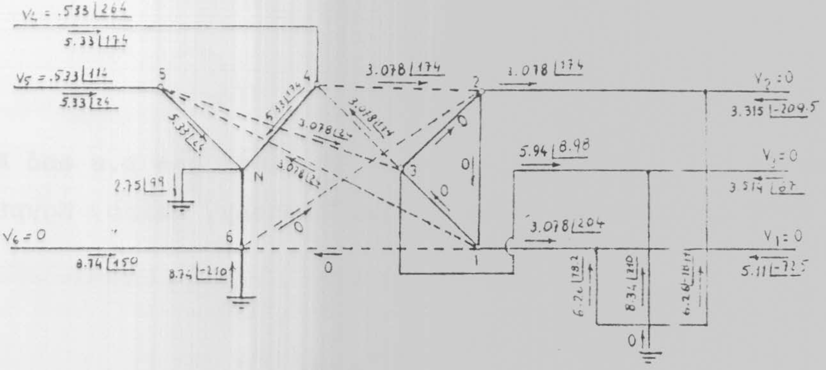

Fig.(4) Yd9 transformer under simultaneous fault condition.

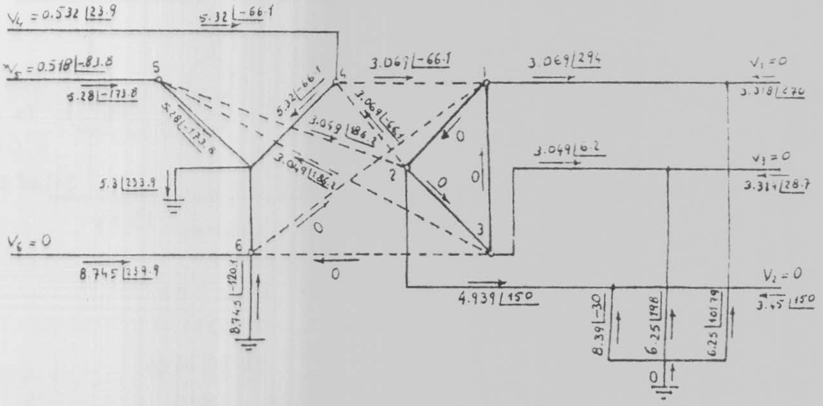

Fig.(5) Yd1 transformer simultaneous fault condition.

Example 2

The following example represents a part of the Alexandria Power System (Saouf Thermal Power Station).

The system is shown schematicallq in Fig. (6), it comprises 4-Generators, 8 Transformers, and one cable (considering $Z_o = Z_1$ for generators).

Source impedance Z_s is a measure of the fault level at the bus bar and defined as:

$$Z_s = \frac{MVA_{s.c}}{(KV)_b^2}$$

The values of MVA_{sc} for 1-ph and 3-ph at Siouf gas p.s and Abis s.s are taken from Engineering Power System Company, Cairo, Egypt.

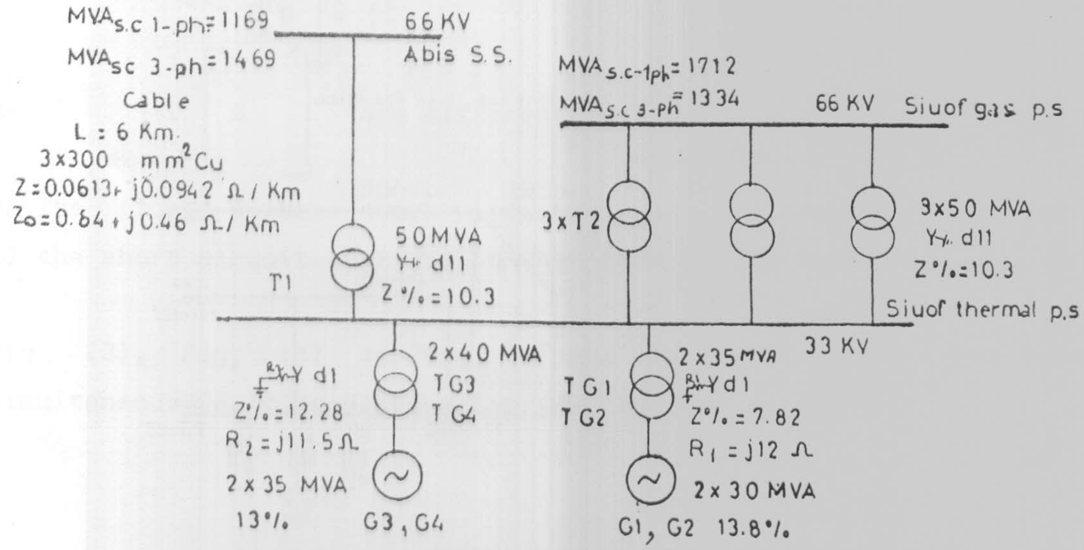

Fig. (6) Part of Alexandria Power System

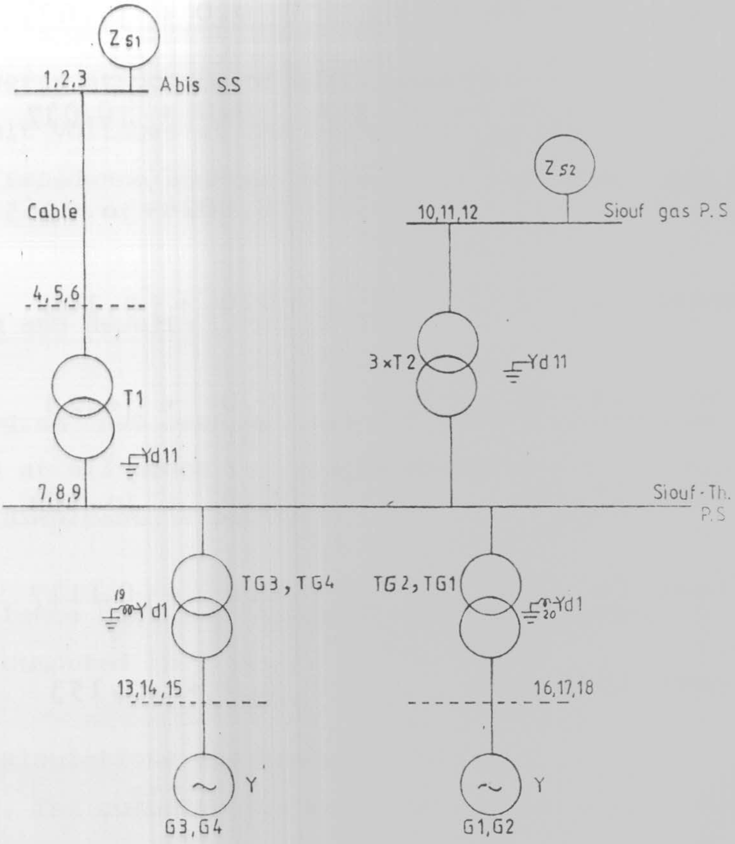

Fig.(7) EXAMPLE 2 SHOWING NUMBERING FOR PHASE-COORDINATE POLYPHASE.

Table (1)

element	$Z_1 = Z_2$	Zo
Source impedance at Abis	0.0 + jo.034	0.0 + jo.06
" " at Siouf gas	0.0 + j0.037	0.0 + jo.0126
Cable	0.0024+jo.0065	0.0579+jo.0317
Transformer T1, T2	0.0 +j 0.103	0.0 + jo.103
Generator G1, G2	0.0 + jo.23	0.0 + jo.23
Generator G3, G4	0.0 + jo.184	0.0 + jo.184
Transformer TG1,TG2	0.0 +j 0.1117	0.0 + jo.1117
Transformer TG3, TG4	0.0 + j0.153	0.0 + jo.153
Reactor R_1	----	0.0 + j0.551
Reactor R_2	----	0.0 + j0.520

Fig. (7) shows numbering for phase-coordinate poly-phase network of 3-phase, 18-Busbar. The per unit data referred to rated voltage 33 KV at Siuf thermal p.s system bus and 50 MVA are given in table (1).

Fig. (8) represefts structure of admittance matrices of elements and values of prefault voltages at the nodes, noting that Z_{s1} and Z_{s2} are represented impedance sources at Abis s.s and Siouf gas p.s 66 kV respectively.

Computer Program and Results

A computer program has been written for computing the short circuit currents levels at all nodes for single phase to ground fault, and at specified nodes simultanegus faults.

The nodal admittance matrix Y (table 2) was constructed, and then the inverse of Y is computed in Z (Table 3).

Fault levels calculations are computed following the steps quoted in section 2, [11]. The computer run has been carried out on IBM-PC. The results are given as follows:
Fig. (9) represents the values of single phase MVA short circuit at the nodes.

Fig. (10) Example 2 (Siouf thermal Power Station) under simultaneous faults condition (3-phase short circuit at nodes 4,5,6 and single phase to earth at node 9).

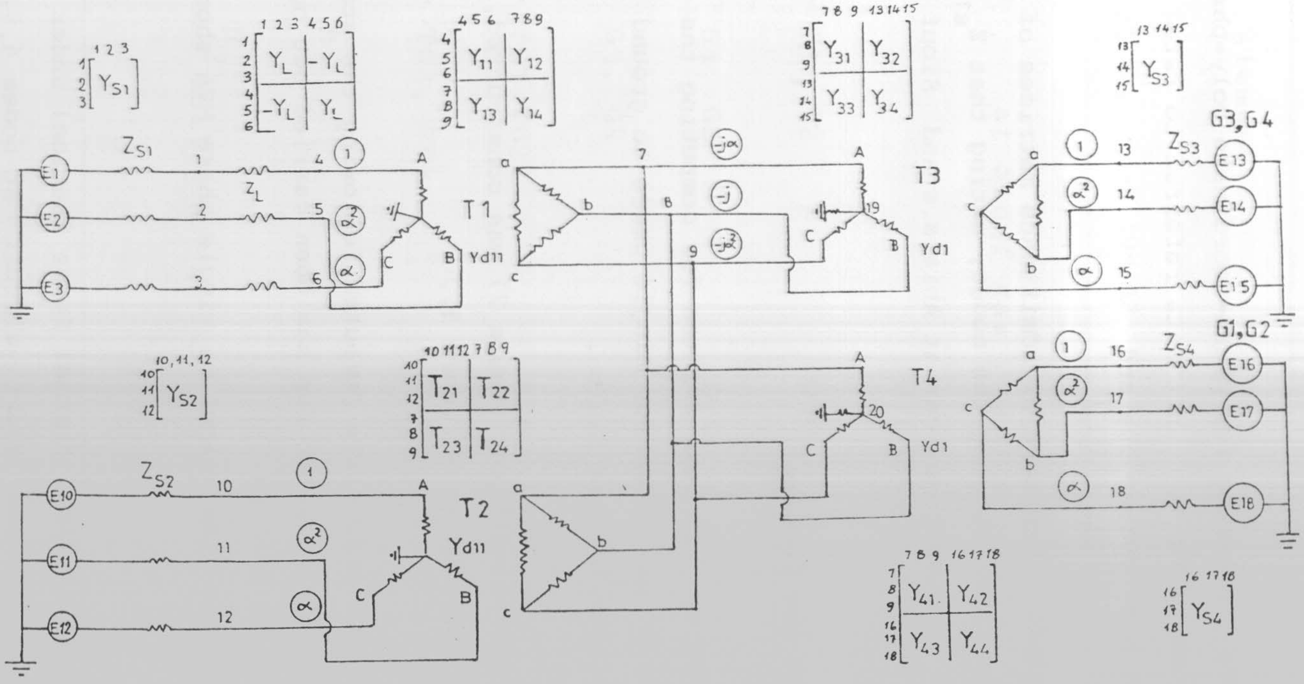

Fig (8) schematic representation of example 2 showing phase busbar numbering sequence and admittance matrices of elements and prefault voltages at different buses.

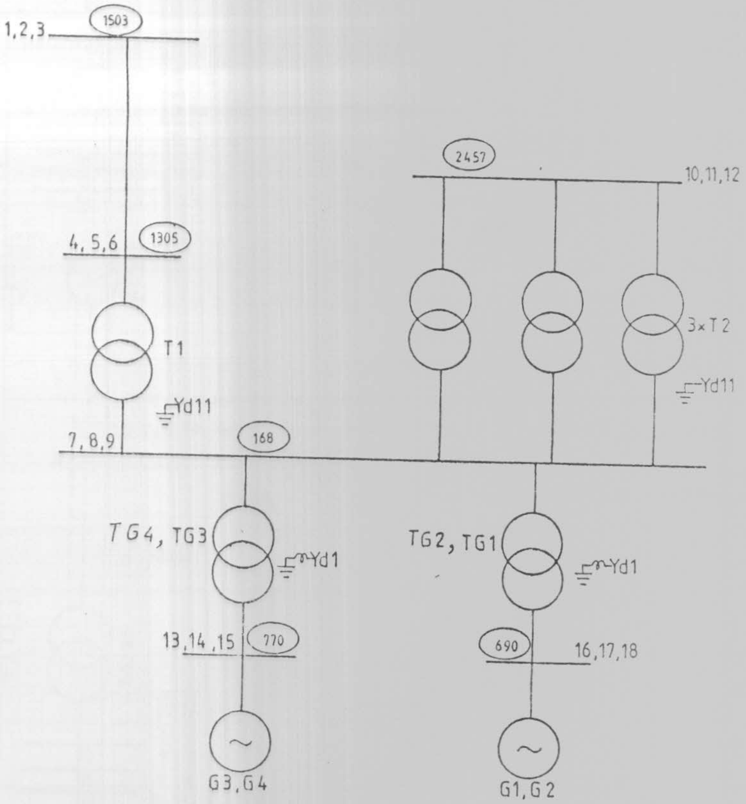

Fig. (9) SINGLE PHASE MVA$_{s.c}$ AT DIFFERENT BUSES BY USING PHASE CO-ORDINATES

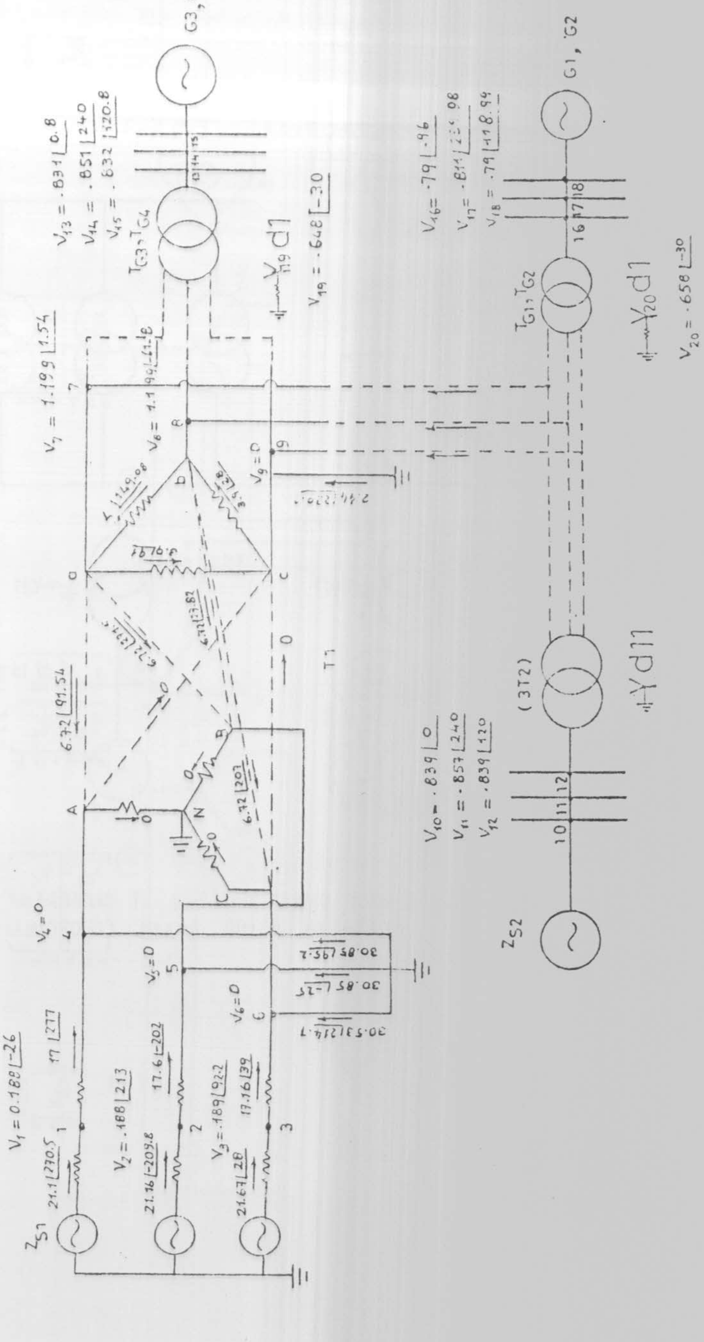

Fig.(10)Siouf thermal power station under simultaneous fault condition.

Table (2)

=

	1	2	3	4	5	6	7	8	9	10
1	51.377-j100.05	-18.97+j38.05	-18.97+j38.05	-51.377+j74.889	18.97-j33.801	18.97-j33.801				
2	-18.97+j38.05	51.377-j100.05	-18.97+j38.05	18.97-j33.801	-51.377+j74.883	18.97-j33.801				
3	-18.97+j38.05	-18.97+j38.05	51.377-j100.05	18.97-j33.801	18.97-j33.801	-51.377+j74.88				
4	-51.377+j74.889	18.97-j33.801	18.97-j33.801	51.377-j84.599	-18.97+j33.081	-18.97+j33.08	0+j5.61		0-j5.61	
5	18.97-j33.801	-51.377+j74.889	18.97-j33.801	-18.97+j33.081	51.377-j84.599	-18.97+j33.081	0-j5.61	0+j5.61		
6	18.97-j33.801	18.97-j33.801	-51.377+j74.883	-18.97+j33.081	-18.97+j33.081	51.377-j84.599		0-j5.61	0+j5.61	
7				0+j5.61	0-j5.61		0-j57.002	0+j13.036	0+j13.036	0+j17
8					0+j5.61	0-j5.61	0+j13.036	0-j57.002	0+j13.036	
9				0-j5.61		0+j5.61	0+j13.036	0+j13.036	0-j57.002	0-j17
10							0+j17		0-j17	0-j73.88
11							0-j17	0+j17		0-j17.44
12								0-j17	0+j17	0-j17.44
13							0+j7.557		0-j7.557	
14							0-j7.557	0+j7.557		
15								0-j7.557	0+j7.557	
16							0+j10.32		0-j10.32	
17							0-j10.32	0+j10.32		
18								0-j10.32	0+j10.32	
19							0+j13.073	0+j13.073	0+j13.073	
20							0+j17.857	0+j17.857	0+j17.857	

Table(2) continue

11	12	13	14	15	16	17	18	19	20
0-j17		0+j7.557	0-j7.557		0+j10.32	0-j10.32		0+j13.073	0+j17.857
0+j17	0-j17		0+j7.557	0-j7.557		0+j10.32	0-j10.32	0+j13.073	0+j17.857
	0+j17	0-j7.557		0+j7.557	0-j10.32		0+j10.32	0+j13.073	0+j17.857
0-j17.44	0-j17.44								
0-j73.88	0-j17.44								
0-j17.44	0-j73.88								
		0-j19.495	0+j4.357	0+j4.357					
		0+j4.357	0-j19.495	0+j4.357					
		0+j4.357	0+j4.357	0-j19.495					
					0-j20.595	0+j5.952	0+j5.952		
					0+j5.952	0-j20.595	0+j5.952		
					0+j5.952	0+j5.952	0-j20.595		
								0-j41.142	
									0-j55.386

Table (3.)

Z =

	1	2	3	4	5	6	7	8	9	10
1	·002+j·0328	·0019+j·0046	·0019+j·0046	-·0032+j·027	-·0027+j0	-·0027+j0	-·0001+j·0043	0	·0001-j·0043	-·0001+j·0026
2	·0019+j·0046	·002+j·0328	·0019+j·0046	-·0027+j0	-·0032+j·027	-·0027+j0	-·0001-j·0043	-·0001+j·0043	0	0-j·0013
3	·0019+j·0046	·0019+j·0046	·002+j·0328	-·0027+j0	-·0027+j0	-·0032+j·027	0	·0001-j·0043	-·0001+j·0043	0-j·0013
4	-·0032+j·027	-·0077+j0	-·0027+j0	·0061+j·038	·0034+j·0057	·0034+j·0057	·0004+j·0051	0	-·0004-j·0051	·0003+j·0031
5	-·0027+j0	-·0032+j·027	-·0027+j0	·0034+j·0057	·0061+j·038	·0034+j·0057	-·0004-j·0051	·0004+j·0051	0	-·0001-j·0015
6	-·0077+j0	-·0027+j0	-·0032+j·027	·0034+j·0057	·0034+j·0056	·0061+j·038	0	-·0004-j·0051	·0004+j·0051	-·0001-j·0015
7	-·0001+j·0043	-·0001-j·0043	0	·0004+j·0051	-·0004-j·0051	0	·0001+j·2991	-·0001+j·2684	-·0001+j·2684	·0001+j·0092
8	0	-·0001+j·0043	·0001-j·0043	0	·0004+j·0051	-·0004-j·0051	-·0001+j·2684	·0001+j·2991	-·0001+j·2684	0
9	·0001-j·0043	0	-·0001+j·0043	-·0004-j·0051	0	·0004+j·0051	-·0001+j·2684	-·0001+j·2684	·0001+j·2991	-·0001-j·0092
10	-·0001+j·0026	0-j·0013	0-j·0013	·0003+j·0031	-·0001-j·0015	-·0001-j·0015	·0001+j·0097	0	-·0001-j·0092	0+j·0204
11	0-j·0013	-·0001+j·0026	0-j·0013	-·0001-j·0015	·0003+j·0031	-·0001-j·0015	-·0001-j·0092	·0001+j·0092	0	0-j·0056
12	0-j·0013	0-j·0013	-·0001+j·0026	-·0001-j·0015	-·0001-j·0015	·0003+j·0031	0	-·0001-j·0092	·0001+j·0092	0-j·0056
13	-·0001+j·0027	0-j·0014	0-j·0014	·0003+j·0032	-·0001-j·0016	-·0001-j·0016	·0001+j·0098	-·0001-j·0094	0+j·0003	0+j·0056
14	0-j·0014	-·0001+j·0027	0-j·0014	-·0001-j·0016	·0003+j·0032	-·0001-j·0016	0+j·0001	·0001+j·0098	-·0001-j·0094	0-j·0029
15	0-j·0014	0-j·0014	-·0001+j·0027	-·0001-j·0016	-·0001-j·0016	·0003+j·0032	-·0001-j·0096	0+j·0001	·0001+j·0098	0-j·0029
16	-·0001+j·0033	0-j·0017	0-j·0017	·0003+j·004	-·0002-j·002	-·0002-j·002	·0001+j·0119	0	-·0001-j·0119	0+j·0072
17	0-j·0017	-·0001+j·0033	0-j·0017	-·0002-j·002	·0003+j·004	-·0002-j·002	-·0001-j·0119	·0001+j·0119	0	0-j·0036
18	0-j·0017	0-j·0017	-·0001+j·0033	-·0002-j·002	-·0002-j·002	·0003+j·004	0	-·0001-j·0119	·0001+j·0119	0-j·0036
19	0	0	0	0	0	0	0+j·265	0+j·265	0+j·265	0
20	0	0	.0	0	0	0	0+j·269	0+j·265	0+j·265	0

Table(3) continue

11	12	13	14	15	16	17	18	19	20
0-j.0013	0-j.0013	-.0001+j.0027	0 j.0014	0 j 0014	-.0001+j.0033	0-j.0017	0-j 0017	0	0
-.0001+j.0026	0-j.0013	0-j.0014	.0001+j.0027	0 j.0014	0-j 0017	-.0001+j 0033	0 j.0017	0	0
0-j.0013	-.0001+j.0026	0 j 0014	0 .0014	.0001+j 0027	0 j 0017	0 0017	-.0001+j.0033	0	0
-.0001-j.0015	-.0001-j.0015	.0003+j.0032	.0001-j.0016	.0001-j 0016	.0003+j.004	-.0002 j.002	-.0002-j.002	0	0
.0003+j.0031	-.0001-j.0015	-.0001-j.0016	.0003+j.0032	-.0001-j.0016	-.0002-j.002	.0003+j.004	-.0002 j.002	0	0
-.0001-j.0015	.0003+j.0031	-.0001-j.0016	-.0001 j.0016	.0003+j.0032	-.0002-j.002	-.0002-j.002	.0003+j.004	0	0
-.0001-j.0092	0	.0001+j.0098	-.0001-j.0094	0+j.0003	.0001+j 0119	-.0001-j.0119	0	0+j.265	0+j.269
.0001+j.0092	-.0001-j.0092	0+j 0001	.0001+j 0098	.0001-j.0094	0	.0001+j.0119	-.0001-j.0119	0+j.265	0+j.269
0	.0001+j.0092	-.0001-j.0096	0+j 0001	.0001+j 0098	-.0001-j.0119	0	.0001+j.0119	0+j.265	0+j.269
0-j.0056	0-j.0056	0+j.0056	0-j.0029	0-j.0029	0+j 0072	0-j.0036	0-j 0036	0	0
0+j.0204	0-j.0056	0-j.0029	0+j.0056	0-j.0029	0-j.0036	0+j.0072	0-j.0036	0	0
0-j.0056	0+j.0204	0-j.0029	0 j.0029	0+j 0056	0-j.0036	0-j.0036	0+j.0072	0	0
0-j.0029	0-j.0029	0+j.065	0+j.0139	0+j.0139	0+j.0076	0-j.0038	0-j 0038	0	0
0+j.0056	0-j.0029	0+j.0139	0+j.065	0+j.0139	0-j 0038	0+j.0076	0-j.0038	0	0
0-j.0029	0+j.0056	0+j.0139	0+j.0139	0+j.065	0-j.0038	0-j.0038	0+j 0076	0	0
0-j.0036	0-j.0036	0+j.0076	0-j.0038	0-j.0038	0+j.0727	0+j.0212	0+j.0212	0	0
0+j.0072	0-j.0036	0-j.0038	0+j.0076	0-j.0038	0+j.0212	0+j.0727	0+j.0212	0	0
0-j.0036	0+j.0072	0-j.0038	0-j.0038	0+j.0076	0+j.0212	0+j.0212	0+j.0727	0	0
0	0	0	0	0	0	0	0	0+j.2775	0+j.2569
0	0	0	0	0	0	0	0	0+j.2569	0+j.2787

Table (2) represents admittance matrix Y

Table (3) represents impedance matrix Z

Conclusion

The method of phase co ordinates for analysis of fault studies in complex unbalanced problems is shown clearly in the presented examples, using nodal admittance matrices [11] with the vector group of power transformers in polyphase network being known.

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Appendix (1)

Admittance matrix of example 1 (Yd3 transformer)

		1	2	3	4	5	6	7	8	9	10
	1	-14.445	-1.111	-1.111	0	5.773	-5.773	0	0	0	16.667
	2	-1.111	-14.445	-1.111	-5.773	0	5.773	0	0	0	16.667
	3	-1.111	-1.111	-14.445	5.773	-5.773	0	0	0	0	16.667
	4	0	-5.773	5.773	-23.071	3.595	3.595	13.071	-3.595	-3.595	0
Y= j	5	5.773	0	-5.773	3.595	-23.071	3.595	-3.595	13.071	-3.595	0
	6	-5.773	5.773	0	3.595	3.595	-23.071	-3.595	-3.595	13.071	0
	7	0	0	0	13.071	-3.595	-3.595	-33.904	7.761	7.761	0
	8	0	0	0	-3.595	13.071	-3.595	7.761	-33.904	7.761	0
	9	0	0	0	-3.595	-3.595	13.071	7.761	7.761	-33.904	0
	10	16.667	16.667	16.667	0	0	0	0	0	0	-83.334

Impedance matrix of example 1 (Yd3 transformer)

		1	2	3	4	5	6	7	8	9	10
	1	0.13	0.01	0.01	0	0.0346	-.0346	0	0.0139	-.0139	0.03
	2	0.01	0.13	0.01	-.0346	0	0.0346	-.0139	0	0.0139	0.03
	3	0.01	0.01	0.13	0.0346	-.0346	0	0.0139	-.0139	0	0.03
	4	0	-.0346	0.0346	0.0771	-.0029	-.0029	.0289	-.003	-.003	0
Z= j	5	0.0346	0	-.0346	-.0029	.0771	-.0029	-.003	.0289	-.003	0
	6	-.0346	.0346	0	-.0029	-.0029	0.0771	-0.003	-0.003	.0289	0
	7	0	-.0139	0.0139	.0289	-0.003	-0.003	0.0451	0.0083	0.0083	0
	8	.0139	0	-.0139	-.003	.0289	-.003	.0083	.0451	.0083	0
	9	-.0139	0.0139	0	-.003	-.003	0.0289	0.0083	0.0083	0.0451	0
	10	0.03	0.03	0.03	0	0	0	0	0	0	0.03

Admittance matrix of example 1 (Yd1 transformer)

$$Y = j \begin{array}{c|cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
1 & -14.445 & -1.111 & -1.111 & 5.775 & 0 & -5.775 & 0 & 0 & 0 & 16.667 \\
2 & -1.111 & -14.445 & -1.111 & -5.775 & 5.775 & 0 & 0 & 0 & 0 & 16.667 \\
3 & -1.111 & -1.111 & -14.445 & 0 & -5.775 & 5.775 & 0 & 0 & 0 & 16.667 \\
4 & 5.775 & -5.775 & 0 & -23.071 & 3.595 & 3.595 & 13.071 & -3.595 & -3.595 & 0 \\
5 & 0 & 5.775 & -5.775 & 3.595 & -23.071 & 3.595 & -3.595 & 13.071 & -3.595 & 0 \\
6 & -5.775 & 0 & 5.775 & 3.595 & 3.595 & -23.071 & -3.595 & -3.595 & 13.071 & 0 \\
7 & 0 & 0 & 0 & 13.071 & -3.595 & -3.595 & -33.904 & 7.761 & 7.761 & 0 \\
8 & 0 & 0 & 0 & -3.595 & 13.071 & -3.595 & 7.761 & -33.904 & 7.761 & 0 \\
9 & 0 & 0 & 0 & -3.595 & -3.595 & 13.071 & 7.761 & 7.761 & -33.904 & 0 \\
10 & 16.667 & 16.667 & 16.667 & 0 & 0 & 0 & 0 & 0 & 0 & -83.334
\end{array}$$

Impedance matrix of example 1 (Yd1 transformer)

$$Z = j \begin{array}{c|cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
1 & 0.1302 & 0.01 & 0.0098 & 0.035 & 0 & -0.0347 & 0.014 & 0 & -.0139 & 0.03 \\
2 & 0.01 & 0.1299 & 0.01 & -0.0345 & 0.0345 & 0 & -0.0136 & 0.0139 & 0 & 0.03 \\
3 & 0.0097 & 0.01 & 0.1302 & 0 & -0.035 & 0.0347 & 0 & -0.0141 & 0.0138 & 0.03 \\
4 & 0.0347 & -0.0347 & 0 & 0.0772 & -0.0028 & -0.0029 & 0.029 & -0.003 & -0.003 & 0 \\
5 & 0 & 0.0347 & -.0347 & -.0023 & 0.0772 & -.0029 & -0.0023 & 0.029 & -0.0029 & 0 \\
6 & -0.035 & 0 & 0.035 & -00037 & -0.0036 & 0.0772 & -.0034 & -.0033 & 0.029 & 0 \\
7 & .0137 & -.0139 & 0 & 0.0287 & -.0033 & 0.003 & 0.045 & 0.0082 & 0.0083 & 0 \\
8 & 0 & 0.0138 & -.0138 & -.0031 & 0.0287 & 0.0031 & 0.0085 & 0.0451 & 0.0084 & 0 \\
9 & -.0144 & 0 & 0.0144 & -0.0043 & -.0043 & 0.0291 & 0.0078 & 0.0079 & 0.0452 & 0 \\
10 & 0.03 & 0.03 & 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0.03
\end{array}$$

Admittance matrix of example 1 (Yd9 transformer)

		1	2	3	4	5	6	7	8	9	10
	1	-14.445	-1.111	-1.111	0	-5.775	5.775	0	0	0	16.667
	2	-1.111	-14.445	-1.111	5.775	0	-5.775	0	0	0	16.776
	3	-1.111	-1.111	-14.445	-5.775	5.775	0	0	0	0	16.776
	4	0	5.775	-5.775	-23.07	3.595	3.595	13.071	-3.595	-3.595	0
	5	-5.775	0	5.775	3.595	-23.07	3.595	-3.595	13.071	-3.595	0
Y=j	6	5.775	-5.775	0	3.595	3.595	-23.07	-3.595	-3.595	13.071	0
	7	0	0	0	13.071	-3.595	-3.595	-33904	7.761	7.761	0
	8	0	0	0	-3.595	13.071	-3.595	7.761	-33904	7.761	0
	9	0	0	0	-3.595	-3.595	13.071	7.761	7.761	-33904	0
	10	16.667	16.667	16.667	0	0	0	0	0	0	-83.334

Impedance matrix of example 1 (Yd9 transformer)

		1	2	3	4	5	6	7	8	9	10
	1	0.130	0.010	0.010	0	-.0347	0.0347	0	-.0139	0.0139	0.03
	2	0.010	0.130	0.010	0.0347	0	-.0347	0.0139	0	-.0139	0.03
	3	0.010	0.010	0.130	-.0347	0.0347	0	-.0139	0.0139	0	0.03
	4	0	0.0347	-.0347	0.0772	-.0029	-.0029	0.029	-.0031	-.0031	0
	5	-.0347	0	0.0347	-.0029	0.0772	-.0029	-.0031	0.029	-.0031	0
Z=j	6	0.0347	-.0347	0	-.0029	-.0029	0.0772	-.0031	-.0031	0.029	0
	7	0	0.0139	-.0139	0.029	-.0031	-.0031	0.0451	0.0083	0.0083	0
	8	-.0139	0	0.0139	-.0031	0.029	-.0031	0.0083	0.0451	0.0083	0
	9	0.0139	-.0139	0	-.0031	-.0031	0.029	0.0083	0.0083	0.0451	0
	10	0.030	0.030	0.030	0	0	0	0	0	0	0.03