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ANALYSIS OF CONCRETE SHORT COLUMNS SUBJECTED TO AXIAL LOAD AND BIAXIAL BENDING

MOHIE EL-DIN S. SHUKRY

Structural Engineering Department
Faculty of Engineering, Alexandria University
Alexandria, EGYPT

ABSTRACT

This paper presents an approach to ultimate strength calculations of reinforced concrete sections subjected to compression loads with uniaxial or biaxial eccentricity. The analysis is performed in accordance with the Draft Egyptian Concrete Code [1]. The concrete stress-strain diagram is a parabola-rectangle while the steel has an elastic-plastic relationship with well-defined yield point. The analysis is developed particularly for rectangular, circular, T and L sections. A computer program was written to implement the proposed method of analysis. The program has the capability to design rectangular column sections subject to biaxial eccentricity. Numerical examples are presented to check the validity of the proposed analysis.

NOTATION

A_c = area of concrete in compression zone

A_{ep} = area of concrete in elasto-plastic region

A_s = cross sectional area of individual steel bar

a = value of the intersection between the neutral axis and the x-axis

b = rectangular column width

b' = effective column width = b - concrete cover

E_c, E_s = modulus of elasticity of concrete and steel

e = eccentricity

e_x, e_y = eccentricity measured parallel to x-axis and y-axis respectively

$\grave{e}_x, \grave{e}_y$ = eccentricities measured from origin (see Fig. 5)

f_c = concrete compressive stress

f_{cd} = concrete maximum compressive stress

f_{cu} = characteristic cube strength of concrete

f_y = characteristic yield stress of steel

h = value of the intersection between the neutral axis and the y-axis

N_o = short column axial load capacity (no bending)

N_u = factored axial load for which the column is to be designed

N_{ux} = factored axial load capacity (M_{ux} acting)

N_{uy} = factored axial load capacity (M_{uy} acting)

M_{ox} = uniaxial x-axis column moment capacity (N_u acting)

M_{oy} = uniaxial y-axis column moment capacity (N_u acting)

M_{ux} = x-axis bending moment for which the column is to be designed

M_{uy} = y-axis bending moment for which the column is to be designed

t = rectangular column depth or diameter of circular section

t' = rectangular column effective depth = t - concrete cover

x_o = depth of neutral axis

x_1 = distance between origin and a point in the section (Fig. 5)

α = inclination of neutral axis with respect to x-axis

γ_c, γ_s = partial safety factors for concrete and steel

ϵ_c, ϵ_s = concrete and steel strain

ϵ_{cu} = failure strain of concrete in compression

θ = ratio of acting moments = M_y / M_x

ρ = ratio of steel reinforcement = ΣA_s / (b t)

1. INTRODUCTION

In analysis and design of structural frames, columns that are subjected to axial load and biaxial bending are frequently encountered. Biaxial bending occurs in corner columns, in exterior columns and in interior columns due to load imbalance of adjacent spans. Also, bridge piers are often subjected to biaxial bending.

The common approachs to the design of biaxially loaded columns are:

1) to assume a section and its reinforcement and then compute the capacity of this particular section under given loads. Successive corrections must then be made until the capacity of the section is in reasonable agreement with design values.

2) to use design-aid methods which involve the use of a three-dimensional interaction surface (such that shown in Fig.1-a) generated from the axial-bending moment interaction diagrams arising from two uniaxial bending cases. The most common used interaction surfaces were proposed by Bresler [2]. Theses surfaces are defined as follows:

$$\frac{1}{N_u} = \frac{1}{N_{ux}} + \frac{1}{N_{uy}} - \frac{1}{N_o} \qquad \text{.......... (1)}$$

$$\left(\frac{M_{ux}}{M_{ox}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{oy}}\right)^{\alpha_n} = 1 \qquad \text{.......... (2)}$$

where α_n is a constant depending on material strengths and bar pattern. For design purpose, Prame et al [3] extended Eq.2 to the following form:

when M_{uy}/M_{oy} is less than M_{ux}/M_{ox}

$$\frac{M_{ux}}{M_{ox}} + \frac{M_{uy}}{M_{oy}} \; \frac{1-\beta}{\beta} = 1 \qquad \text{.......... (3.a)}$$

when M_{uy}/M_{oy} exceeds M_{ux}/M_{ox} then

$$\frac{M_{uy}}{M_{oy}} + \frac{M_{ux}}{M_{ox}} \; \frac{1-\beta}{\beta} = 1 \qquad \text{.......... (3.b)}$$

where β is a factor depending on the ratio N_u/N_o , material and cross

section properties and was found [3] to range from 0.55 to 0.70.
When rectangular sections are used with reinforcement distributed uniformly along the faces, the ratio of M_{oy}/M_{ox} will approximately equal to b/t ; thus Eqs.3 take the form:

$$M_{uy} + M_{ux} \frac{b}{t} \frac{1-\beta}{\beta} \simeq M_{oy} \quad \text{for} \quad \frac{M_{uy}}{M_{ux}} \geq \frac{b}{t} \quad \text{......... (4.a)}$$

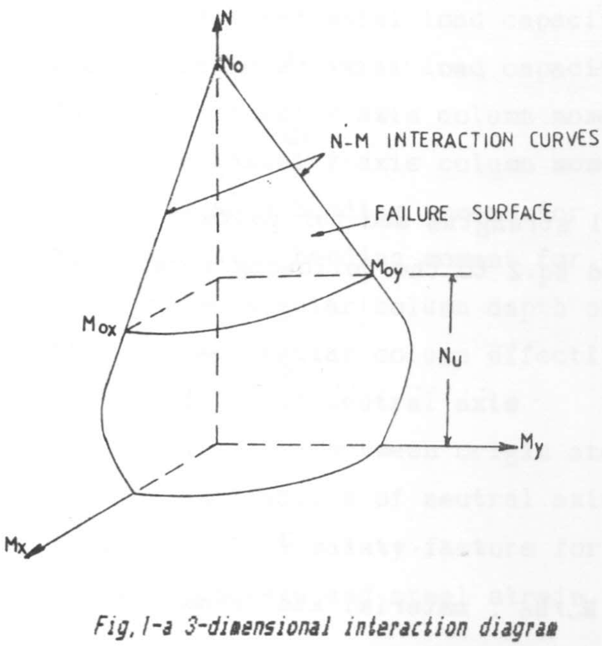
$$M_{ux} + M_{uy} \frac{t}{b} \frac{1-\beta}{\beta} \simeq M_{ox} \quad \text{for} \quad \frac{M_{uy}}{M_{ux}} \leq \frac{b}{t} \quad \text{......... (4.b)}$$

BS 8110:85 [4] proposed two equations similar to Eqs.4 using a coefficient β' in place of the value $(1-\beta)/\beta$ in the above two equations. The BS equations (which are also proposed by the Draft Egyptian Code) take the following form :

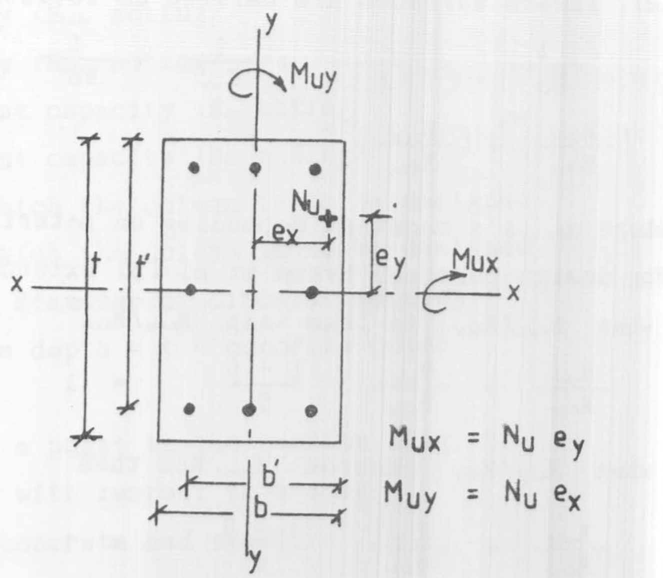
$$M_{uy} + M_{ux} \beta' \frac{b'}{t'} = M_{oy} \quad \text{for} \quad \frac{M_{ux}}{M_{uy}} < \frac{t'}{b'} \quad \text{.......... (5.a)}$$

$$M_{ux} + M_{uy} \beta' \frac{t'}{b'} = M_{ox} \quad \text{for} \quad \frac{M_{ux}}{M_{uy}} \geq \frac{t'}{b'} \quad \text{.......... (5.b)}$$

where t' and b' are the effective depths (see Fig.1-b) and the factor β' depends on the value N_u/btf_{cu}.



Fig, 1-a 3-dimensional interaction diagram



Fig, 1-b Column section subjected to eccentric load

From Eqs. 4 and 5, the value of M_{ox} or M_{oy} is computed and using uniaxial design aids, the column section and reinforcement satisfying N_u and M_{ox} or M_{oy} is determined.

This paper presents a method for analysis of reinforced concrete columns of general shape; rectangular, circular, T and L sections, and suggestions for the design of rectangular sections are presented.

2. THE ANALYSIS

In this section the ultimate strength of a section subjected to axial compression load with uniaxial or biaxial eccentricity is presented. The analysis is based on the recommendations of the Egyptian Code for Reinforced Concrete Structures [1] with the following assumptions :

1- plane sections remain plane after deformation

2- the reinforcement is subjected to the same variation in strain as the adjacent concrete

3- the tensile strength of concrete is neglected

4- the maximum compressive strain of the concrete is taken to be 0.003 in bending (simple or with axial compression) and 0.002 in axial compression. For calculating the resisting load-effect it is assumed that the strain diagram must pass through point A or B in Fig.2 and to be proportional to the stress.

5- the stress in steel is assumed to be proportional to the strain up to the design stress; f_y / γ_s where γ_s is the partial safety factor for steel defined as :

$$\gamma_s = 1.36 - 0.43\ (e/t) \geq 1.15 \qquad \text{......... (6.a)}$$

After yield, the reinforcement stress is considered independent of strain and is equal to: f_y / γ_s. The stress-strain diagram of steel is as shown in Fig.3.

6- the concrete stress-strain relationship is assumed to be a parabola-rectangle such that shown in Fig.4. The partial safety factor for concrete; γ_c, is defined as:

$$\gamma_c = 1.75 - 0.5\ (e/t) \geq 1.50 \qquad \text{......... (6.b)}$$

For rectangular sections subjected to biaxial eccentricity, the value of e/t in Eqs.6 is the maximum of e_x/b and e_y/t where e_x and e_y are the eccentricity about y and x axis respectively. For L-sections, the following values were adopted for the partial safety factors $\gamma_c = 1.5$ and $\gamma_s = 1.15$.

7- compressive stresses and strains for concrete and tensile stresses and strains for steel are considered positive .

8- the origin is located at the point of maximum concrete strain ϵ_{cu}.

Mathematical Formulation:

Strain

For a section subjected to biaxial eccentricity, the strain at a point, defined by the coordinates x and y, is

$$\epsilon_c = \epsilon_{cu}\,(1 - x/a - y/h) \quad \text{..........} \quad (7.a)$$

where a and h are the intersection points of the neutral axis with the x and y axes (see Fig.5).

For a section subjected to uniaxial eccentricity, ϵ_c is defined as:

$$\epsilon_c = \epsilon_{cu}\,(1 - x/x_o) \quad \text{..........} \quad (7.b)$$

where x_o is the depth of the neutral axis measured normal to it.

When the whole of the section is in compression, the strain diagram passes through point B (see Fig.2) and the maximum strain in concrete is defined as:

$$\epsilon_{cu} = 0.003 - 0.001\,(x_o - t_1)\,/\,(x_o - t_1/3) \quad \text{..........} \quad (8)$$

where t_1 is the distance between the origin and the least compressed point in the section and measured normal to the neutral axis.

Concrete Stresses

In the elasto-plastic region (parabolic zone of the diagram), the concrete stress at any point in the compression zone is defined as:

$$f_c = \frac{0.67\,f_{cu}}{\gamma_c}\left(1 - \frac{(x_1 - x_o/3)^2}{(2x_o/3)^2}\right) \quad \text{..........} \quad (9.a)$$

where for any point, in the cross section, with coordinates x, y:

$$x_1 = x \sin\alpha + y \cos\alpha \quad \text{..........} \quad (9.b)$$

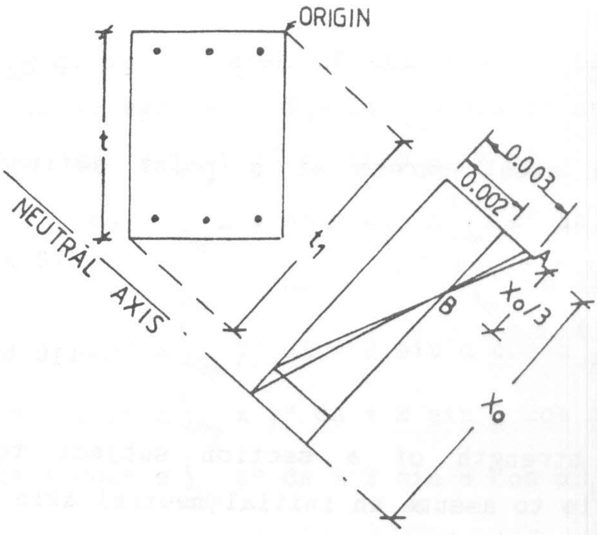


Fig.2 Strain diagram when whole section is under compression

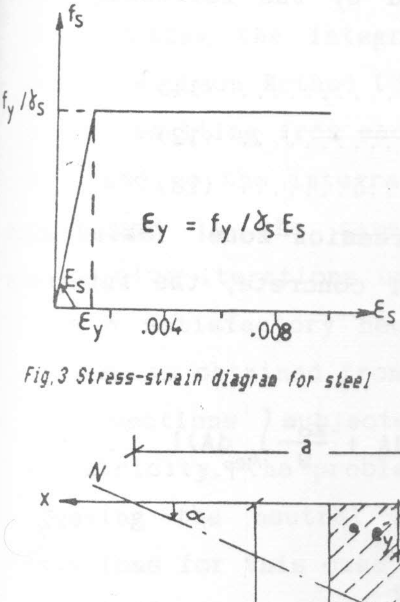


Fig.3 Stress-strain diagram for steel

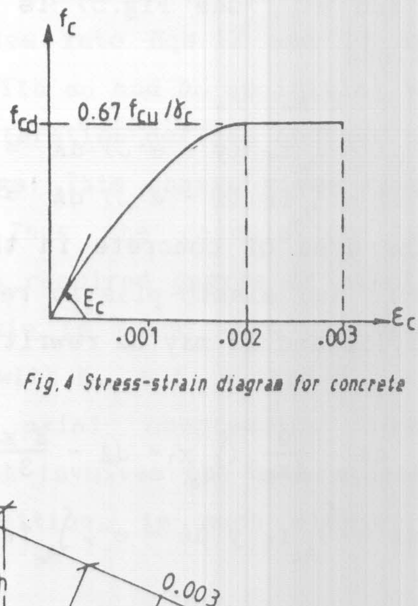


Fig.4 Stress-strain diagram for concrete

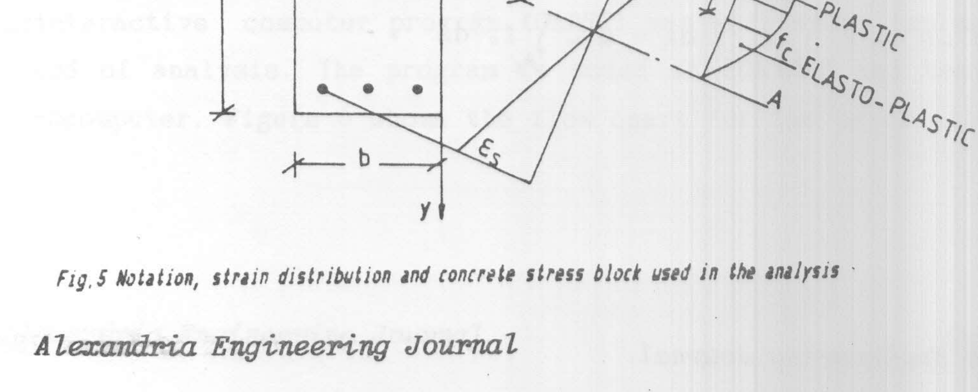


Fig.5 Notation, strain distribution and concrete stress block used in the analysis

and α is the inclination of the N.A. with respect to the x-axis

In the plastic region:

$$f_c = f_{cd} = 0.67\ f_{cu}/\gamma_c \qquad \text{(9.c)}$$

Steel stresses

In the elastic region, the steel stress at a point defined by the coordinates x,y is given by:

$$f_s = E_s\ \epsilon_s = E_s\ \epsilon_{cu}\ (x/a + y/h - 1) \qquad \text{(10.a)}$$

In the plastic region

$$f_s = f_y/\gamma_s \qquad \text{(10.b)}$$

Ultimate strength of section

To calculate the ultimate strength of a section subject to biaxial eccentricity, the first step is to assume an initial neutral axis position ($a = a_o$, $h = h_o$). For this position, the magnitude of the strength load N_u (located at e'_x and e'_y, see Fig.5) is obtained by the following three equilibrium equations:

$$N_u = \int_{Ac} f_c\ dA - \Sigma\ A_s\ f_s \qquad \text{(11)}$$

$$\Sigma\ A_s\ f_s\ (y - e'_y) - \int_{Ac} f_c\ (y - e'_y)\ dA = 0 \qquad \text{(12)}$$

$$\Sigma\ A_s\ f_s\ (x - e'_x) - \int_{Ac} f_c\ (x - e'_x)\ dA = 0 \qquad \text{(13)}$$

where $\int_{Ac} dA$ is the area of concrete in the compression zone. Taking into account the plastic and elasto-plastic regions of concrete, the integrals over Ac in Eqs.11, 12 and 13 may be rewritten as:

$$\int_{Ac} f_c\ dA = f_{cd}\ [\int_{Ac} dA - \frac{9}{4x_o^2}(\int_{Aep} x_1^2\ dA - \frac{2\ x_o}{3}\int_{Aep} x_1\ dA + \frac{x_o^2}{9}\int_{Aep} dA)] \qquad \text{(14)}$$

$$,\int_{Ac} f_c\ (y - e'_y)\ dA = \int_{Ac} f_c\ y\ dA - e'_y \int_{Ac} f_c\ dA \qquad \text{(15)}$$

where

$$\int_{Ac} f_c\ y\ dA = f_{cd}\ [\int_{Ac} y\ dA - \frac{9}{4\ x_o^2}\ (\int_{Aep} x_1^2\ y\ dA - \frac{2\ x_o}{3}\int_{Aep} x_1\ y\ dA + \frac{x_o^2}{9}\int_{Aep} y\ dA)]$$

$$,\int_{Ac} f_c\ (x - e'_x)\ dA = \int_{Ac} f_c\ x\ dA - e'_x \int_{Ac} f_c\ dA \qquad \text{(16)}$$

where

$$\int_{Ac} f_c\, x\, dA = f_{cd}\, [\int_{Ac} x\, dA - \frac{9}{4\, x_o^2}\, (\int_{Aep} x_1^2\, x\, dA - \frac{2\, x_o}{3} \int_{Aep} x_1\, x\, dA + \frac{x_o^2}{9} \int_{Aep} x\, dA)]$$

where $\int_{Aep} dA$ is the area of concrete in the elasto-plastic region. From Eq.9.b, the integrals in Eqs.14, 15 and 16 are given by:

$$\int_{Aep} x_1\, dA = \cos\alpha \int_{Aep} y\, dA + \sin\alpha \int_{Aep} x\, dA$$

$$\int_{Aep} x_1\, x\, dA = \cos\alpha \int_{Aep} x\, y\, dA + \sin\alpha \int_{Aep} x^2\, dA$$

$$\int_{Aep} x_1\, y\, dA = \cos\alpha \int_{Aep} y^2\, dA + \sin\alpha \int_{Aep} x\, y\, dA$$

$$\int_{Aep} x_1^2\, dA = \cos^2\alpha \int_{Aep} y^2\, dA + 2 \sin\alpha \cos\alpha \int_{Aep} x\, y\, dA + \sin^2\alpha \int_{Aep} x\, dA$$

$$\int_{Aep} x_1^2\, x\, dA = \cos^2\alpha \int_{Aep} x\, y^2\, dA + 2 \sin\alpha \cos\alpha \int_{Aep} x^2\, y\, dA + \sin^2\alpha \int_{Aep} x^3\, dA$$

$$\int_{Aep} x_1^2\, y\, dA = \cos^2\alpha \int_{Aep} y^3\, dA + 2 \sin a \cos\alpha \int_{Aep} x\, y^2\, dA + \sin^2\alpha \int_{Aep} x^2\, y\, dA$$

These values of integrals are denominated mechanical characteristics of the reinforced concrete section and depend on the position of the neutral axis. Substituting the integral values into Eqs.12 and 13 and solving by the Newton Raphson Method [5,6,7] with a_o and h_o as initial values. The pair of roots resulting from each new iteration defines another position of neutral axis and so the integrals change. This change gives rise to new equations processed in the same way. Thus the calculation is carried out by successive iterations until the required degree of exactitude is attained. Once a satisfactory neutral axis is found, the ultimate strength of the section is obtained from Eq.11 with $M_{ux} = N_u\, e_y$ and $M_{uy} = N_u\, e_x$.

For sections subjected to axial compression load with uniaxial eccentricity, the problem, which involves the determination of one unknown defining the neutral axis position, is much simpler. The equilibrium equations for this case are:

$$\Sigma\, A_s\, f_s\, (x - e') - \int f_c\, (x - e')\, dA = 0 \quad \text{..........} \quad (17)$$

$$N_u\, e' = \int f_c\, x\, dA - \Sigma\, A_s\, f_s\, x \quad \text{..........} \quad (18)$$

An interactive computer program (CADSC) was written to implement the above method of analysis. The program is coded in FORTRAN and tested on IBM AT microcomputer. Figure 6 shows the flow chart for the calculation process

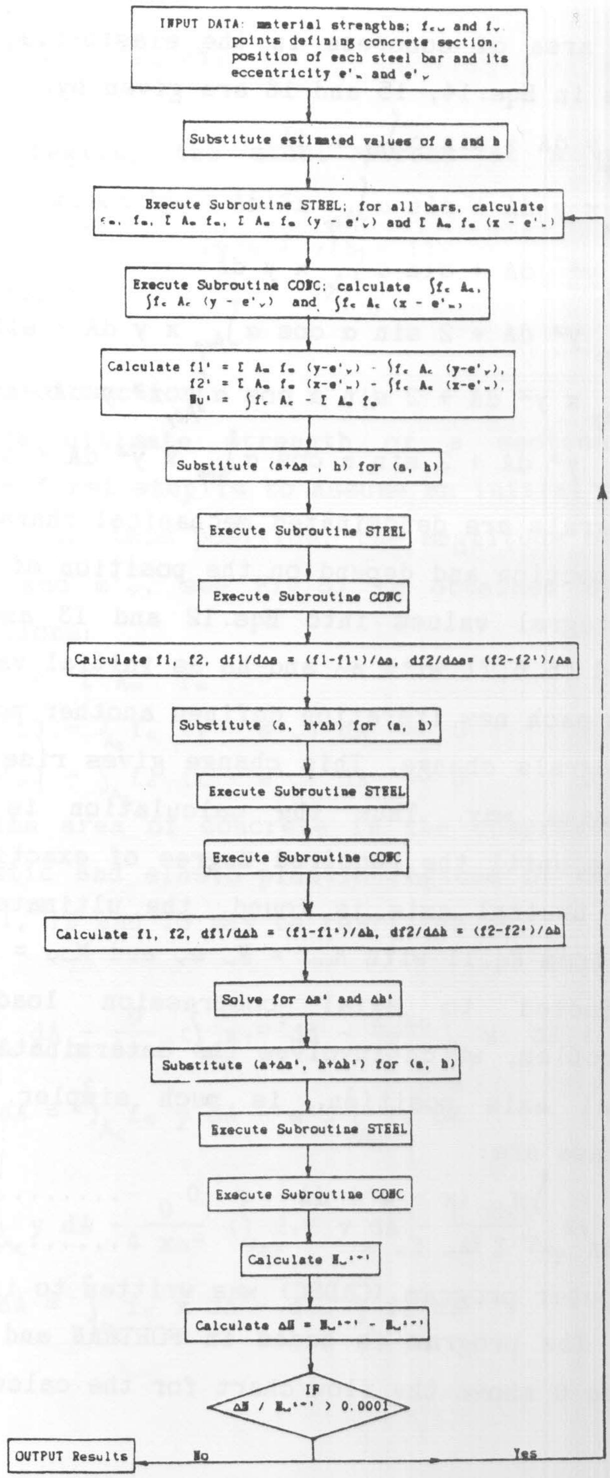


Fig. 6 Main flow chart for a section subject to axial compression load and biaxial eccentricity

of the biaxially loaded section.

The program includes the following options:

a. Serviceability Limit State (SLS) or Ultimate Limit State (ULS) analysis.

b. sections with symmetric or biaxial bending

c. columns with rectangular, circular, T or L section.

For the SLS, the results include the neutral axis position and concrete and steel stresses. Holes ,in the concrete sections, are allowed for the SLS analysis.

3. DESIGN OF RECTANGULAR SECTIONS

For a given loading and chosen quantities of the materials to be used, the main task of a design method is to determine the concrete dimensions and steel reinforcement for the assumed cross section. In practice, this is usually achieved by choosing concrete section and calculating the necessary reinforcement. The concrete section may be based on the design of the column as axially loaded by multipling the factored load N_u by a coefficient depending whether the column is interior, facade or corner (see BS 8110 [4]). After the choice of the concrete section the reinforcement ratio ρ is designed to carry N_u, M_{ux} and M_{uy}.

Program CADSC has the capability to calculate ρ for a given section under given factored loads. This is also achieved by using the compatiability relations together with the three equilibrium equations. These equations are:

$$\Sigma A_s f_s - \int f_c \, dA + N_u = 0 \quad .... (11)$$

$$\Sigma A_s f_s y - \int f_c \, dA \, y - M_{ux} = 0 \quad .... (19)$$

$$\Sigma A_s f_s x - \int f_c \, dA \, x + M_{uy} = 0 \quad .... (20)$$

Equations 11, 19 and 20 are solved (using Newton Raphson Method) for the neutral axis position (i.e. a and h) and ρ. A similar approach is suggested by Vanluchene [8], who presented a set of algorithms, based on Frame equations, to solve the nonlinear design equations.

Reinforcing steel is assumed to be in the form of individual bars symmetrically disrtibuted in the section while the number of bars is either

6 or 8 or 10 (see Fig.7). In the next section an example on the reinforcement design for a given section is presented.

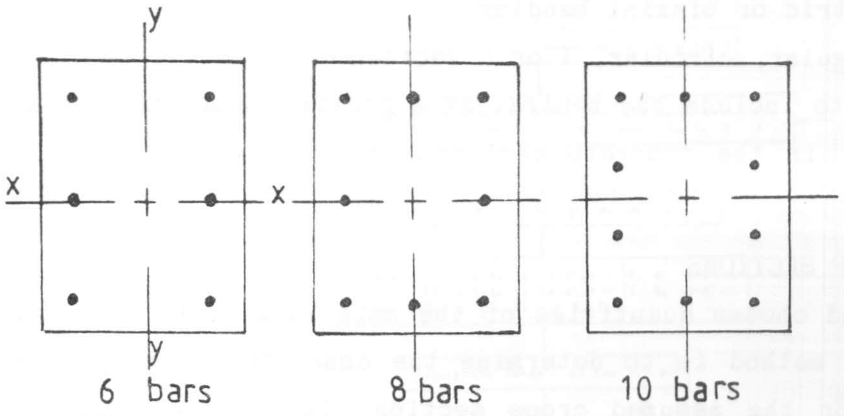


Fig. 7 Bar arrangement used in the design option of program CADSC

4. **NUMERICAL EXAMPLES**

1. An L-section, with dimensions, material strength and reinforcement shown in Fig. 8, is analysed under the following working loads:

i) $N = 25.0$ ton , $M_x = 2.25$ t.m. and $M_y = 2.00$ t.m. ; $\theta = M_y/M_x = 0.89$

ii) $N = 25.0$ ton , $M_x = 4.75$ t.m. and $M_y = 1.43$ t.m. ; $\theta = 0.30$

the results of the analysis are shown in Fig. 8.

Figure 8 also displays the results of the ULS analysis for the two cases of θ; $\theta = 0.89$ and $\theta = 0.30$.

2. The present analysis is used to predict the interaction diagram for a circular column section subject to an eccentric compression load. Figure 9 shows the interaction diagrams for different reinforcement ratios.

3. Figure 10 shows a rectangular section, with dimensions, material strength and reinforcement shown in figure, subjected to compression load at eccentricities $e_x = 12.50$ cm and $e_y = 21.05$ cm. The present analysis predicts the ultimate strength as follows:

$N_u = 106.53$ ton, $M_{ux} = 22.43$ t.m. and $M_{uy} = 13.32$ t.m.

The adequecy of the section is checked using the equations developed by

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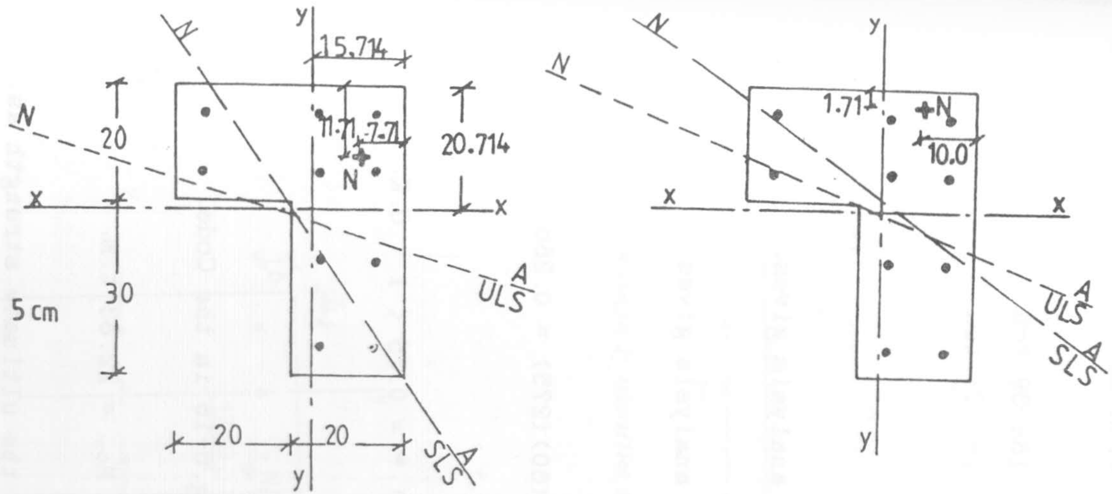
A_s = 2.0 cm^2

cover to all bars = 5 cm



f_{cu} = 250 kg/cm², E_c = 221 t/cm², f_y = 2400 kg/cm², E_s = 2000 t/cm²

SLS	Given:	N = 25 ton	N = 25 ton
		M_x = 2.25 t.m.	M_x = 4.75 t.m.
		M_y = 2.00 t.m.	M_y = 1.43 t.m.
	Output:	a = 34.26 cm	a = 43.81 cm
		h = 50.37 cm	h = 29.88 cm
		f_{cmax} = 79.65 kg/cm²	f_{cmax} = 119.10 kg/cm²
ULS	Given:	e'_x = 7.71 cm	e'_x = 10.00 cm
		e'_y = 11.71 cm	e'_y = 1.71 cm
	Output:	a = 94.35 cm	a = 71.03 cm
		h = 27.90 cm	h = 28.91 cm
		N_u = 79.18 ton	N_u = 71.20 ton

Fig. 8 Example 1; L-section subject to eccentric load (SLS & ULS)

Bresler (Eqs.1 and 2), the Egyptian Code formulas (Eqs.5) and the equations developed by Prame et al (Eqs.3). The present analysis is used to obtain some of the results required to use these formulas.

i) Equation 1:

for e_y = 21.05 cm, e_x = 0.0 cm ..... N_{ux} = 184.39 ton

for e_x = 12.50 cm, e_y = 0.0 cm ..... N_{uy} = 166.07 ton

$N_o = 0.4\, f_{cu}\, A_c + 0.75\, f_y\, A_s$ = 354.90 ton

from Eq.1, N_u = 115.91 ton > 106.53 ton .. O.K.

ii) Equation 2

for N_u = 106.53 ton acting on y-axis, the analysis gives:

e_y = 43.27 cm and M_{ox} = 46.09 t.m.

for N_u = 106.53 ton acting on x-axis, the analysis gives

e_x = 22.60 cm and M_{oy} = 24.08 t.m.

for N_u/N_o = 106.53/354.9 = 0.3 and

q (reinforcement index) = (40)(4130)/(35)(60)(275) = 0.286

$\therefore \beta$ = 0.286 (Ref.3) and α_n = 1.5

from Eq.2

$$[22.43/46.09]^{1.5} + [13.32/24.08]^{1.5} = 0.75 < 1 \quad \text{O.K.}$$

iii) Egyptian Code (Eqs.5)

t' = 54 cm, b' = 29 cm, t'/b' = 1.86

M_{ux}/M_{uy} = 1.68 < t'/b'

N_u/btf_{cu} = 0.184 ... β' = 0.79 (from Fig.6-18 in the Code)

from Eq.5.a M_{oy} = 22.83 t.m.

check the section for N_u = 106.53 ton and M_{oy} = 22.83 t.m.

i.e. e_x = 21.43 cm

for this eccentricity, the analysis gives the ultimate strength as:

N_u = 111.43 ton > 106.53 ton ..O.K. and

M_{oy} = 23.88 t.m.

iv) Equations 4

M_{uy} / M_{ux} = 0.59 > b / t = 0.58

M_{oy} = 21.00 t.m. (Eq.4-a)

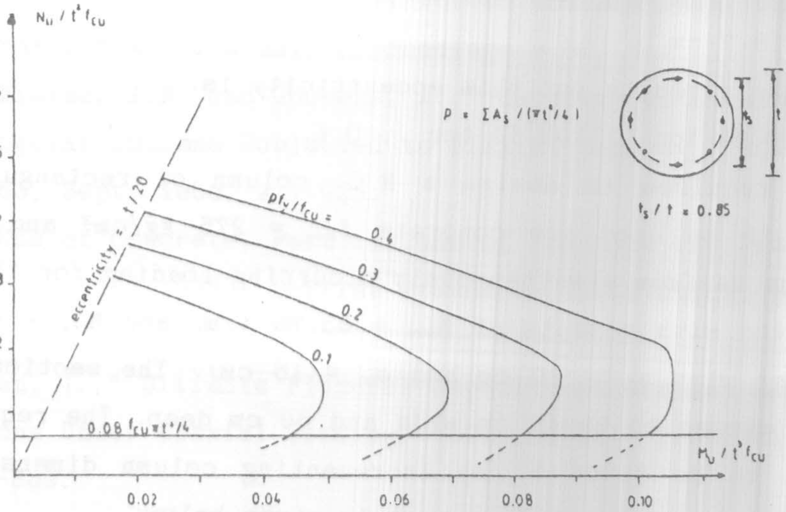


Fig.9 Example 2; interaction diagrams for circular section subject to eccentric load

f_{cu} = 275 kg/cm², f_y = 4130 kg/cm², E_s = 2000 t/cm²

Σ = 40.0 cm²

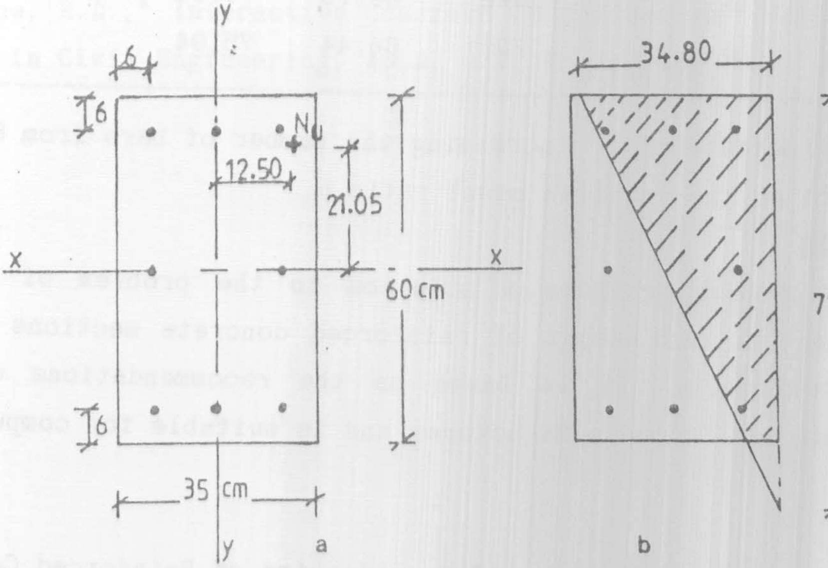


Fig.10 Example 3; rectangular section subject to biaxial eccentric load
a) geometry b) neutral axis location

check the section for N_u = 106.53 ton and M_{oy} = 21.00 t.m.
i.e. e_x = 19.7 cm
the ultimate strength for this eccentricity is:
N_u = 119.64 ton > 106.53 ton .. O.K.

4. It is required to design a R.C. column of rectangular section. The materials to be used are concrete f_{cu} = 275 kg/cm^2 and steel f_y = 4130 kg/cm^2. The maximum simultaneously occurring loading for the column is: N_u = 225 ton with moments of M_{ux} = 33.75 t.m. and M_{uy} = 45.0 t.m. (i.e. a load acting with e_x = 20 cm and e_y = 15 cm). The section dimensions were initially chosen to be 45 cm wide and 60 cm deep. The required steel ratio was found to be 3.804 %. By incrementing column dimensions, a table of design solutions for reinforcement is given below:

section	no. of bars	ρ, %	a, cm	h, cm	no. of iterations
45 x 60 cm	8	3.804	42.08	98.55	4
50 x 60 cm	8	2.870	49.30	90.72	5
60 x 60 cm	8	1.671	63.79	79.94	5
60 x 60 cm	10	1.706	66.44	75.94	5

The results indicate that increasing the number of bars from 8 to 10 has a slight effect on the required steel ratio ρ.

5. CONCLUSIONS

The present method provides a solution to the problem of the ultimate strength analysis and design of reinforced concrete sections subjected to biaxial eccentricity. It is based on the recommendations of the Draft Egyptian Code for Concrete Structures and is suitable for computer usage.

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