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THE ACCURACY OF TWO APPROXIMATE TILE DRAINAGE DESIGN EQUATIONS

Helmi M. Hathoot

Professor of Hydraulic Engineering,

Faculty of Engineering, Alexandria University

Alexandria, Egypt.

Abstract

The accuracy of an approximate drain discharge equation and that of an approximate drain spacing equation are investigated in this paper.

The case considered herein is that of draining a top clay cap underlain by an impervious substratum by a system of equally spaced drain tubes. The clay cap is considered to be homogeneous and isotropic.

Two computer programs are prepared to evaluate exact and approximate discharges as well as spacings for a wide range of variables covering many practical conditions.

Graphs for discharge and spacing are plotted and curves according to exact and approximate equations are put side by side for comparison purposes.

Practical limits for fairly accurate results, which can be obtained when approximate equations are applied, are suggested according to the

graphical analysis presented in this paper.

In fact, the approximate equations, when applied within their range of validity, are of major importance to designers since the equations are simple and practical.

Nomenclature

D	Top clay thickness below drains
d	Drain diameter
H	Height of water table above drains midway between two drains
H_o	Depth of drains below ground level
K	Hydraulic conductivity of clay
L	Spacing between drains
Q	Discharge reaching each unit length of drain
T	Specified time period
t	Time
μ	Drainable porosity of clay.

Introduction

Many investigation have attempted problems of designing tile drainage systems on different lines of approach [1,2]. The problem of draining a top clay cap by a system of equidistant drain tubes, Fig. 1, has been investigated by researchers such as Kirkham [3,4], Glover [5], Hammad [6,7,8] and Hathoot [9,10,11,12].

In an earlier paper Hathoot [9] presented an exact drain discharge equation which was simplified to yield another approximate one. On the basis of the approximate discharge equation and according to the

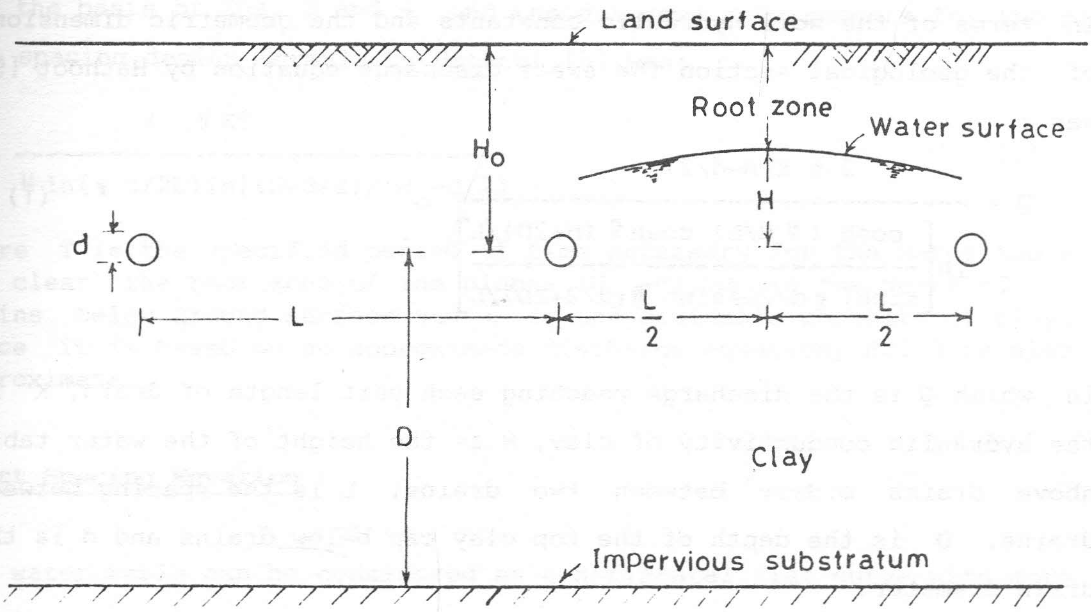

Fig.1: Geological Section .

drawdown requirements of plant Hathoot [9] provided an aproximate drain spacing equation.

The objective of this paper is to investigate the accuracy and range of validity of the approximate equations.

Drain Discharge Equations

In terms of the soil hydraulic constants and the geometric dimensions of the geological section the exact discharge equation by Hathoot [9] was :

$$Q = \frac{2\pi K(H-d/2)}{\ln\left[\dfrac{\cosh(\pi H/L)\,\cosh\pi(H+2D)/L}{\sinh(\pi d/2L)\,\sinh\pi(d/2+2D)/L}\right]} \qquad (1)$$

in which Q is the discharge reaching each unit length of drain, K is the hydraulic conductivity of clay, H is the height of the water table above drains midway between two drains, L is the spacing between drains. D is the depth of the top clay cap below drains and d is the drain diameter.

Hathoot introduced the following approximations to simplify Eq. 1:

$\cosh(\pi H/L) \simeq 1.0$

$\cosh \pi(H+2D)/L \simeq \frac{1}{2}\exp\ (H+2D)/L$

$\sinh \pi(d/2+2D)/L \simeq \frac{1}{2}\exp\ (d/2+2D)/L$, and

$\sinh(\pi d/2L) \simeq \pi d/2L$, and the resulting approximate discharge

equation was

$$Q = \frac{2\pi K(H-d/2)}{\pi(H-d/2)/L - \ln(\pi d/2L)} \qquad (2)$$

Drain Spacing Equation

On the basis of Eqs. 2 and 4 and the drawdown reqyirements for the plant, the spacing design formula by Hathcot [9] was:

$$L = \frac{2\pi KT}{\mu \ln(\pi d/2L)\ln[(H-d/2)/(H_o-d/2)} \qquad (3)$$

where T is the specified period of time necessary for the water table to clear the root zone of the plants (H_o -H), H_o is the depth of drains below ground surface and μ is the drainable porosity of clay. Since it is based on an approximate discharge equation, Eq. 3 is also approximate.

Exact Spacing Equation

The water table can be considered as a horizontal flat curve with more or less local depressions above drain tubes [8, 13]. After a heavy rain or after irrigation it is necessary to lower the water table from H_o to H within a specified period of time, T, otherwise the plant would die. The root zone and the period, T, depend on the kind of plant. The differential equation approximating the unsteady movement of the water table may be put in the from:

$$Q = -\mu L(dH/dt) \qquad (4)$$

in which t is the time.

Substituting Q as given by Eq. 1 into Eq. 4:

$$\frac{2\pi K(H-d/2)}{\ln\left[\dfrac{\cosh(\pi H/L)\cosh\pi(H+2D)/L}{\sinh(\pi d/2L)\sinh\pi(d/2+2D)/L}\right]} = -\mu L(dH/dt)$$

Therefore:

$$\int_0^T dt = -(\mu L/2\pi K)\int_{H_0}^{H} \ln\left[\frac{\cosh(\pi H/L)\cosh\pi(H+2D)/L}{\sinh(\pi d/2L)\sinh\pi(d/2+2D)/L}\right]\frac{dH}{(H-d/2)}$$

Integrating and rearranging:

$$= -(2\pi KT/\mu)/\int_{H_0}^{H} \ln\left[\frac{\cosh(\pi H/L)\cosh\pi(H+2D)/L}{\sinh(\pi d/2L)\sinh\pi(d/2+2D)/L}\right]\frac{dH}{(H-d/2)} \quad (5)$$

Drain spacings evaluated by Eq. 5 are referred to as exact spacings although they are subject to a small error due to the assumption of a flat horizontal water table.

Two computer programs. Prepared by the author, are applied to evaluate exact and approximate values of discharge and spacing. In designing computer programs, a wide practical range of the variables affecting discharge and spacing are considered.

Accuracy of the Approximate Discharge Equation

Figure 2 is plotted to investigate the characteristics of the discharge equations, the solid lines are for the exact equation and the dashed lines for the approximate one. The practical spacing ratio, L/d = 310.0, is considered. The drain diameter is taken as d = 0.1 m, since this diameter is extensively used in tile drainage practice. Figure 2 shows that discharges computed according to the approximate Eq. 2 are independent of the clay thickness below drains. In all cases approximate discharges are greater than exact ones. For small D/L ratios, differenes between exact discharges and approximate ones are considerable and as D/L increases, differences become smaller. To inspect the accuracy of the approximate discharge Eq. 2, Fig. 3 is plotted. Percentage difference in discharge ratio versus clay thickness ratio is shown plotted in Fig. 3. It is clear that the water table height ratio, H/d, has a negligible effect on the percentage difference in discharge ratios. Though at D/L = 0.129 the difference is about 7 %, yet at clay thickness ratios as small as D/L = 0.2 the difference is less than 3 %. At D/L = 0.3 the difference is about 1 %. The above figures indicate that as D/L increases the accuracy of the approximate equation rapidly improves.

Accuracy of the Approximate Spacing Equation

To study the characteristics of the exact and approximate spacing equations, Figs. 4,5 and 6 are plotted., Solid lines indicate exact spacings while dashed lines represent approximate ones. In Figs. 4,5 and 6 spacing ratio $L/(2\pi KT/\mu)$ versus clay thickness ratio D/L is plotted. The water table height ratios H_o = 6.0. 12.0 and 18.0 are considered in Figs. 4,5, and 6, respectively. A common practical ratio

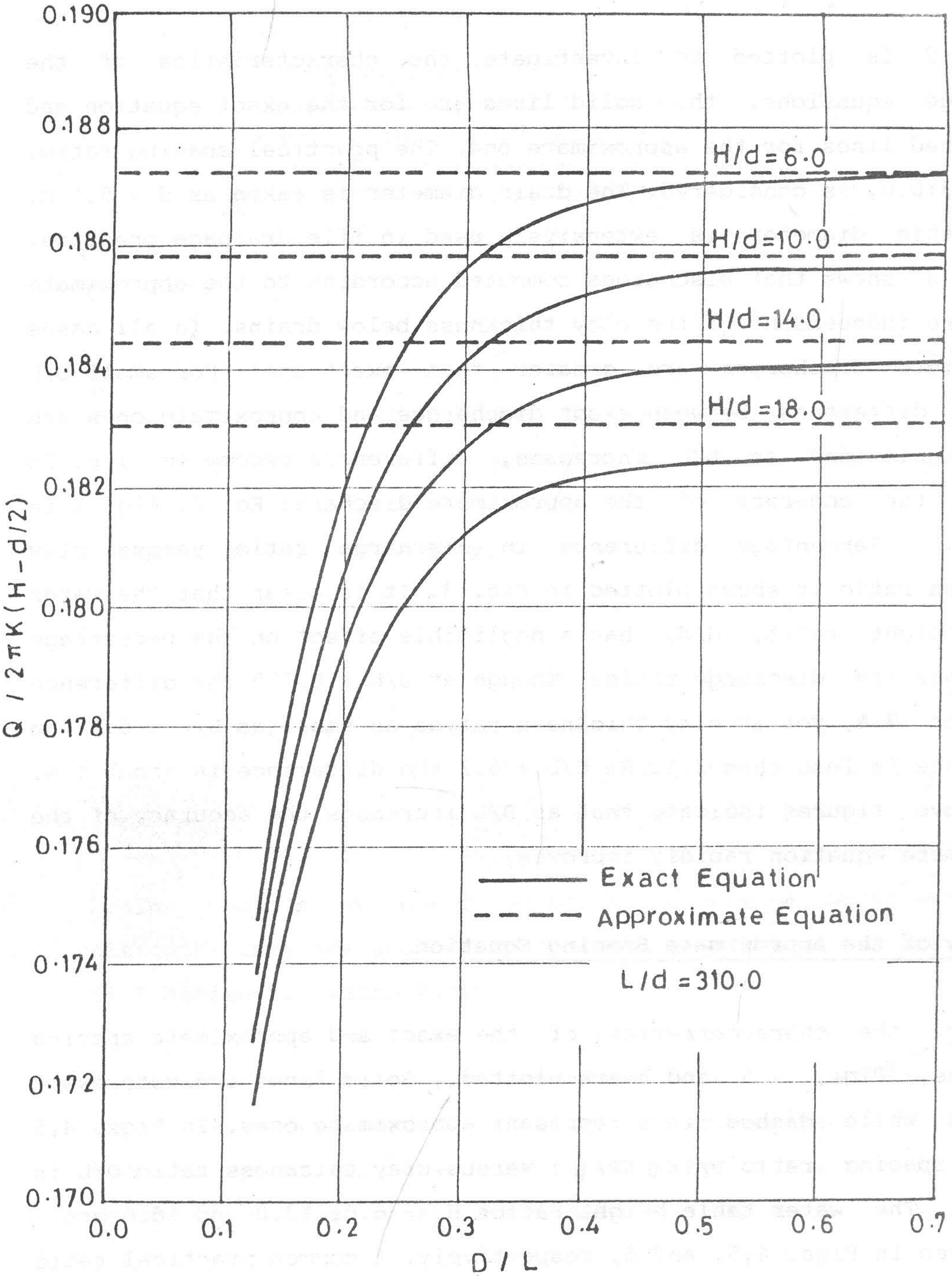

Fig.2: Discharge Ratio Versus Clay Thickness Ratio.

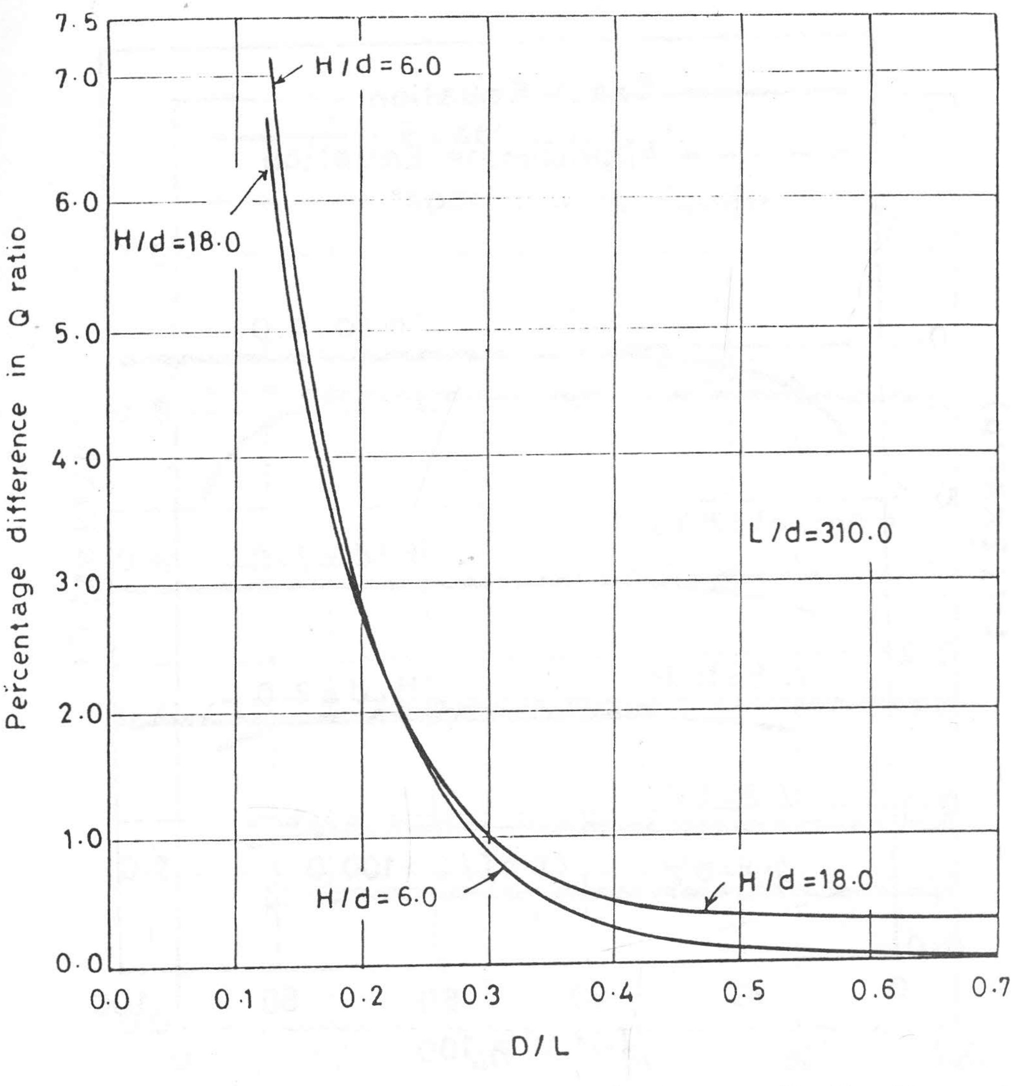

Fig. 3: Percentage Difference in Discharge Ratio Versus Clay Thickness Ratio.

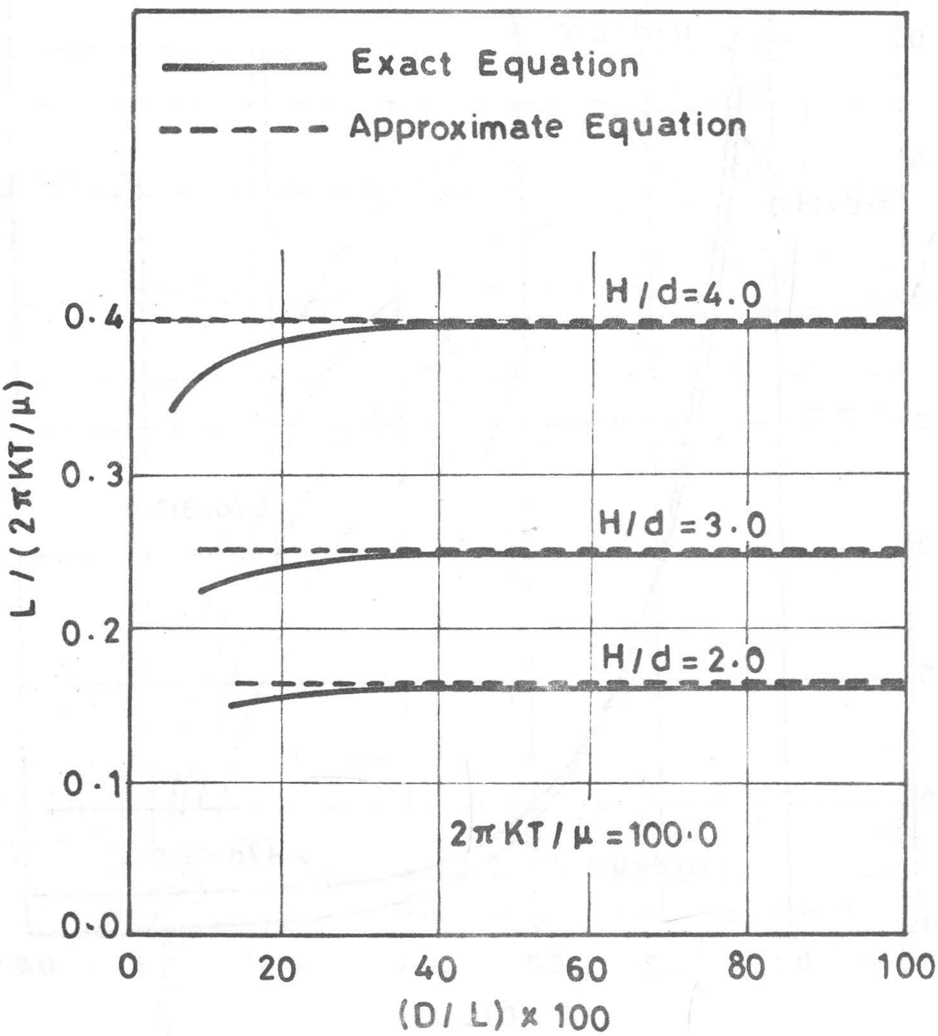

Fig.4: Spacing Ratio Versus Clay Thickness Ratio ($H_o/d = 6.0$).

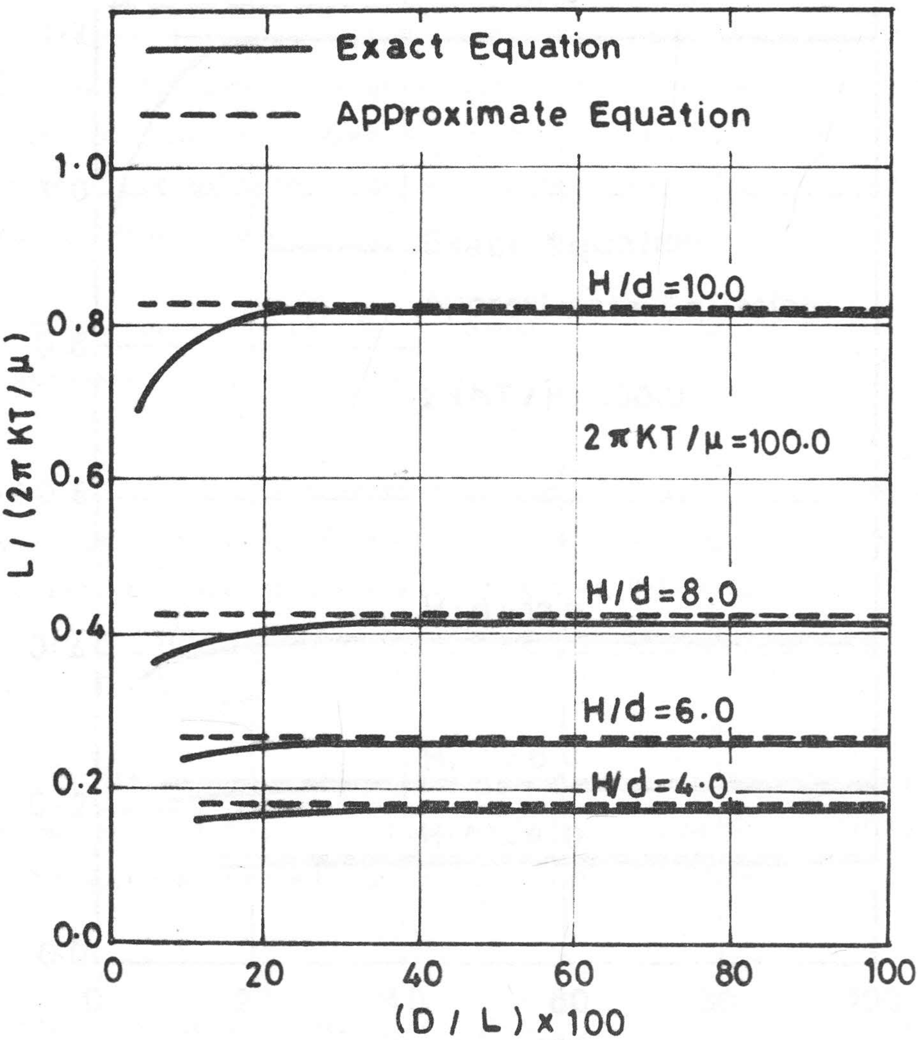

Fig. 5: Spacing Ratio Versus Clay Thickness Ratio ($H_o/d = 12.0$).

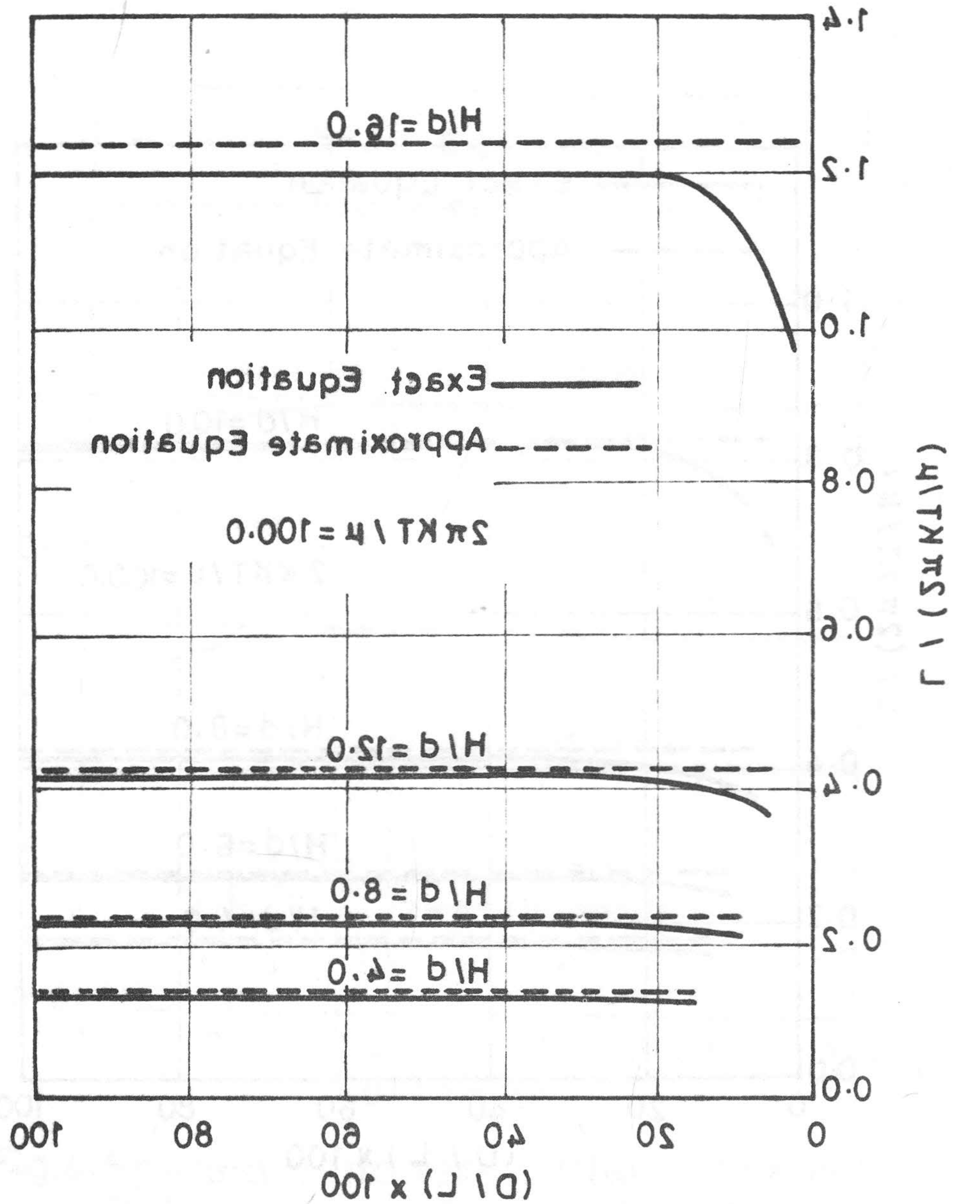

Fig. 6: Spacing Ratio Versus Clay Thickness Ratio (H_o/d = 18.0) .

$(2\pi KT/\mu) = 100.0$ is taken into account in all cases. As it is evident from Figs. 4,5 6, approximate spacings are greater than exact ones, and it is also clear that approximate spacings are independent of the clay thickness. In cases of small clay thickness ratios D/L, differences between exact and approximate spacing ratios are noticeable. As D/L increases differences decrease and hence the accuracy of the approximate equation improves. Differences between exact and approximate spacing ratio versus clay thickness ratio is plotted in Figs. 7 and 8.

Fig. 7 is devoted to drain depth ratio $H_o/d = 6.0$ while Fig. 8 corresponds to $H_o/d = 12.0$ and 18.0.

In Fig. 7 for clay thickness ratios as small as 0.1 the difference between exact and approximate spacing ratios ranges between 9 % and 11 %. Differences are of the order of 3 % at D/L = 0.3 and D/L = 0.2 for H/d = 2.0 and H/d = 4.0, respectively. For both curves differences are below 2 % for $D/L \geq 0.5$.

In Fig. 8 differences between exact and approximate spacing ratios are as high as 9 % for D/L = 0.1, considering H/d = 10, and H/d = 16.0. For the above mentioned curves, differences are less than 3 % for $D/L \geq 0.2$.

For the two curves corresponding to H/d = 4.0 the difference ranges between 6.3 % and 8 % at D/L = 0.2 while at D/L = 0.3, differences are between 4% and 6.4 %. It is worthy to note that in Fig. 8 for H/H_o = 0.22, differences are 5% while for $H/H_o = 0.33$ differences are over 3%, in the range $D/L \geq 0.5$.

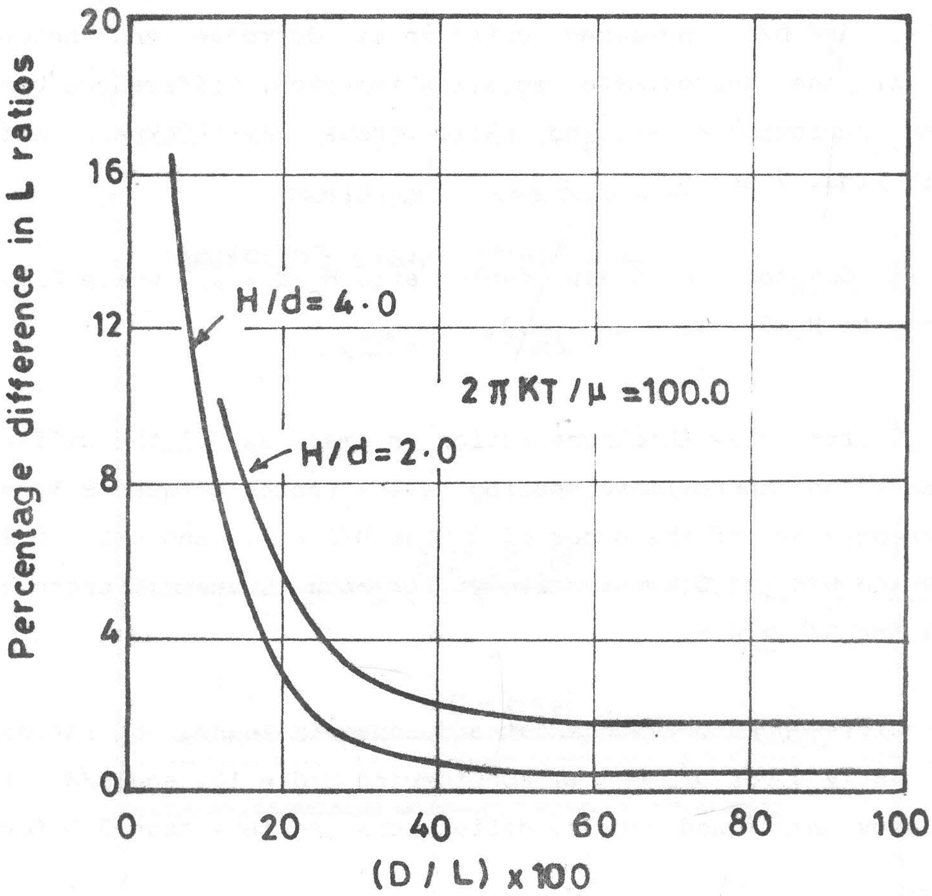

Fig.7: Percentage Difference in Spacing Ratio Versus Clay Thickness Ratio ($H_o/d = 6.0$).

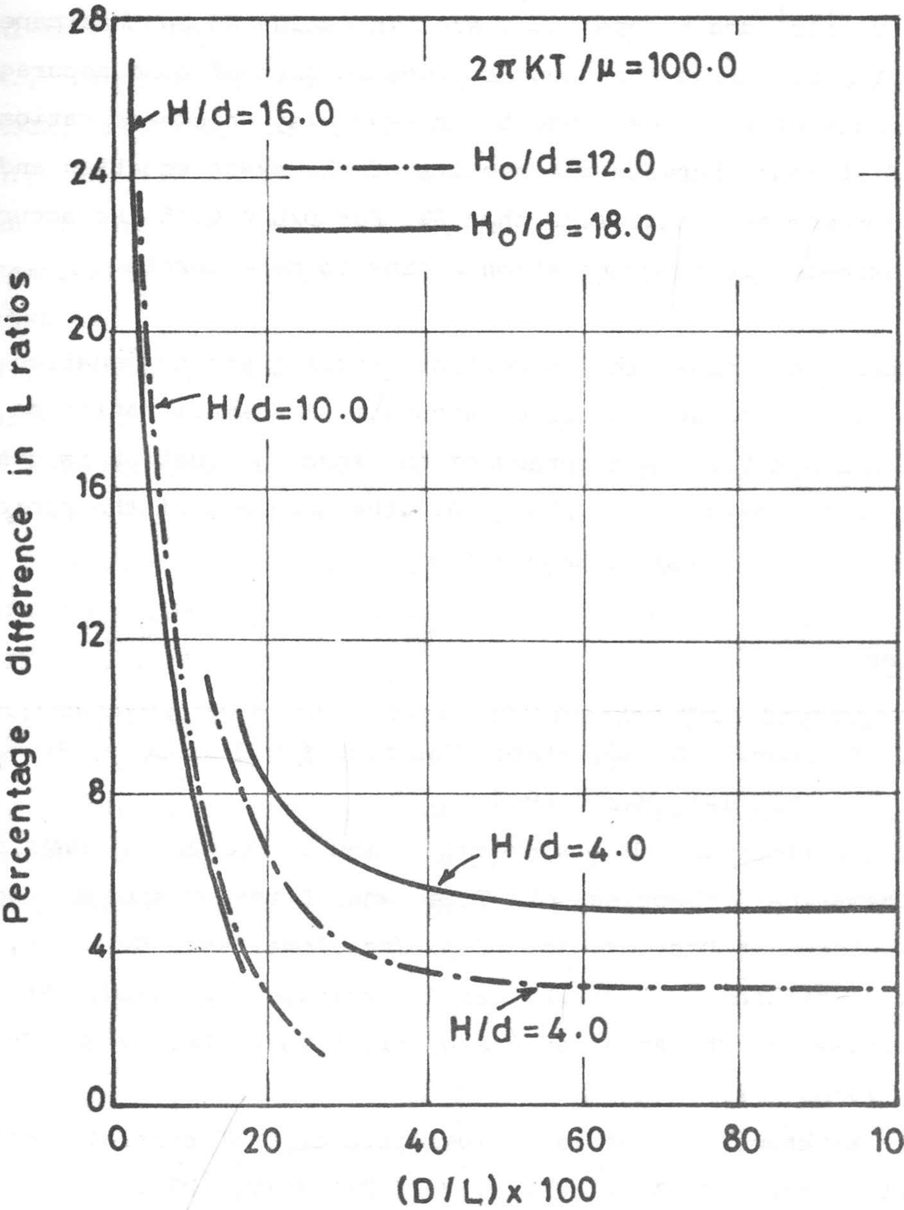

Fig. 8: Percentage Difference in Spacing Ratio Versus Clay Thickness Ratio (H_o/d=12.0 and 18.0)

Conclusions

The accuracy of two approximate equations, one for drain discharge and the other for drain spacing, are investigated in this paper. The approximate discharge equation provides results of good accuracy over a wide range of practical conditions. For clay thickness ratios $D/L \geq 0.2$ the difference between the results of the exact equation and those of the approximate one is less than 3%. For $D/L < 0.15$ the accuracy of the approximate discharge equation begins to deteriorate.

In most practical cases the approximate spacing design equation can be applied with adequate degrees of accuracy. For small ratios as $H/H_o < 0.2$ and $D/L < 0.2$ the accuracy of the spacing equation is generally low. In all cases for $D/L > 0.3$ the accuracy of the approximate spacing equation is higher than 6.4 %.

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