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SYNTHESIS OF AN OPTIMUM ENERGY-ABSORBING SYSTEM FOR AUTOMOTIVE APPLICATIONS

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Abstract

The kinematic properties of nonlinear spring mass system possessing nonlinear damping force function was investigated in order to design an optimum energy dissipation system during impact. Four types of damping force function were considered: coulomb, viscous, quadratic and mixed coulomb_viscous damping. Cushioning material properties are synthesized such that the impacting mass does not rebound, the peak deceleration is a minimum and the absorption efficiency is maximum. It was found that the optimum energy dissipation system is the mixed coulomb and viscous critically damped system and characterized by high nonlinear spring rate. The optimum system parameters were determined and tabulated.

Nomenclature

E	Dissipated energy with respect to the initial kinetic energy
f(v)	Dimensionless damping force.
g	Gravitational Constant.
k	Linear spring rate.
m	Mass.
PI	Performance index.
T	Kinetic energy.
$\bar{t}$	Time.
t	Dimensionless time.
$\bar{v}$	Velocity.
v	Dimensionless velocity.
v'	Dimensionless acceleration.
$\bar{x}$	Displacement.
x	Dimensionless displacement.
β	Nonlinear spring rate.
ε	Dimensionless nonlinear spring rate.
ξ_o, ξ_1, ξ_2	Coulomb, viscous, and quadratic damping parameters.

Subscripts

f	Final
max.	Maximum
o	Initial

Introduction

The shock and vibration hazards experienced by goods during shipment result from the vibration of a cargo-carrying vehicle, the impact of railroad cars, and the shock caused by handling (e.g. dropping). To provide the necessary isolation and protection of packages, a resilient means, known as package cushioning, is interposed between the equipment and the container. Similar isolation is also needed to mitigate the effects of vehicles striking a barrier or vehicle collisions. The most important materials used as a cushion are polymers, various plastic foams and latex hair [1].

The kinematic properties of systems possessing a nonlinear damping-force function play an important role in the selection of shock-absorbing materials. Lakin and Sachs [2] studied the energy dissipation of nonlinear critically damped systems during the impact assuming linear-spring behavior. They showed that the design of an optimum energy-dissipation system depends on particular forms of the damping force function and particular values of the system parameters. Sachs [3] studied the characteristics of a nonlinear supercritical damped mass elastic system with linear spring rate. Iwata and Kobori [4] investigated the response of a single-degree-of freedom nonlinear spring mass system with viscous damping when an impulsive force acts on a vibrating system. Kulagin and Prourzin [5] formulated the optimal control and spatial motion of a shock absorbing rigid body. The quality criteria were taken to be maximum absolute value of the deflection of the cushion mass from the moving base and the maximum absolute acceleration of the cushioned mass. Rice et al [6] proposed design guidelines with a view to optimization of the system parameters and the selection of spring type for linear damped absorber.

Although cushioning materials exhibit linear force-deflection characteristics for small deflections, efficient package design involves large deflections and consequent nonlinearity of cushioning materials [1,7,8]. In the meantime, these materials obey the nonlinear damping force law. The previous literature review reveals that the selection of shock absorbing material was proceeded by considering either nonlinear damping function or nonlinear spring rate.

The purpose of this work is to synthesize the optimum energy dissipation system parameters considering the nonlinearity of both the spring and the damping. The optimization will be based on critical damping condition, lower peak deceleration of the impacting mass, and the maximum efficiency of energy dissipation.

Method of Approach

Since the cushioning material is characterized by its softening behavior and non-linear damping, the normalized equations of motion of free falling mass upon a viscoelastic foundation or a vehicle striking a barrier have the form

$$\frac{dv}{dt} = - f(v) - x + \varepsilon x^3 \qquad (1)$$

$$\frac{dx}{dt} = v \qquad (2)$$

and the initial conditions at the time of impending impact (t=0) are:

$$v(0) = v_o , \quad x(0) = 0 \qquad (3)$$

where $t = \omega \bar{t}$, $\omega^2 = k/m$, $x = \bar{x}/x_{stat}$,

$$x_{stat} = mg/k\ , \quad v_o = \bar{v}_o/\omega\, x_{stat}\ , \quad \varepsilon = \beta\, x_{stat}^2/k$$

and $\bar{t}$, x, v_o, g, k, , f(v) are time, displacement, initial velocity, gravitational constant, linear spring rate, nonlinear spring rate and dimensionless damping force, respectively.

The general form of the velocity dependent resistance law can be written as

$$f(v) = \sum_{o}^{n} 2\, \xi_n\, v^{n+1} / |v|\, v_o^{n-1} \qquad (4)$$

Criteria for the selection of cushioning materials properties

An optimum energy absorption system is synthesized based on the following criteria.

1. **No Rebound of Impact Mass (critical damping)**

Criteria of critical damping is introduced to describe the limit of oscillatory and nonoscillatory motion. It can be deduced from a qualitative study of the phase-plane trajectories in the neighborhood of singular points (focus, node, or saddle point). Lakin and Sachs [2] showed that if the phase-plane trajectories of a nonlinear system have a singular point which has the same geometrical properties as those of a critically damped linear system, one may infer that the nonlinear system satisfies the criteria of critical damping of a linear system. This means that for critical damping, the trajectories are confined to the half planes ($\pm$ v(t), x(t)) and at the limit, the singular point (v=x=0) is approached from the direction -x(t).

2. Peak Deceleration, $\dot{v}'_{max}$

The fragility index of equipment [1] as estimated from experience in dropping similar equipment applies only to the peak deceleration. In general, the natural frequencies of important component of the equipment are substantially greater than the natural frequency of the package cushioning. Under these conditions, damage tends to be directly proportional to the maximum deceleration. Therefore the form of the dissipation function and the nonlinear spring rate will be determined to produce lower peak deceleration values.

3. Energy Dissipation Efficiency

The energy dissipation within the absorber with respect to the initial kinetic energy is

$$E = (T_o - T_f)/T_o$$

$$= 1 - \left(\frac{v_f}{v_o}\right)^2$$

where v_f is the rebound velocity, and T_o, T_f are the initial and final kinetic energy.

Defining the performance index PI of the absorber as,

$$PI = \left[1 - \left(\frac{v_f}{v_o}\right)^2 \right] / \dot{v}'_{max} \qquad (5)$$

the optimum energy absorbing system is determined upon maximizing the performance index.

Results and Discussions

Equations (1) and (2) are solved numerically using Runge-Kutta fourth order technique for dimensionless nonlinear spring rate values range from 0 to 0.7, and for different forms of dissipation functions (coulomb, viscous, quadratic, and mixed coulomb-viscous damping). The results are illustrated in Figures 1 through 5.

Criteria of Critical Damping

Figure 1 presents the relation between the ratio of the rebound to initial velocity and the damping parameter. The figure indicates that both the rebound velocity and the critical damping value slightly decrease with the increase in the nonlinear spring rate. As ε increases from 0 to 0.7, the critical coulomb damping parameter is reduced from 0.176 to 0.154, while the critical viscous damping decreases from 1.0 to 0.93. The values of critical coulomb damping can also be determined analytically using equations (A-5) and (A-7) of the appendix. The effect of nonlinear spring rate is negligible when the quadratic damping is considered. It is worthnoting that there is no finite critical quadratic damping parameter. This means that the singular point of such a system is focus point which is characterized the motion by being subcritical which agrees with reference 2 results for ε =0. The proof is given in the Appendix.

As for the mixed coulomb-viscous damping, there exists an infinite number of parameter combination (ξ_o and ξ_1) that result in critical damping. These values are plotted in Figure 2.

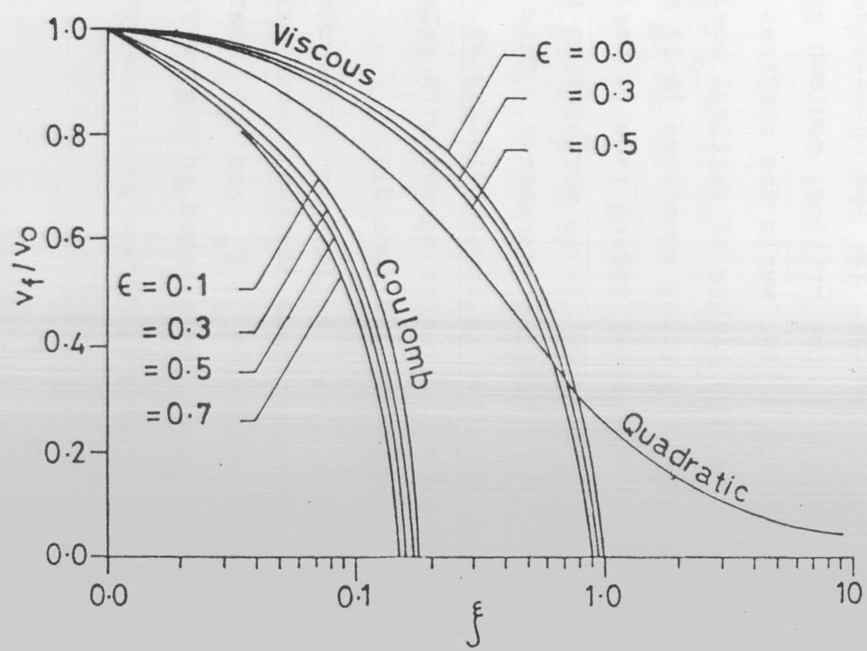

Figure 1: Effect of damping parameters and spring nonlinearity on the rebound velocity.

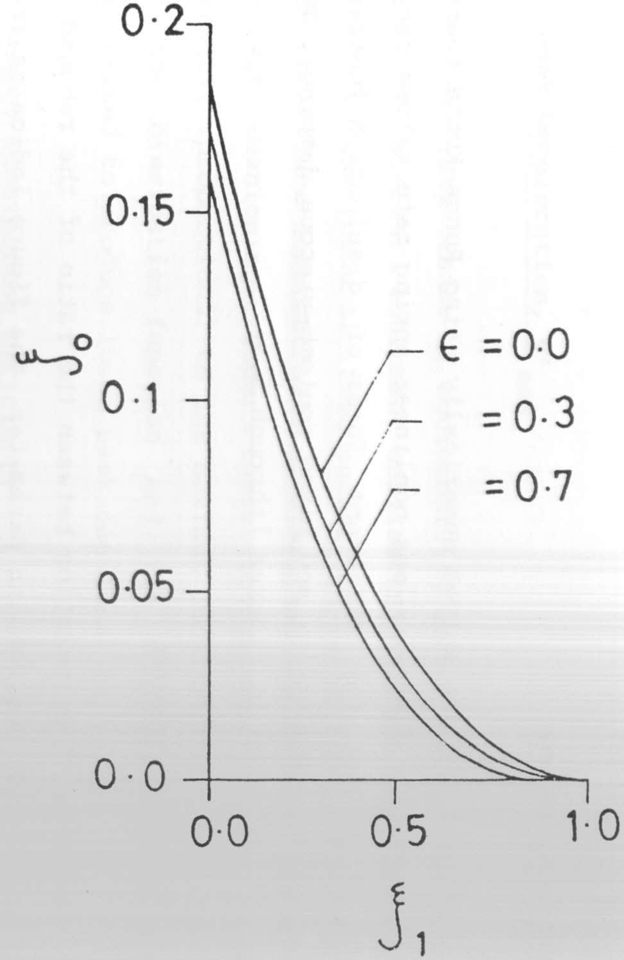

Figure 2: Critical damping parameters for mixed coulomb-viscous damped system

Peak Deceleration

The absolute values of the peak deceleration as a function of the damping parameter are shown in Figure 3, and lower peak deceleration values are reported in Table 1.

Table 1: System with minimum peak deceleration

Damping Form	ε	0.0	0.1	0.3	0.5	0.7
Coulomb	$\lvert v'_{max} \rvert$	1.0	0.93	0.742	0.624	0.54
	ξ_o	0	0	0	0	0
Viscous	$\lvert v'_{max} \rvert$	0.813	0.716	0.712	0.57	0.5106
	ξ_1	0.3	0.25	0.02	0	0
Quadratic	$\lvert v'_{max} \rvert$	0.707	0.68	0.6	0.6	0.51
	ξ_2	0.354	0.32	0.3	0.3	0

For coulomb damping, the lower peak deceleration values occur at ξ_o = 0 and it decreases from 1.0 (at ε=0) to 0.54 (at ε=0.7). When the viscous damping is considered, it can be noticed from the figure that the damping parameters that produce lower peak deceleration values tend to decrease with the increase in ε. In the case of quadratic damping, there exists a minimum value for v'_{max} around ξ_2 = 0.3 and it is decreased from 0.707 to 0.6 as ε changes from 0 to 0.5. As ε increases to 0.7, a minimum value of 0.51 occurs at ξ_2 = 0. For

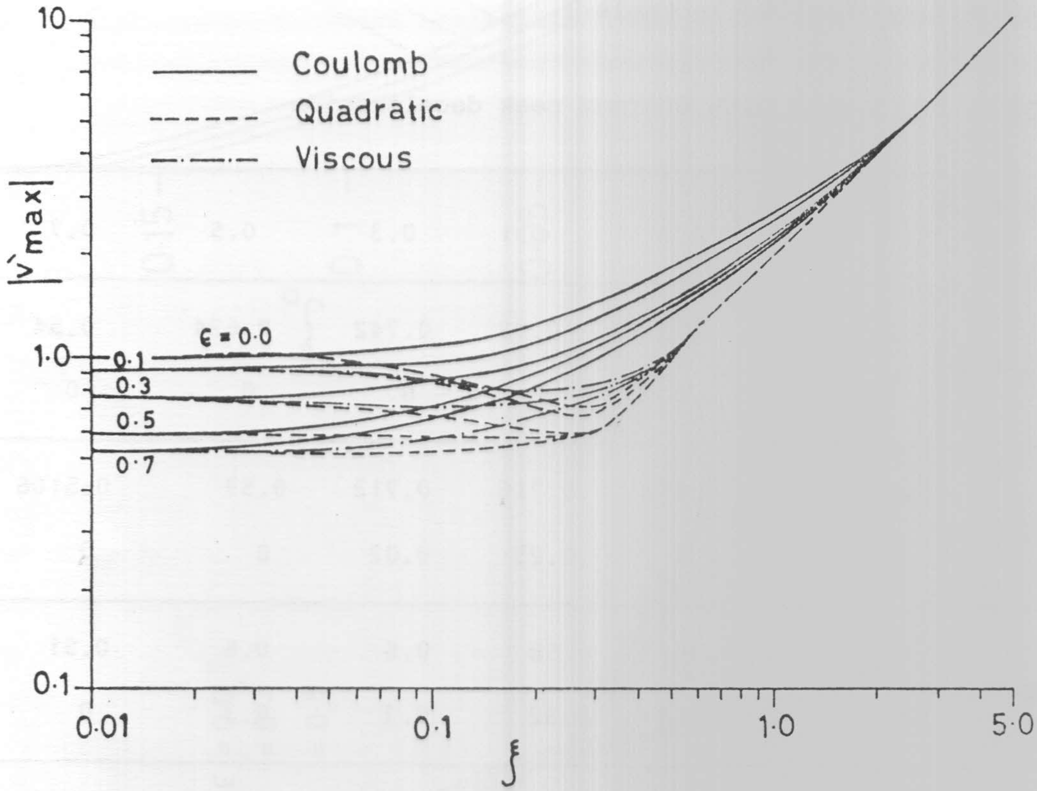

Figure 3: Effect of damping parameters and spring nonlinearity on the peak deceleration

the same value of ε, it can be observed that the quadratic dissipation function produces the minimum peak deceleration.

Energy Dissipation Efficiency

Energy dissipation PI values are reported in Figure 4 for the case of coulomb, viscous, and quadratic damping and in Figure 5 for the case of mixed columb-viscous damping.

Comparing the maximum performance index values in Figure 4, it appears that the coulomb damping results in the highest PI while the viscous damping produces the lowest values. It can also be noticed that PI_{max} increases with the increase in the nonlinear spring rate. For example, as ε increases from 0 to 0.7, PI_{max} for coulomb damping increases from 1.032 to 1.29 and it occurs at the critical damping parameters. For the viscous damping, PI_{max} occurs around $\xi_1 = 0.5$ and its value increases from 0.685 to 0.785. Meanwhile PI_{max} increases from 0.935 to 1.18 and occurs around $\xi_2 = 0.3$ in the case of the quadratic damping.

Considering the critical parameter combination of mixed coulomb-viscous damping, one may notice from Figure 5 that it results in the maximum PI_{max}. At $\varepsilon = 0.0$, PI_{max} is 1.1 and at $\varepsilon = 0.5$, PI_{max} is equal to 1.195 as compared to 1.032 and 1.16 respectively for coulomb damping.

Conclusions

An optimum energy absorbing system has been synthesized based on:

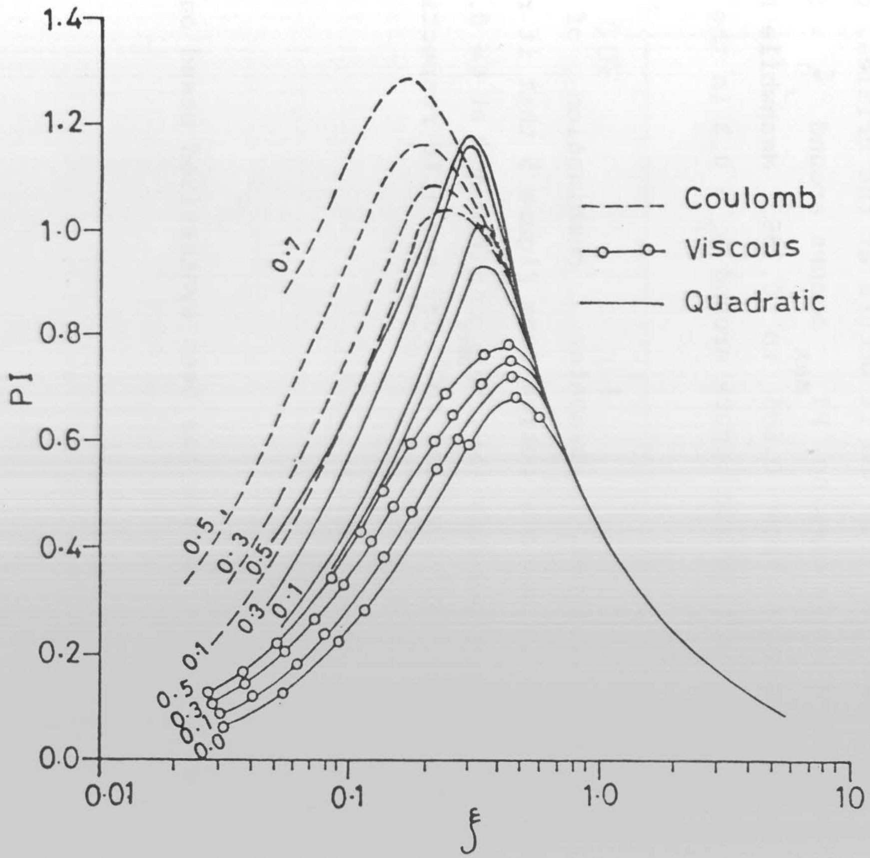

Figure 4: Energy dissipation performance index for coulomb, viscous or quadratic damped system.

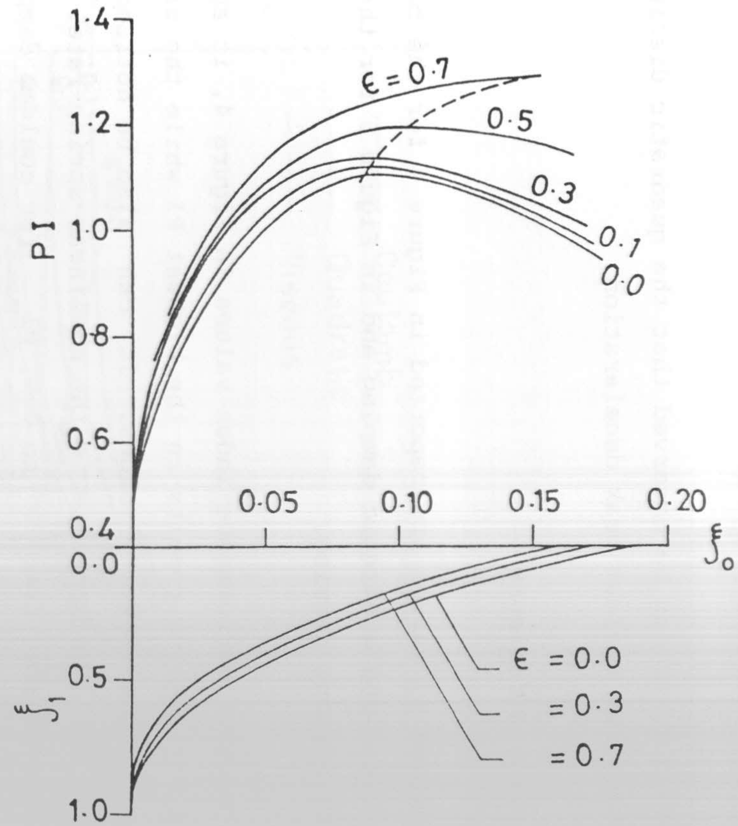

Figure 5: Energy dissipation performance index for mixed coulomb-viscous damped system.

1. No rebound of impacting mass
2. Lower peak deceleration
3. Maximum absorption efficiency

Four forms of damping force function, (coulomb, viscous, quadratic and mixed coulomb and viscous damping), were investigated, when the dimensionless nonlinear spring constant varies from 0 to 0.7.

It was found that there exists no finite critical quadratic damping parameter. Therefore, the motion with quadratic damping is always subcritical and renders a rebound velocity. The critical coulomb damping results in higher absorption efficiency and lower peak deceleration than the critical vicous damping. Meanwhile, the mixed coulomb-viscous critically damped system produces the best efficiency as compared to the coulomb critically damped one.

It is concluded that the optimum energy dissipation system, that satisfies all the design criteria, is the mixed coulomb and viscous critically damped system and whose parameters are reported in Table (2)

Table (2): Optimum energy dissipation system parameters

ε	0	0.1	0.3	0.5	0.7
ε_o	0.085	0.09	0.095	0.105	0.15
ξ_1	0.27	0.25	0.2	0.16	0

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Appendix: Closed-form expression for critical damping parameter

1. Coulomb damping

Considering only the dry friction, the normalized damping force will be

$$(f(v) = 2 \xi_o v_o v / |v| \qquad \text{(A-1)}$$

Substituting f(v) in equation (1) and dividing equation (1) by (2) give,

$$\frac{dv}{dx} = (\mp 2 \xi_o v_o - x + \varepsilon x^3)/v \qquad \text{(A-2)}$$

Equation (A-2) has the solution in the form

$$v^2 = \mp 4 \xi_o v_o x - x^2 + \varepsilon x^4/2 + C \qquad \text{(A-3)}$$

where the upper sign holds for $v > 0$, and the lower sign for $v < 0$.

For $v > 0$, and using the initial conditions $[v(0)=v_o, x(0)=0]$, the integration constant C_1 is

$$C_1 = v_o^2 \qquad \text{(A-4)}$$

At the end of the first quarter cycle, $[v=0, x = x_{max}]$, equations (A-3) and (A-4) result in

$$x_{max} = \sqrt{\frac{1}{2\varepsilon} + \left[\frac{1}{2\varepsilon} + v_o\sqrt{\frac{2}{\varepsilon}}\,(2 \xi_o + \sqrt{4 \xi_o^2 + 1})\right]^{\frac{1}{2}}} \qquad \text{(A-5)}$$

The begining of the second quarter $v < 0$, is the point $(x_{max}, 0)$, which determins the integration constant C_2 as

$$C_2 = v_o^2 - 8\,\xi_o\,v_o\,x_{max} \qquad (A-6)$$

The condition of the critical damping implies that, at the end the second quarter, the displacement as well as the velocity must vanish. Therefore:

$$\xi_{o,critical} = \frac{v_o}{8\,x_{max}} \qquad (A-7)$$

i.e. There exists a finite parameter $\xi_{o,critical}$ and its value depends on the dimensionless nonlinear spring rate constant ε .

2. Quadratic damping

For the quadratic damping,

$$F(v) = -\,2\,\xi_2\,v^3 / |\,v\,|\,v_o$$

the normalized equations of motion can be written in the form

$$\frac{dv^2}{dx} \pm 4\,(\xi_2/v_o)\,v^2 = -\,2x + 2\,\varepsilon\,x^3 \qquad (A-8)$$

For $v > 0$, and the initial point $(0, v_o)$, the solution will be

$$v^2=v_o^2\left[1-\frac{1}{8\xi_2^2}\left(1-\frac{3\varepsilon v_o^2}{8\xi_2^2}\right)\right]e^{-4(\xi_2/v_o)x}+\frac{\varepsilon v_o}{2\xi_2}x^3$$

$$-\frac{3\varepsilon v_o^2}{8\xi_2^2}x^2+\left(\frac{3v_o^3\varepsilon}{16\xi_2^3}-\frac{v_o}{2\xi_2}\right)x-\frac{3\varepsilon v_o^4}{64\xi_2^4}+\frac{v_o^2}{8\xi_2^2} \qquad \text{(A-9)}$$

For $v < 0$, the solution is

$$v^2=C_2e^{4(\xi_2/v_o)x}-\frac{\varepsilon v_o}{2\xi_2}x^3-\frac{3\varepsilon v_o^2}{8\xi_2^2}x^2-\left(\frac{3\varepsilon v_o}{16\xi_2^3}-\frac{v_o}{2\xi_2}\right)x$$

$$+\frac{v_o^2}{8\xi_2^2}\left(1-\frac{3\xi v_o^2}{8\xi_2^2}\right) \qquad \text{(A-10)}$$

where

$$C_2=e^{-4(\xi_2/v_o)x_{max}}\left[\frac{\varepsilon v_o}{2\xi_2}x_{max}^3+\frac{3\varepsilon v_o^2}{8\xi_2^2}x_{max}\right.$$

$$\left.+\left(\frac{3\varepsilon v_o^3}{16\xi_2^3}-\frac{v_o}{2\xi_2}\right)x_{max}-\frac{v_o^2}{8\xi_2^2}\left(1-\frac{3\varepsilon v_o^2}{8\xi_2^2}\right)\right]$$

and x_{max} can be determined from equation (A-9) and $v = 0$. At the end of the second quarter, x is equal to zero, and equation (A-10) gives

$$v_f^2=C_2+\frac{v_o^2}{8\xi_2^2}\left(1-\frac{3\varepsilon v_o^2}{8\xi_2^2}\right)\neq 0 \qquad \text{(A-11)}$$

i.e. The quandratic damping is subcritical.