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PROBLEM IN GENERALIZED THERMOELASTICITY FOR A HALF SPACE SUBJECT TO SMOOTH HEATING OF ITS BOUNDARY

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Abstract

The distribution of temperature, stress and displacement in a homogenecus isotropic solid occupying the half space and subjected to a smooth time dependent heating effect, only at its bounding surface, are investigated. The problem is formulated in the context of generalized thermoelasticity with one relaxation time.

The Laplace transform with respect to time is used to obtain the solution. Inversion of the resulting expressions is carried out using the small values of time approximation as well as numerical inversion formula. Numerical results for a particular case are given.

Nomenclature

T absolute temperature

σ_{ij} components of stress tensor

ρ density

k thermal conductivity

t time

λ, μ Lamé's constants

α_t coefficient of linear thermal expansion

C_E specific heat for processes with invariant strain tensor

γ $(3\lambda + 2\mu)\alpha_t$

T_o reference temperature chosen such that $|(T - T_o)/T_o| << 1$

θ $(T - T_o)/T_o$

β $[(\lambda + 2\mu)/\mu]^{\frac{1}{2}}$

b $\gamma T_o/\mu$

g $\gamma/\rho C_E$

τ_o relaxation time

ε gb/β^2

Introduction

The classical theory of heat conduction predicts that the effects of any thermal disturbances will propagate with infinite speed through the conducting medium. That is, such disturbances are felt instantaneously at distances far away from the source. This situation contradicts physical observations.

The theory of generalized thermoelasticity with one relaxation time, based on a modified Fourier's law of heat conduction, was developed by Lord and Shulman [1].

This theory allows for the so-called second sound effects in solids, which lead to propagation of thermal disturbances with finite wave speeds. This remedies the previously unrealistic situation in classical thermoelasticity theory.

The mathematical model of the generalized thermoelasticity theory is of a complicated nature which renders the derivation of closed form solution a difficult task. However, several particular situations were investigated, among which, the case of a spherically symmetric problem with a point source of heat by Sherief [2], the case of cylindrically symmetric problem with a line source of heat by Sherief and Anwar [3], the case of an infinite medium with a cylindrical hole subject to either a unit step in stress and zero temperature change, or a unit step in temperature and zero stress at the boundary of the hole by Sharma [4]. The case of plane wave propagation in a thermoelastic half-space under Smooth heating of its boundary was considered by Gladysz [5] in the context of classical theory of thermoelasticity. The present paper investigates the solution of the generalized thermoelasticity problem for a solid occupying the half space, and subjected to a smooth heating of its boundary which is mantained free from any mechanical loads.

Mathematical Formulation of the Problem

In the present paper, a homogeneous isotropic solid occupying the half space $z \geq 0$ is considered. The medium is initially quiescent while the plane $z = 0$, which bounds the half space $z > 0$, is maintained free from mechanical loads but subjected to a smooth time dependent heating.

In the absence of body forces and heat sources or sinks, the general equations of generalized thermoelasticity with one relaxation time are as follows [3],

a) the equations of motion

$$(\lambda + \mu)\, u_{j,ij} + \mu u_{i,jj} - \gamma T_{,j} = \rho \ddot{u}_i \quad , \tag{1}$$

b) the energy equation

$$k T_{,ii} = \rho C_E (\dot{T} + \tau_o \ddot{T}) + \gamma T_o (\dot{e}_{kk} + \tau_o \ddot{e}_{kk}) \quad , \tag{2}$$

c) the constitutive equations

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma (T - T_o)\, \delta_{ij} \quad , \tag{3}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \; . \tag{4}$$

The dots denote differentiation with respect to time and the comma denotes material derivative; the summation convention is used and δ_{ij} denotes the Kronecker delta symbol .

Introducing the dimensionless variables ;

$$x_i' = v\eta x_i \;, \quad t' = v^2 \eta t, \quad \theta = \frac{T - T_o}{T_o}$$

$$u_i' = v\eta u_i \;, \quad \tau_o' = v^2 \eta \tau_o \,, \quad \sigma_{ij}' = \frac{1}{\mu} \sigma_{ij}$$

where

$$v = \left(\frac{\lambda + 2}{\rho} \right)^{\frac{1}{2}} , \quad \eta = \frac{\rho C_E}{k} ;$$

into equations (1-4), dropping the dashes for convenience, we arrive at the following dimensionless equations:

The equations of motion

$$\beta^2 \ddot{u}_i = \beta^2 u_{j,ij} - b\theta , \tag{5}$$

The equation of energy

$$\theta_{,ii} = \dot{\theta} + \tau_o \ddot{\theta} + g(\dot{u}_{i,i} + \tau_o \ddot{u}_{i,i}) . \tag{6}$$

The constitutive equations

$$\sigma_{ij} = u_{i,j} + u_{j,i} + (\beta^2 - 2) u_{k,k} \delta_{ij} - b\theta \delta_{ij} . \tag{7}$$

We have made use of the corresponding dimensionless version of equation (4).

Since the solid occupies the half space with the plane $z=0$ as boundary ; the components of the displacement vector are taken as

$$u_1 = u_2 = 0 \,, \quad u_3 = u\,(z,t) \,.$$

Equations (5-6) reduce to

$$\beta^2 \ddot{u} = \beta^2 D^2 u - b\,\theta \tag{8}$$

$$D^2\theta = \dot{\theta} + \tau_o \ddot{\theta} + g D\,(\dot{u} + \tau_o \ddot{u}) \,, \tag{9}$$

$$\sigma_{xx} = (\beta^2 - 2)\, D u - b\,\theta \,, \tag{10}$$

$$\sigma_{yy} = (\beta^2 - 2)\, D u - b\theta \,, \tag{11}$$

$$\sigma_{zz} = D u - b\theta \,, \tag{12}$$

where σ_{xx} (z,t), σ_{yy} (z,t) and σ_{zz} (z,t) are the three components of the stress, (the remaining components are equal to zero), Furthermore , $D = \partial / \partial z$.

The boundary conditions for $t \geq 0$ at the plane $z = 0$ are taken as

(i) $\sigma_{zz}(0,t) = 0$ (13.a)

(ii) $\theta(0,t) = f(t)$ (13.b)

where f(t) represents the smooth heating effect, to be specified. The medium is assumed to be initially quiescent which leads to

$$\sigma_{zz}(z,0) = \dot{\sigma}_{zz}(z,0) = \theta(z,0) = \dot{\theta}(z,0) = 0 \tag{14}$$

Furthermore, for physically acceptable solutions the usual regularity conditions for the corresponding field variables must be imposed as $z \to \infty$.

Solution Of The Problem

The solution of the problem formulated in the previous section and described by equations (8-14) is carried out by introducing the thermoelastic potential function φ defined as ,

$$u = D\varphi \quad . \tag{15}$$

Using equation (1), equations (8) - (12) become ,

$$\theta = \frac{\beta^2}{b}(D^2\varphi - \ddot{\varphi}) \tag{16}$$

$$D^2\theta = \dot{\theta} + \tau_o\ddot{\theta} + g\, D^2(\dot{\varphi} + \tau_o\ddot{\varphi}) \tag{17}$$

$$\sigma_{xx} = (\beta^2 - 2)\, D^2\varphi - b\theta \tag{18}$$

$$\sigma_{yy} = (\beta^2 - 2)\, D^2\varphi - b\theta \tag{19}$$

$$\sigma_{zz} = D^2\varphi - b\theta \tag{20}$$

Using equation (16) then, equation (17) becomes

$$D^4\varphi - (1+\varepsilon)\, D^2\dot{\varphi} - [(1+\varepsilon)\tau_o + 1]\, D^2\ddot{\varphi} + \dddot{\varphi} + \tau_o\ddddot{\varphi} = 0 \ , \tag{21}$$

where $\varepsilon = gb/\beta^2$

The boundary conditions at $z = 0$ and as $z \to \infty$ follow from equations (13), (14) and (20); and from the regularity conditions respectively. These will be discussed in more details in subsequent paragraph. The initial conditions for φ and its corresponding time derivatives are all homogeneous at $t = 0$, this follows from the assumption that the medium is initially quiescent.

Taking the Laplace transform, defined by,

$$\overline{W}(p) = \int_0^\infty W(t)\, e^{-pt}\, dt$$

of both sides of equation (21) and considering the homogeneous initial conditions for φ and its time derivatives we get,

$$(D^2 - k_1^2)(D^2 - k_2^2)\, \overline{\varphi}(z, p) = 0 \ , \tag{22}$$

where k_1 and k_2 are the positive roots of the characteristic equations of (21) namely

$$k^4 - [p(1+\varepsilon)(1+\tau_o p) + p^2]k^2 + (p^3 + \tau_o p^4) = 0, \quad (23)$$

The solution of equation (22) that satisfies the regularity condition as z tends to infinity is given by,

$$\bar{\varphi}(z,p) = Ae^{-k_1 z} + \frac{B}{k_2^2 - k_1^2} \cdot e^{-k_2 z} \quad (24)$$

where A and B are parameters to be determined.

Taking the Laplace transform of both sides of equation (16); assuming the homogeneous initial conditions for φ and substituting for $\bar{\varphi}$ given by equation (24), we get

$$\bar{\theta}(z,p) = \frac{\beta^2}{b} [A (k_1^2 - p^2) e^{-k_1 z} + \frac{B(k_2^2 - p^2)}{k_2^2 - k_1^2} e^{-k_2 z}] \quad (25)$$

Similarly, taking the Laplace transform of both sides of equation (20) and using equations (24) and (25), we obtain

$$\bar{\sigma}_{zz}(z,p) = A [k_1^2 - \beta^2 (k_1^2 - p^2)] e^{-k_1 z} + \frac{B}{k_2^2 - k_1^2} [k_2^2 - \beta^2 (k_2^2 - p^2) e^{-k_2 z}] \quad (26)$$

To determine the two parameters A and B, we take the Laplace transform of the boundary conditions at z = 0 given by equations (13) and (14). Thus using the expresions $\bar{\theta}(0,p)$ and $\bar{\sigma}_{zz}(0,p)$ obtained from equations (25) and (26) above we arrive at

$$A = \frac{b}{\beta^2} F(p) \frac{[k_2^2 - \beta^2 (k_2^2 - p^2)]}{p^2(k_1^2 - k_2^2)}$$

and

$$B = \frac{b}{\beta^2} F(p) \frac{[k_1^2 - \beta^2 (k_1^2 - p^2)]}{p^2}$$

where F(p) denotes the Laplace transform of the function f(t) in equation (13.b) .

Hence the corresponding expressions for $\bar{\varphi}(z,p)$, $\bar{\theta}(z,p)$ and $\bar{\sigma}_{zz}(z, p)$ are given by

$$\bar{\varphi}(z,p) = \frac{b}{\beta^2} \cdot \frac{F(P)}{p^2(k_1^2-k_2^2)} \{ [k_2^2 - \beta^2 (k_2^2 - p^2)] e^{-k_1 z} - [k_1^2 - \beta^2 (k_1^2 - p^2)] e^{-k_2 z} \} \quad (27)$$

$$\bar{\theta}(z,p) = \frac{F(P)}{p^2(k_1^2-k_2^2)} \{(k_1^2-p^2) [k_2^2 - \beta^2 (k_2^2 - p^2)] e^{-k_1 z} -(k_2^2 - p^2)[k_1^2 - \beta^2 (k_1^2 - p^2)] e^{-k_2 z} \} \quad (28)$$

$$\bar{\sigma}_{zz}(z,p) = \frac{b}{\beta^2} \cdot \frac{F(p)}{p^2(k_1^2-k_2^2)} [k_1^2 - \beta^2(k_1^2-p^2)] . [k_2^2 - \beta^2(k_2^2-p^2)](e^{-k_1 z} - e^{-k_2 z}) \quad (29)$$

The Laplace transform of the displacement component is obtained from equations (15) and (27) as

$$\bar{u}(z,p) = \frac{b}{\beta^2} \cdot \frac{F(p)}{p^2(k_1^2-k_2^2)} \{ -k_1^2 [k_2^2 - \beta^2 (k_2^2 - p^2)] e^{-k_1 z}$$

$$+ k_2 [k_1^2 - \beta^2 (k_1^2 - p^2)] e^{-k_2 Z} \} \quad (30)$$

Similar expressions can be obtained for $\bar{\sigma}_{xx}(z,p)$ and $\bar{\sigma}_{yy}(z,p)$ using equations (18), and (19) and the corresponding expressions for $\bar{\varphi}$ and $\bar{\theta}$.

Inversion Of The Laplace Transforms

In order to obtain the temperature distribution $\theta(z,t)$, the stress component $\sigma_{zz}(z,t)$ and the displacement $u(z,t)$ in the physical z-t domain, we need to invert the Laplace transforms given by equations (28), (29) and (30) respectively.

In the present paper two approaches are adopted for such inversion. Because of the well known short duration of the second sound effects in generalized thermoelasticity with one relaxation time, the first approach is based on the small time approximations proposed by Paria [6] and Hetnarski [7], which corresponds to large values of p.

The second approach is based on the numerical inversion technique developed by Honig and Hirdes [8], which is suitable for subsequent values of time.

Inversion Of The Laplace Transforms For Small Values Of Time

The method is based on expressing the various terms in equations (28), (29) and (30) as truncated series of powers of p in a consistent manner depending on the desired degree of accuracy as will be noted in the following analysis.

The expressions of k_1 and k_2 obtained from the characteristic equation (23) are given by

$$k_i = p\, h_i^{\frac{1}{2}}\,(1/p)\ ; \qquad i = 1,2 \tag{31}$$

where

$$h_{1,2}(1/p) = \frac{1}{2}\left\{1 + (1+\varepsilon)(1/p + \tau_o) \pm \left[1 + 2(\varepsilon - 1)(1/p + \tau_o) + (\varepsilon + 1)^2 (1/p + \tau_o)^2\right]^{\frac{1}{2}}\right\}$$

Writing the Maclaurin series for h_i, $i = 1,2$ and retaining terms up to order $1/p^4$, we get

$$n_i\left(\frac{1}{p}\right) = \sum_{j=0}^{4} a_{ij}\frac{1}{p^j} \qquad i = 1,2 \tag{32}$$

where

$$\left.\begin{aligned}
a_{10} &= \frac{1}{2}\left[1 + (1+\varepsilon)\tau_o + a\right] \\
a_{11} &= \frac{1}{2}\left[1+\varepsilon + \frac{\varepsilon - 1 + (\varepsilon+1)^2\tau_o}{a}\right] \\
a_{12} &= \frac{\varepsilon}{a^3} \\
a_{13} &= \frac{-\varepsilon\left[\varepsilon - 1 + (\varepsilon+1)^2\tau_o\right]}{a^5} \\
a_{14} &= \frac{\varepsilon\left[\varepsilon^2 - 2\varepsilon + 1 + 2(\varepsilon-1)(\varepsilon+1)^2\tau_o + (\varepsilon+1)^4\tau_o^2\right]}{a^7} \\
a_{20} &= a_{10} - a \\
a_{21} &= 1 + \varepsilon - a_{11}
\end{aligned}\right\} \tag{33}$$

$$a_{2j} = - a_{1j}\,, \quad j = 2,3,4$$

and

$$a = \left[1 + 2(\varepsilon - 1)\tau_o + (\varepsilon+1)^2\tau_o^2\right]^{1/2}$$

Hence in a similar manner equation (31) leads to

$$k_i = p \sum_{j=0}^{3} b_{ij} \frac{1}{p^j} , \quad i = 1,2 \tag{34}$$

where

$$\begin{aligned}
b_{i0} &= a_{i0}^{\frac{1}{2}} \\
b_{i1} &= \frac{a_{i1}}{2a_{i0}^{\frac{1}{2}}} \\
b_{i2} &= \frac{4a_{i2}a_{i0} - a_{i1}^2}{8a_{i0}^{3/2}} \\
b_{i3} &= \frac{8a_{i3}a_{i0}^2 - 4a_{i0}\, a_{i1}a_{i2} + a_{i1}^3}{16\, a_{i0}^{5/2}}
\end{aligned} \tag{35}$$

And using the Maclaurin expansion for $\frac{1}{k_1^2-k_2^2}$ we obtain

$$\frac{1}{k_1^2-k_2^2} = \frac{1}{p^2} \sum_{j=0}^{4} b_j \frac{1}{p^j} \tag{36}$$

where

$$\left.\begin{aligned}
b_o &= \frac{1}{a} \\
b_1 &= - \frac{1}{a^3}[\, \varepsilon - 1 + (\, \varepsilon + 1)^2\, \tau_o]
\end{aligned}\right\} \tag{37}$$

$$\left.\begin{aligned}
b_2 &= \frac{1}{a^5}[\varepsilon^2 - 4\varepsilon + 1 + 2(\varepsilon - 1)(\varepsilon + 1)^2 \tau_o + (\varepsilon + 1)^4 \tau_o^2] \\
b_3 &= \frac{-1}{a^7}[(\varepsilon - 1)(\varepsilon^2 - 8\varepsilon + 1) + 3(\varepsilon + 1)^2(\varepsilon^2 - 4\varepsilon + 1)\tau_o \\
&\quad + 3(\varepsilon - 1)(\varepsilon + 1)^4 \tau_0^2 + (\varepsilon + 1)^6 \tau_0^3] \\
b_4 &= \frac{1}{a^9}[\varepsilon^4 - 16\varepsilon^3 + 36\varepsilon^2 - 16\varepsilon \\
&\quad + 1 + 4(\varepsilon - 1)(\varepsilon + 1)^2(\varepsilon^2 - 8\varepsilon + 1)\tau_o \\
&\quad + 6(\varepsilon + 1)^4(\varepsilon^2 - 4\varepsilon + 1)\tau_0^2 \\
&\quad + 4(\varepsilon - 1)(\varepsilon + 1)^6 \tau_0^3 + (\varepsilon + 1)^8 \tau_0^4]
\end{aligned}\right\} \quad (37)$$

Using the expansions given in equations (34) and (36), and in reference to the expresions for $\bar{\theta}$, $\bar{\sigma}_{zz}$ and $\bar{u}$, we can arrive at

$$\frac{(k_i^2 - p^2)[k_j^2 - \beta^2 (k_j^2 - p^2)]}{p^2(k_1^2 - k_2^2)} = \sum_{n=0}^{4} B_{ijn} \cdot \frac{1}{p^n} \qquad (38)$$

where $i, j = 1, 2$ and $i \neq j$

$$B_{ijn} = \sum_{m=0}^{n} b_m A_{ij(n-m)}, \qquad n = 0, 1, \ldots, 4$$

$$A_{ijk} = \sum_{\ell=0}^{k} \hat{a}_{i\ell} \tilde{a}_{j(k-\ell)}, \qquad k = 0, 1, \ldots, 4$$

$$\left.\begin{aligned} \hat{a}_{io} &= a_{io} - 1 \\ \hat{a}_{i\ell} &= a_{i\ell} \\ \tilde{a}_{io} &= a_{io} - \beta^2 (a_{io} - 1) \end{aligned}\right\} , \quad \ell = 1,2,3,4$$

$$\tilde{a}_{i\ell} = a_{i\ell} (1 - \beta^2) , \quad \ell = 1,2,3,4$$

and

$$\frac{[k_i^2 - \beta^2 (k_i^2 - p^2)][k_j^2 - \beta^2 (k_j^2 - p^2)]}{p^2 (k_1^2 - k_2^2)} = \sum_{n=0}^{4} C_n \frac{1}{p^n} \tag{39}$$

where $i \neq j$, $i,j = 1,2$

$$C_n = \sum_{m=0}^{n} \tilde{b}_{im} \tilde{a}_{j(n-m)}, \qquad n = 0,1,\dots,4$$

$$\tilde{b}_{ik} = \sum_{\ell=0}^{k} b_\ell \tilde{a}_{i(k-\ell)} , \qquad k = 0,1,\dots,4$$

It should be mentioned that the above coefficients are symmetric for i and j, $(i \neq j)$.

Similarly, we have

$$\frac{k_i [k_j^2 - \beta^2 (k_j^2 - p^2)]}{p^2 (k_1^2 - k_2^2)} = \sum_{n=0}^{3} D_{ijn} \frac{1}{p^{n+1}} \tag{40}$$

where $i \neq j$, $i,j = 1,2$

$$D_{ijn} = \sum_{m=0}^{n} b_{im} \tilde{b}_{j(n-m)}, \qquad n = 0,1,2,3$$

Substituting from equations (38), (39) and (40) into equations (28), (29) and (30) respectively, we obtain

$$\bar{\theta}(z,p)=F(P)\left[\left(\sum_{n=0}^{4} B_{12n}\cdot\frac{1}{p^n}\right)e^{-k_1 z}-\left(\sum_{n=0}^{4} B_{21n}\frac{1}{p^n}\right)e^{-k_2 z}\right], \tag{41}$$

$$\bar{\sigma}_{zz}(z,p)=\frac{b}{\beta^2}F(p)\left(\sum_{n=0}^{4} C_n\cdot\frac{1}{p^n}\right).\left[e^{-k_1 z}-e^{-k_2 z}\right] \tag{42}$$

and

$$\bar{u}(z,p)=\frac{b}{\beta^2}F(p)\left[-\left(\sum_{n=0}^{3} D_{12n}\frac{1}{p^{n+1}}\right)e^{-k_1 z}\right.$$

$$\left.+\left(\sum_{n=0}^{3} D_{21n}\frac{1}{p^{n+1}}\right)e^{-k_2 z}\right]. \tag{43}$$

In order to invert the Laplace transforms in equations (41) - (43) for small values of time we have to know the expression of the heating effect f(t) at the boundary z = 0 in equation (14).

Thus we consider the case

$$f(t)=t^2 e^{-\alpha t} \quad \text{for } t \geq 0, \alpha > 0 \tag{44}$$

the parameter α is responsible for the velocity of changes in the temperature on the boundary.

Taking the Laplace transform of equation (44) we get

$$F(p)=\frac{2}{(p+\alpha)^3} \tag{45}$$

Substituting from equation (45) into equations (41) - (43), and since only small values of time are considered, which corresponds to large values of p, it follows from equation (34) that it is sufficient to take k_i, i=1,2 appearing in the exponential functions as

$$k_i = b_{io}\, p + b_{i1},$$

thus

$$e^{-k_i z} = e^{-b_{i1} z} \cdot e^{-b_{io} p z}$$

Having done such substitutions, the convolution theorem of the Laplace transform is then repeatedly used to invert the resulting expresions. After some lengthy algebraic manipulations we arrive at

$$\theta(z,t) = [\sum_{n=0}^{4} B_{12n}\, \phi_{1n}(z,t)]\, e^{-b_{11} z}\, H(t-b_{10}z)$$

$$- [\sum_{n=0}^{4} B_{21n}\, \phi_{2n}(z,t)]\, e^{-b_{21} z}\, H(t-b_{20}z) \qquad (46)$$

$$\sigma_{zz}(z,t) = \frac{b}{\beta^2} [\sum_{n=0}^{4} C_n\, \phi_{1n}(z,t)]\, e^{-b_{11} z}\, H(t-b_{10}z)$$

$$- [\sum_{n=0}^{4} C_n\, \phi_{2n}(z,t)]\, e^{-b_{21} z}\, H(t-b_{20}z) \qquad (47)$$

$$u(z,t) = \frac{b}{\beta^2} \{-[\sum_{n=0}^{3} D_{12n}\, \phi_{1(n+1)}(z,t)]\, e^{-b_{11} z}\, H(t-b_{10}z)]$$

$$+ [\sum_{n=0}^{3} D_{21n}\, \phi_{2(n+1)}(z,t)]\, e^{-b_{21} z}\, H(t-b_{20}z)]\} \qquad (48)$$

where

$$\psi_{io}(z,t) = \frac{2}{\alpha^2} e^{-\alpha(t - b_{io}z)} [1 - e^{-\alpha(t-b_{io}z)}]$$

$$\psi_{i1}(z,t) = e^{-\alpha t} [t^2 I_{io} - 2t I_{i1} + I_{i2}]$$

$$\psi_{i2}(z,t) = e^{-\alpha t} [-b_{io} z t^2 I_{io} + (t^2 + 2b_{io} zt) I_{i1} - (2t + b_{io} z) I_{i2} + I_{i3}]$$

$$\psi_{i3}(z,t) = \frac{1}{2} e^{-\alpha t} [b_{io}^2 z^2 t^2 I_{io} - 2b_{io} zt (t + b_{io} z) I_{i1}$$

$$+ (t^2 + 4b_{io} zt + b_{io}^2 z^2) I_{i2} - 2(t + b_{io} z) I_{i3} + I_{i4}]$$

$$\psi_{i4}(z,t) = \frac{1}{6} e^{-\alpha t} [-b_{io}^3 z^3 t^2 I_{io} + b_{io}^2 z^2 t (3t + 2b_{io} z) I_{i1}$$

$$- b_{io} z (3t^2 + 6b_{io} zt + b_{io}^2 z^2) I_{i2} + (t^2 + 6b_{io} zt + 3b_{io}^2 z^2) I_{i3}$$

$$- (2t + 3 b_{io} z) I_{i4} + I_{i5}]$$

where

$$I_{io} = \int_{b_{io}z}^{t} e^{\alpha x} dx = \frac{1}{\alpha} (e^{\alpha t} - e^{\alpha b_{io} z})$$

$$I_{in} = \int_{b_{io}z}^{t} x^n e^{\alpha x} dx = \frac{(t^n e^{\alpha t} - b_{io}^n z^n e^{\alpha b_{io} z})}{\alpha} - \frac{n}{\alpha} I_{i(n-1)}$$

$$n = 1,2,3,4,5$$

$$i = 1,2$$

and $H(t-b_{io}z)$ is the Unit step function defined by

$$H(t-b_{io}z) = \begin{cases} 0 & t < b_{io}z \\ 1 & t > b_{io}z \end{cases}$$

It is apparent that the solutions of the present generalized thermoelasticity problem with one relaxation time, for small value of time consist of two waves like expresions; this is due to the presence of the unit step functions. The above functions vanish identically for $z > t/b_{20}$. Hence the effect of heating source is confined to a bounded but time dependent region of space. This means that we have a signal propagating with a finite speed.

Numerical Inversion Of Laplace Transform

In order to extend the range of application of the expressions for $\bar{\theta}$, $\bar{\sigma}_{zz}$ and $\bar{u}$ given by equations (28), (29) and (30), respectively, for subsequent values of time, a numerical method for the inversion of Laplace transforms is adopted.

It is well known that there exists a number of numerical inversion methods, among these are the methods based on Fourier series approximations. Durbin [9] derived the approximation formula

$$f(t) = \frac{e^{vt}}{T}\left[-\frac{1}{2} \mathrm{Re}\left\{ F(v) \right\} + \sum_{k=0}^{\infty} \mathrm{Re}\left\{ F(v+i\frac{k\pi}{T}) \right\} \cos\frac{k t \pi}{T} - \sum_{k=o}^{\infty} \mathrm{Im}\left\{ F(v+\frac{ik\pi}{T}) \right\} \sin\frac{k\pi t}{T} \right] - \hat{F}(v,t,T) \tag{49}$$

for $0 < t < 2T$, where $\hat{F}(v,t,T)$ is the discretization error given by,

$$\hat{F}(v,t,T) = \sum_{k=1}^{\infty} e^{-2vkT} f(2kT+t),$$

and v is a free positive parameter. Durbin [9] showed that the discretization error can be made arbitrarily small by choosing v sufficiently large. As the infinite series in equation (49) can only be summed up to a finite number N of terms, hence the approximate inversion formula for f(t) is,

$$f_N(t) = \frac{e^{vt}}{T}\left[-\frac{1}{2} \mathrm{Re}\left\{ F(v) \right\} + \sum_{k=0}^{N} \mathrm{Re}\left\{ F(v+i\frac{k\pi}{T}) \right\} \cos\frac{k\pi t}{T} - \sum_{k=o}^{N} I_m\left\{ F(v+\frac{ik\pi}{T}) \right\} \sin\frac{k\pi t}{T} \right] \tag{50}$$

Thus a " truncation" error is introduced. A disadvantage of such inversion method is the dependence of discretization and turncation errors on the choice of the free parameter. Honig and Hirdes [8] were able to remove such disadvantage by the simultaneous application of a procedure for the reduction of the discretization error, a method for accelerating the convergene of the Fourier series and a procedure that computes approximately the "best" choice of the free parameter. For details of their work the reader is referred to the above reference. For the sake of illustration, the special case of copper material ($\varepsilon = 0.0168$, $\beta^2 = 3.94$ and $\tau_o = 0.1$) is considered. The procedure outlined in [8] is adopted for the numerical inversion of the Laplace transforms given by (28), (29) and (30) to obtain the temperature distribution θ, stress distribution σ_{zz} and displacement u, respectively. Figures (1), (2) and (3) represent their profiles at various points away from the boundary; these profiles are produced for different values of time and choice of the parameter α in the heating effect.

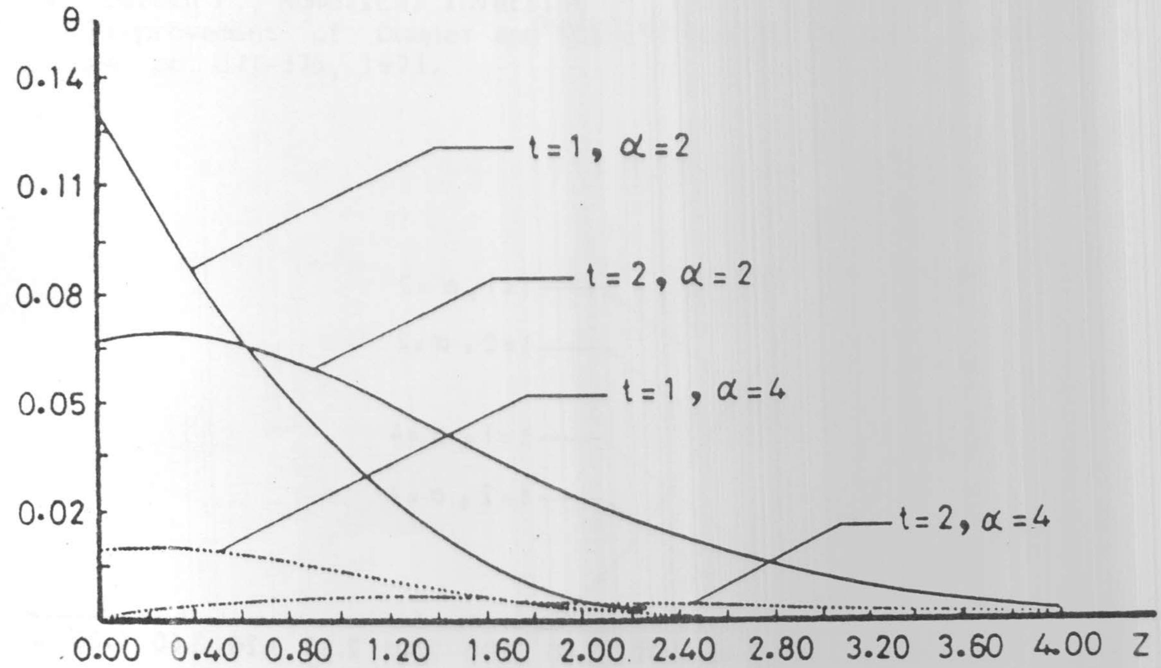

Fig. 1-Temprature distribution

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