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INVESTIGATION INTO THE PROBLEM OF CONTROL OF SHIP STEERING

Aly M. El-Iraki and M. Hanafi

Department of Marine Engineering and Naval Architecture

Faculty of Engineering, Alexandria University

Egypt

Abstract

The turning and change of direction manoeuvres were analysed through a linearized mathematical model for coupled yaw and sway motions of a ship. Alternative time constants to those of Nomoto were given. The effect of changing ship's and rudder's parameters on the delay and rise times, maximum drift angle and its peak time, rudder deflection duration, turning radius and time to complete one turn were investigated. It was found that, among the various parameters, besides the rudder area, the length-draft ratio has displayed the predominant effect on the response of the ship, and in an opposing sense to the effect of length-beam ratio.

Nomenclature

A_r	rudder area
B	ship's breadth
C.P.	center of pressure of ship's hull
D	ship's draft
d	longitudinal distance between C.G. of ship and the center of pressure positive if forward of C.G.
F	propeller thrust
f_m	peak value of yaw rate for unit step input
f_{100}	the steady state value of yaw rate for unit step rudder deflection
J	polar mass moment of inertia of ship about a vertical axis through C.G. including added moment of inertia due to yaw
K	reciprocal of one of Nomoto's time constants
K_{cL}	hydrodynamic lateral force on rudder per unit radian of rudder deflection
K_D	total drag force on ship acting in the center of pressure
K_f	hydrodynamic damping torque coefficient for ship in yawing
K_L	hydrodynamic lateral force on ship per unit radian of drift angle; acting in the center of pressure
L	ship's length
l	longitudinal distance between C.G. of ship and point of action of hydrodynamic force on rudder
L.C.G.	longitudinal position of the center of gravity of ship; positive if aft of midship
m	mass of ship including added mass in sway
R	radius of steady turn
R.A.R.	rudder area ratio = A_r / LD
s	laplacian operator

T_1, T_2
T_3, T_a time constants
T_1', T_2'
t time
t_m time for maximum yaw rate for step input
t_{max} time for maximum drift angle in change of direction manoeuvre
t_{100} time required for yaw rate to reach 100 % of its steady state value for the first time for step input.
α drift angle
α_1, α_2 poles of the transfer functions
α_{max} maximum drift angle in change of direction manoeuvre
δ_r rudder angle
$\bar{\delta}_r$ amplitude of rudder angle
ε rudder deflection duration
ξ damping ratio
θ_m yaw angle
$\bar{\theta}_m$ required change of course
θ_p course angle
τ_1, τ_2, τ_3 Nomoto time constants
ω_c, ω_d frequencies of the system
ω_L, ω_n
ω_s
$(\ )^{\cdot}$ differentiation with respect to time

1. Introduction

The turning and change of direction manoeuvres of ships are considered using a linearized mathematical model [1] for the coupled yaw and sway motions of the ship during steering. This mathematical model represents the transfer functions for the yaw rate, yaw, course and drift angles with respect to rudder deflection. Analytical solution for step and pulse rudder inputs were derived. A set of time constants, alternative to those suggested by Nomoto, were given, and the correlation to Nomoto's constants is given. The effect of the nature of the poles and zeros and their relative locations, in the s-plane, on the ship's response is discussed.

Variation of both ship's and rudder's parameters, namely length-breadth ratio, length-draft ratio, location of the center of gravity, location of the center of pressure and the rudder area are taken into account.

The effects of these parameters on the ship's response were evaluted on the computer. Evaluation is based on the time domain response characteristics. For the turning manoeuvre these are: rise and delay times, steady turning radius and time required to complete one turn. For the change of direction manoeuvre the rise and delay times, rudder deflection duration and the maximum drift angle together with its peak time were investigated.

Further analytical investigation for the different relative locations of poles and zeros of the transfer functions was performed.

2. Mathematical Treatment

In [1] the transfer functions for the yaw rate $\dot{\theta}_m$, yaw angle θ_m, course angle θ_p and drift angle α with respect to rudder deflection input were derived, namely:

$$\frac{\dot{\theta}_m(s)}{\delta_r(s)} = \frac{K_{cL}[\, m\ell Vs + (K_L+K_D)d + (K_L+F)\,\ell\,]}{\{JmVs^2+[J(K_L+F)+K_f mV]s+K_f(K_L+F)-mV(K_L+K_D)d\,\}} \qquad (1)$$

$$\frac{\theta_m(s)}{\delta_r(s)} = \frac{K_{cL}[\, m\ell Vs + (K_L+K_D)d+(K_L+F)\,\ell]}{s\,\{JmVs^2+[J(K_L+F)+K_f mV]s + K_f(K_L+F)-mV(K_L+K_D)d\,\}} \qquad (2)$$

$$\frac{\theta_p(s)}{\delta_r(s)} = \frac{K_{CL}[-Js^2-K_f s+(K_L+K_D)d+(K_L+F)\,\ell\,]}{s\,\{JmVs^2+[J(K_L+F)+K_f mV]s+K_f(K_L+F)-mV(K_L+K_D)d\}} \qquad (3)$$

and

$$\frac{\alpha(s)}{\delta_r(s)} = \frac{\theta_m(s)}{\delta_r(s)} - \frac{\theta_p(s)}{\delta_r(s)}$$

$$= \frac{K_{cL}\,[Js + mV\ell + K_f]}{\{JmVs^2+[J(K_L+F)+K_f mV]s+K_f(K_L+F)-mV(K_L+K_D)d\,\}} \qquad (4)$$

In order to formulate the problem in a way displaying its physical nature, the following time constants are introduced

$$T_1 = \frac{mlV}{h} \quad \text{(sec)}$$

$$T_2 = \sqrt{\frac{J(K_L + F) + K_f\, mV}{K_{CL}\, h}} \quad \text{(sec)}$$

$$T_3 = \sqrt[3]{\frac{Jm\, V}{K_{CL}\, h}} \quad \text{(sec)}$$

$$T_a = \frac{[K_f\, (K_L + F) - mV\, (K_L + K_D)d\,]}{K_{CL}\, h} \quad \text{(sec)}$$

$$T_1' = \frac{K_f}{h} \quad \text{(sec)}$$

$$T_2' = \sqrt{\frac{J}{h}} \quad \text{(sec)}$$

where

$$h = [(K_L + K_D)d + (K_L + F)\, 1\,] \, .$$

With these time constants Eqns (1), (2), (3) and (4) can now be put in the form:

$$\frac{\dot{\theta}_m(s)}{\delta_r(s)} = \frac{T_1 s + 1}{T_3^3 s^2 + T_2^2 s + T_a} = \frac{T_1 s + 1}{T_3^3 (s+ \alpha_1)(s+ \alpha_2)} \tag{5}$$

$$\frac{\theta_m(s)}{\delta_r(s)} = \frac{T_1 s + 1}{s(T_3^3 s^2 + T_2^2 s + T_a)} = \frac{T_1 s + 1}{sT_3^3 (s+ \alpha_1)(s+ \alpha_2)} \tag{6}$$

$$\frac{\theta_p(s)}{\delta_r(s)} = \frac{-T_2'^2 s^2 - T_1' s + 1}{s(T_3^3 s^2 + T_2^2 s + T_a)} = \frac{-T_2'^2 s^2 - T_1' s + 1}{2T_3^3\, (s+ \alpha_1)(s+ \alpha_2)} \tag{7}$$

$$\frac{\alpha(s)}{\delta_r(s)} = \frac{T_2'^2 s + T_1 + T_1'}{T_3^3 s^2 + T_2^2 s + T_a} = \frac{T_2'^2 s + T_1 + T_1'}{T_3^3 (s+\alpha_1)(s+\alpha_2)} , \quad (8)$$

where

$$\alpha_1, \alpha_2 = \frac{-T_2^2 \pm \sqrt{T_2^4 - 4\, T_a\, T_3^3}}{2T_3^3}$$

These time constants can also be expressed in terms of the frequencies defined in [1]

$$T_1 = \frac{1}{\omega_L + \left(\frac{\omega_s}{\omega_c}\right)^2 \omega_{cL}} \quad \text{(sec)}$$

$$T_2 = \frac{\sqrt{T_1(\omega_d + \omega_L)}}{\omega_c} \quad \text{(sec)}$$

$$T_3 = \sqrt[3]{\frac{T_1}{\omega_c^2}} \quad \text{(sec)}$$

$$T_a = \frac{T_1(\omega_d \cdot \omega_L - \omega_s^2)}{\omega_c^2} \quad \text{(sec)}$$

$$\frac{\alpha(s)}{\delta_r(s)} = \frac{T_2'^2 s + T_1 + T_1'}{T_3^3 s^2 + T_2^2 s + T_a} = \frac{T_2'^2 s + T_1 + T_1'}{T_3^3 (s+\alpha_1)(s+\alpha_2)}, \tag{8}$$

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$$T_3 = \sqrt[3]{\frac{T_1}{\omega_c^2}} \qquad \text{(sec)}$$

$$T_a = \frac{T_1(\omega_d \cdot \omega_L - \omega_s^2)}{\omega_c^2} \qquad \text{(sec)}$$

$$T_1' = \frac{T_1\ \omega_d \cdot \omega_{cL}}{\omega_c^2} \quad \text{(sec)}$$

$$T_2' = \frac{\sqrt{T_1\ \omega_{cL}}}{\omega_c} \quad \text{(sec)}$$

Eqn. (5), when put in the following form, is well known as Nomoto equation [2]

$$\frac{\dot{\theta}_m(s)}{\delta_r(s)} = \frac{K(\tau_3 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{9}$$

The correlation between the time constants in both models can be expressed as

$$K = \frac{1}{T_a} \tag{10}$$

$$\tau_3 = T_1 \tag{11}$$

$$\tau_1 \cdot \tau_2 = \frac{T_3}{T_a} \tag{12}$$

$$\tau_1 + \tau_2 = \frac{T_2^2}{T_a} \tag{13}$$

Solving for τ_1 and τ_2 from Eqns. (12) and (13) we get

From Eqn. (9) the zero of the system transfer function is

$$Z = -\frac{1}{\tau_3}$$

and the poles are $-\frac{1}{\tau_1}$ and $-\frac{1}{\tau_2}$, where

$$\alpha_1 = \frac{1}{\tau_1}$$

$$\alpha_2 = \frac{1}{\tau_2} \cdot$$

Eqn. (5) or (9) has $(PD)_2$ control property. The poles $-\alpha_1$ and $-\alpha_2$ determine the stability of the system, while the relative position of poles and zeros determine the nature of the response, as may be seen from Fig. (1).

Considering the two poles to be real, Fig. (1a , b , c) the relative position of the zero Z to the poles $-\alpha_1$ and $-\alpha_2$ governs the shape of the response curve. When Z lies to the left of both poles, Fig. (1a), or between them, Fig. (1b), there exists no peak in the response, whereas for Z lying to the left of both poles the response is slower than in the second case. However, if Z lies to the right of both poles, Fig. (1c), the response is characterized by

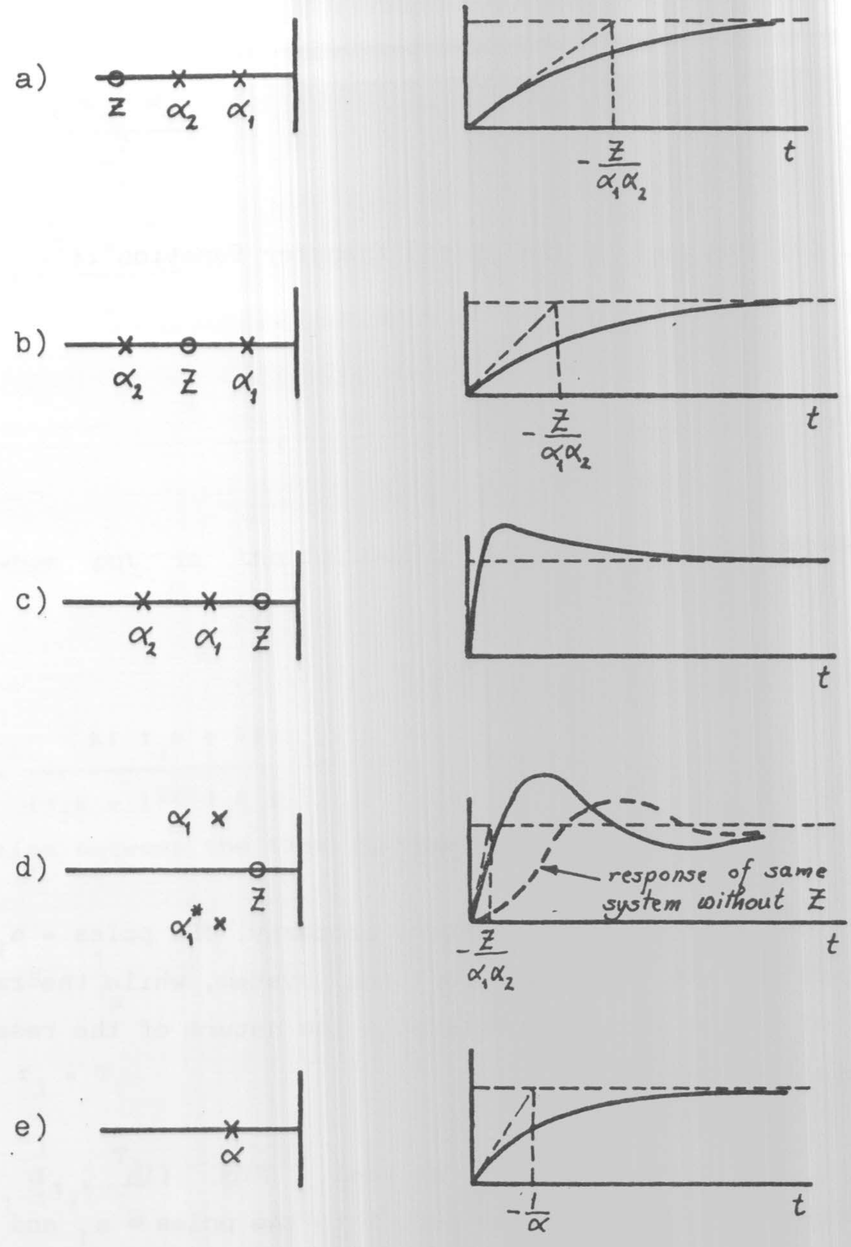

Fig. (1) Pole-zero mapping and transient response of yaw rate for step input the presence of a peak [3].

Second order systems may be approximated by first order ones if damping is considerably large [4]. Considering Eqn. (5) and dividing both numerator and denominator by T_3^3, which is considerably larger than T_1, T_2 and T_a, the equation may be simplified to represent a first order system for which the pole map and response is illustrated in Fig. (1d). Such an approximation is widely used, e.g. in [5].

Fig. (1e) illustrates the pole-zero mapping and the comparison of the transient response of P_2 and $(PD)_2$ - property system when the poles are conjugate complex.

Eqn. (1), with its second order characteristic equation, can be represented in a decomposed block diagram form displaying its dynamic nature, as shown in Fig. (2) in which

$$\omega_n = \sqrt{\frac{[K_f(K_L + F)-mV(K_L + K_D)d]}{JmV}}$$

and

$$\xi = \frac{J(K_L + F) + mVK_f}{\sqrt{4\ JmV[K_f(K_L + F)-mV(K_L + K_D)d]}}$$

2.1 Steady turn response

Considering the transfer function between $\dot{\theta}_m(s)$ and $\delta_r(s)$, Eqn. (5), and for a step input

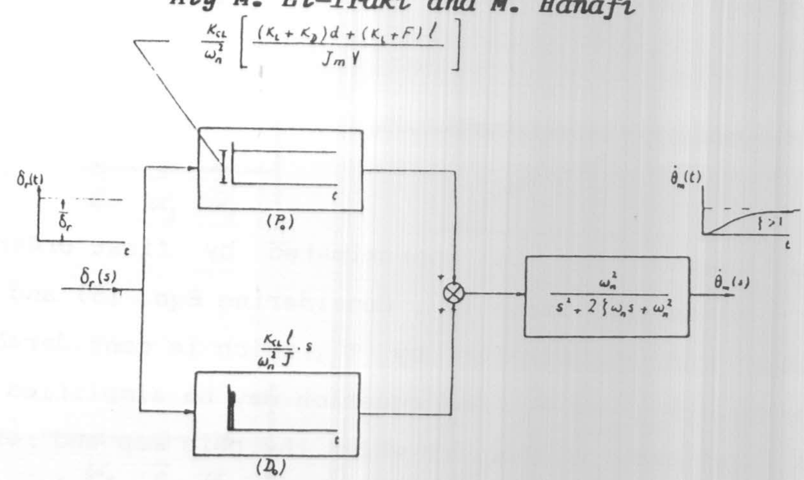

Fig. (2) Decomposition of Eqn. (1) into block diagram form

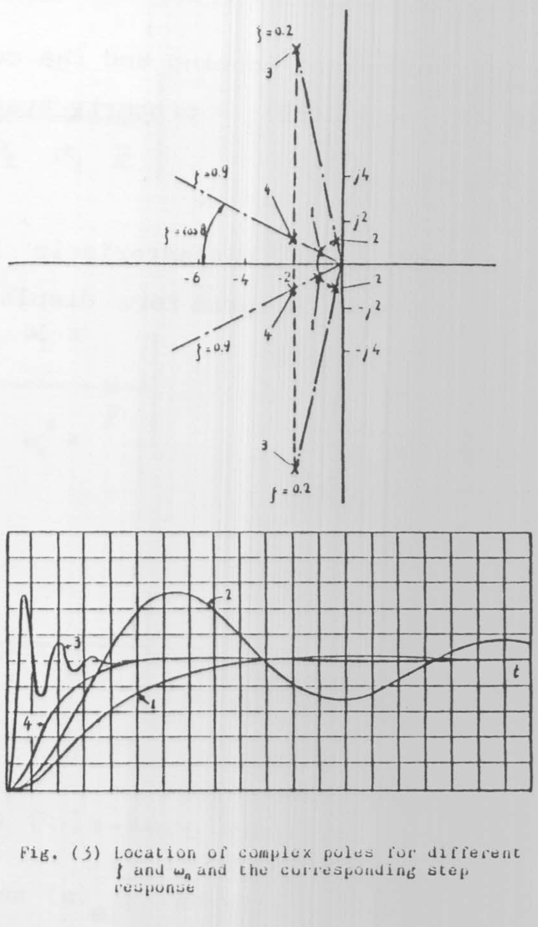

Fig. (3) Location of complex poles for different ζ and ω_n and the corresponding step response

$$\delta_r(t) = \bar{\delta}_r,$$

i.e. $$\delta_r(s) = \frac{\bar{\delta}_r}{s},$$

and using Bromwich's integral formula to obtain

$$\dot{\theta}_m(t) = \frac{1}{2\pi j} \lim_{\beta \to \infty} \int_{\gamma - j\beta}^{\gamma + j\beta} e^{st}\, \dot{\theta}_m(s)\, ds$$

$= \Sigma$ residues of $e^{st}\ \dot{\theta}_m(s)$ at all poles of $\dot{\theta}_m(s)$, where the integral encloses all poles of the transfer function. The poles of yaw rate $\dot{\theta}_m$ (s), zero, $-\alpha_1$ and $-\alpha_2$, are real and distinct.

$$\text{residue}\Big|_{s=0} = \lim_{s \to 0} s.\ \frac{\bar{\delta}_r(T_1 s + 1)}{sT_3^3(s+\alpha_1)(s+\alpha_2)} \cdot e^{st} = \frac{1}{T_3^3\ \alpha_1 \alpha_2}\ \bar{\delta}_r$$

$$\text{residue}\Big|_{s=-\alpha_1} = \lim_{s \to -\alpha_1} (s+\alpha_1).\ \frac{\bar{\delta}_r(T_1 s + 1)}{sT_3^3(s+\alpha_1)(s+\alpha_2)} \cdot e^{st}$$

$$= \frac{(-T_1\ \alpha_1 + 1)}{T_3^3 \alpha_1(\alpha_1 - \alpha_2)}\ \bar{\delta}_r\ e^{-\alpha_1 t}$$

$$\text{residue}\Big|_{s=-\alpha_2} = -\lim_{s \to -\alpha_2} (s+\alpha_2).\ \frac{\bar{\delta}_r(T_1 s + 1)}{sT_3^3(s+\alpha_1)(s+\alpha_2)} \cdot e^{st}$$

$$= \frac{(-T_1\ \alpha_2 + 1)}{\alpha_2(\alpha_2 - \alpha_1)}\ \bar{\delta}_r\ e^{-\alpha_2 t}$$

Hence,

$$\dot{\theta}_m(t) = \frac{\bar{\delta}_r}{T_3^3} \{ A^* + B^* e^{-\alpha_1 t} + C^* e^{-\alpha_2 t} \} \tag{14}$$

where

$$A^* = \frac{1}{\alpha_1 \alpha_2}$$

$$B^* = \frac{-T_1 \alpha_1 + 1}{\alpha_1 (\alpha_1 - \alpha_2)}$$

$$C^* = \frac{-T_1 \alpha_2 + 1}{\alpha_2 (\alpha_2 - \alpha_1)}$$

The yaw angle $\theta_m(t)$ is then obtained through integration

$$\theta_m(t) = \int_o^t \dot{\theta}_m(t)\, dt$$

$$= \frac{\bar{\delta}_r}{T_3^3} [A^* t - \frac{B^*}{\alpha_1} e^{-\alpha_1 t} - \frac{C^*}{\alpha_2} e^{-\alpha_2 t} + \frac{B^*}{\alpha_1} + \frac{C^*}{\alpha_2}]$$

Following a similar procedure for the course angle $\theta_p(t)$ we obtain

$$\theta_p(t) = \frac{\bar{\delta}_r}{T_3^3} [\bar{A} + \bar{B}t + \bar{C}e^{-\alpha_1 t} + \bar{D}e^{-\alpha_2 t}] ,$$

where

$$\bar{A} = \{\alpha_1^2 \alpha_2^2 (\alpha_2 - \alpha_1)(-T_2'^2 - T_1' + 1) - \alpha_1 \alpha_2 (\alpha_2 - \alpha_1)(1+\alpha_1)(1+\alpha_2) - \alpha_2^2(-T_2'^2 \alpha_1^2 + T_1' \alpha_1 + 1)(1+\alpha_2) + \alpha_1^2(-T_2'^2 \alpha_2^2 + T_1' \alpha_2 + 1)(1+\alpha_1) \} / [\alpha_1^2 \alpha_2^2 (\alpha_2 - \alpha_1)(1+\alpha_1)(1+\alpha_2)]$$

$$\bar{B} = \frac{1}{\alpha_1 \alpha_2}$$

$$\bar{C} = \frac{-T'^2_2 \alpha_1^2 + T'_1 \alpha_1 + 1}{\alpha_1^2 (\alpha_2 - \alpha_1)}$$

$$\bar{D} = \frac{-T'^2_2 \alpha_2^2 - T'_1 \alpha_2 + 1}{\alpha_2^2 (\alpha_1 - \alpha_2)}$$

Finally, the drift angle is

$$\alpha(t) = \theta_m(t) - \theta_p(t)$$

To find the time t_m for maximum yaw rate, Eqn. (14) is differentiated with respect to time and equated to zero

$$\ddot{\theta}_m(t_m) = \frac{\bar{\delta}_r}{T_3^3} \left\{ - B^* \alpha_1 e^{-\alpha_1 t_m} - C^* \alpha_2 e^{-\alpha_2 t_m} \right\} = 0 \quad ,$$

from which

$$t_m = \frac{\ln \left(\dfrac{-T_1 \alpha_2 + 1}{T_1 \alpha_1 - 1} \cdot \dfrac{\alpha_1}{\alpha_2} \right)}{(\alpha_2 - \alpha_1)}$$

which only exists if

$$\left(\frac{-T_1}{T_1} \frac{\alpha_2 + 1}{\alpha_1 - 1} \cdot \frac{\alpha_1}{\alpha_2} \right) > 0 .$$

Considering now the corresponding Nomoto equation, Eqn. (4), the peak of the yaw rate, and its time, for the case where Z lies to the right of both poles $-\alpha_1$ and $-\alpha_2$, Fig. (1c), i.e.

$$\frac{\tau_1}{\tau_2} \equiv b > 1$$

$$\frac{\tau_3}{\tau_2} \equiv c > 1,$$

the following relations can be derived for a unit step function [6]. The time t_m, required for the system to reach 100 % of its steady state value for the first time can be calculated from

$$\frac{t_m}{t_{100}} = 1 + \frac{\ln b}{\ln \frac{1-c}{b-c}}$$

and the peak value f_m from

$$\frac{f_m}{f_{100}} = 1 + \frac{b - 1}{[b^p - b] \; b^{\left(\frac{p}{b-1}\right)}}$$

where

$$p = \frac{\frac{t_m}{t_{100}}}{\frac{t_m}{t_{100}} - 1} ,$$

$f_{100} = K.$

To find the radius of steady turn, we first determine the steady state yaw rate as follows

$$\lim_{t \to \infty} \dot{\theta}_m(t) = \lim_{S \to 0} s\,\dot{\theta}_m(s) = \frac{1}{T_3^3\,\alpha_1\,\alpha_2}\,\bar{\delta}_r.$$

The radius R of steady turning is then

$$R = \frac{V\,T_3^3\,\alpha_1\,\alpha_2}{\bar{\delta}_r} \cdot \frac{180}{\pi}$$

where $\bar{\delta}_r$ is in degrees.

It is worth mentioning that for a system with one of its time constants appreciably larger than the others, the delay time may be approximated by the sum of all the smaller time constants, while the rise time corresponds approximately to the value of the largest time constant [7]. This has been verified by the obtained results.

2.2 Change of direction manoeuvre

To change the course of a ship, the rudder is put to an angle for a certain period of time. This may be mathematically represented by a pulse function of height $\bar{\delta}_r$ and duration ε. Such a pulse function, however, is an approximation of the real rudder deflection, as the time required to deflect the rudder is neglected.

Consider the Laplace transform of the pulse function

$$\delta_r(s) = \bar{\delta}_r \cdot \frac{(1 - e^{-\varepsilon s})}{s}$$

and using Taylor expansion for the exponential function $e^{-\varepsilon s}$, we obtain

$$\delta_r(s) = \frac{\bar{\delta}_r}{s}[\, 1 - (1 - \varepsilon s + \frac{\varepsilon^2 s^2}{2!} - \frac{\varepsilon^3 s^3}{3!} + - \ldots .)\,]$$

Considering the expansion up to the linear term in s we get

$$\delta_r(s) \approx \varepsilon \bar{\delta}_r$$

The yaw rate $\dot{\theta}_m(s)$ is then given by, Eqn. (5),

$$\dot{\theta}_m(s) = \frac{T_1 s + 1}{T_3^3 s^2 + T_2^2 s + T_a} \varepsilon \bar{\delta}_r ,$$

Using Heavisides' expansion for $\dot{\theta}_m(s)$ and taking inverse Laplace transform, we obtain,

$$\dot{\theta}_m(t) = \frac{\varepsilon \bar{\delta}_r}{T_3^3} [\hat{A} e^{-\alpha_1 t} + \hat{B} e^{-\alpha_2 t}],$$

where

$$\hat{A} = \frac{-T_1 \alpha_1 + 1}{\alpha_2 - \alpha_1}$$

$$\hat{B} = \frac{-T_1\,\alpha_2 + 1}{\alpha_1 - \alpha_2},$$

and

$$\theta_m(t) = \int_o^t \dot{\theta}_m(t)\,dt$$

$$= -\frac{\varepsilon\bar{\delta}_r}{T_3^3}\left[\frac{\hat{A}}{\alpha_1}e^{-\alpha_1 t} + \frac{\hat{B}}{2}e^{-\alpha_2 t} - \left(\frac{\hat{A}}{\alpha_1} + \frac{\hat{B}}{\alpha_2}\right)\right] \quad (15)$$

The duration ε of the rudder deflection to cause a change in course $\bar{\theta}_m$ of ship can be calculated as follows

$$\bar{\theta}_m = \theta_m(t=\infty) = \frac{\varepsilon\bar{\delta}_r}{T_3^3}\left(\frac{\hat{A}\alpha_2 + \hat{B}\,\alpha_1}{\alpha_1\,\alpha_2}\right)$$

from which

$$\varepsilon = \frac{\alpha_1\;\alpha_2\;T_3^3}{\hat{A}\;\alpha_2 + \hat{B}\;\alpha_1} \cdot \frac{\bar{\theta}_m}{\bar{\delta}_r}$$

$$= \alpha_1\;\alpha_2\;T_3^3 \cdot \frac{\bar{\theta}_m}{\bar{\delta}_r},$$

since $\hat{A}\,\alpha_2 + \hat{B}\,\alpha_1 = 1$.

Following the same procedure to obtain $\theta_p(t)$ using Eqn. (7) after substituting for the rudder pulse input and taking inverse Laplace transform, we get

$$\theta_p(t) = \frac{\varepsilon \bar{\delta}_r}{T_3^3} [\tilde{A} + \tilde{B}.e^{-\alpha_1 t} + \tilde{C}.\ e^{-\alpha_2 t}]$$

where

$$\tilde{A} = \frac{1}{\alpha_1\ \alpha_2}$$

$$\tilde{B} = \frac{-T_2'^2\ \alpha_1^2 + T_1'\ \alpha_1 + 1}{\alpha_1\ (\alpha_1 - \alpha_2)}$$

$$\tilde{C} = \frac{-T_2'^2\ \alpha_2^2 + T_1'\alpha_2 + 1}{\alpha_2(\alpha_2 - \alpha_1)}$$

Finally, the drift angle is

$$\alpha(t) = \theta_m(t) - \theta_p(t)$$

$$= \frac{\varepsilon\ \bar{\delta}_r}{T_3^3} [\bar{\bar{A}}e^{-\alpha_1 t} + \bar{\bar{B}}\ e^{-\alpha_2 t}] ,$$

where $\bar{\bar{A}} = \dfrac{-T_2'^2\alpha_1 + T_1 + T_1'}{(\alpha_2 - \alpha_1)}$

$$\bar{\bar{B}} = \frac{-T_2'^2\ \alpha_2 + T_1 + T_1'}{(\alpha_1 - \alpha_2)}$$

The steady state value of the drift angle, $\alpha(\infty)$, vanishes since once on the new course, the ship will have no drift angle.

3. Oscillation of Ship during Manoeuvring

In the previous analysis the poles of the transfer functions of the ship were assumed to be real and distinct. The existence of complex conjugate poles of the transfer function will cause damped oscillations in the response of the ship, which are not preferable, especially from course keeping point of view, and should be avoided.

The conditions that lead to oscillating response depend on the ship's parameters, which in some extreme cases, e.g. excessive transfer of L.C.G. forward of the center of pressure due to improper loading, may render the poles of the transfer functions complex.

Considering Eqn. (8) relating $\alpha(s)$ to $\delta_r(s)$ with pulse input representing change of direction manoeuvre and assuming complex poles, the drift angle $\alpha(t)$ is then given by

$$\alpha(t) = \varepsilon \bar{\delta}_r \, e^{-C_2 t}(C_1 \cos C_3 t + C_4 \sin C_3 t),$$

where

$$C_1 = \frac{T_2^2}{T_3^3}$$

$$C_2 = \frac{T_2^2}{2T_3^3}$$

$$C_3 = \sqrt{\frac{T_a}{T_3^3} - C_2^2}$$

$$C_4 = \left[\frac{2T_3^3 \, (T_1 + T_1') - T_2'^2 \, T_2^2}{2 \, T_3^6} \right] \Big/ C_3$$

Fig. (3) illustrates the response corresponding to different locations of the complex poles (different ω_n and ξ) of the transfer function in the s-plane [3].

4. Results and Discussion

The methods and equations for computing the ship parameters and hydrodynamic forces and torques were given in [1]. The ship parameters were chosen guided by data presented in [8,9]. A reference ship with the following particulars, including variations in L/B, L/D, R.A.R. and d, was considered in the computations:

L = 200 m
L/B = 6.0 , 6.5 , 7.0
L/D = 16, 19, 22
V = 16 knots
R.A.R.= 1.5 % , 2.5 %
d = o.o33L, 0.026 L

The time domain behavior during both turning and the change of direction manoeuvres was calculated. The numerical calculations were carried out using the PDP-11 computer of the Faculty of Engineering, Alexandria University.

4.1 The turning manoeuvre

For this manoeuvre the rudder input is assumed to be a step function of height $\bar{\delta}_r$. Although the turning manoeuvre will normally be executed at higher rudder angles, an angle of 10° was chosen here in order to preserve consistency with the linearized model. However, similar effects of the various parameters would be expected for larger values of the rudder angle.

Fig. (4) repres ents typical results for the yaw rate, yaw, course and drift angles. The effect of the integrating property is displayed in both the responses of θ_m and θ_p since the function $\theta_m(s)/\delta_r(s)$ is a $(PI)_2$ property and that of $\theta_p(s)/\delta_r(s)$ is $(PI(-D))_2$ property. The (-D) property is attributed to the negative terms in numerator of the transfer function, Eqn. (3), and is displayed through the negative part of the response. Further, the transfer function $\dot{\theta}_m(s)/\delta_r(s)$ and $\alpha_m(s)/\delta_r(s)$ are both of $(PD)_2$ property. The accuracy of the mathematical model is confirmed by the fact that the results obtained for the drift angle α are still in the proximity of the assumed linear range, i.e. $\alpha < 10^{\circ}$.

Figs. (5) and (6) represent the effect of changing L/B on the transient response of both yaw rate $\dot{\theta}_m(t)$ and drift angle α (t), respectively. From both curves it is obvious that the speed of response is almost insensitive to variations in L/B. Increasing L/B reduces the response of both $\dot{\theta}_m(t)$ and $\alpha(t)$. Fig. (5) shows further that changing the L.C.G. to 0.007 L forward decreases the yaw rate. In Fig. (6) the effect of changing the distance between the C.G. and the center of pressure is shown. The effects of L/B on the turning radius together with the effects of both R.A.R. and location of center of

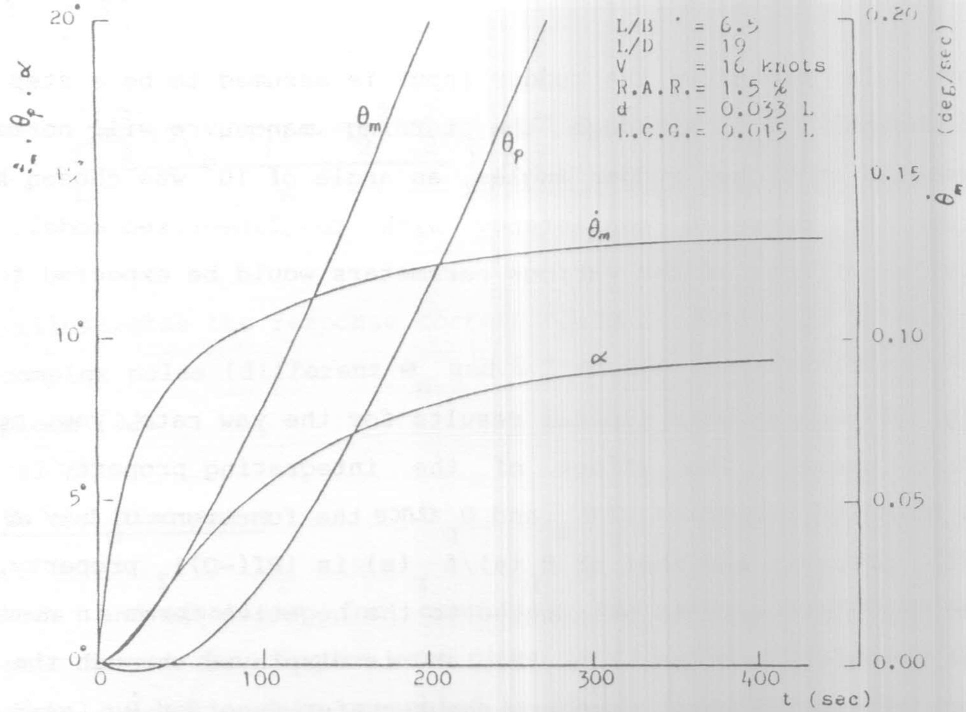

Fig. (4) Typical responses for $\dot{\theta}_m$, θ_m, θ_p and α in turning manoeuvre

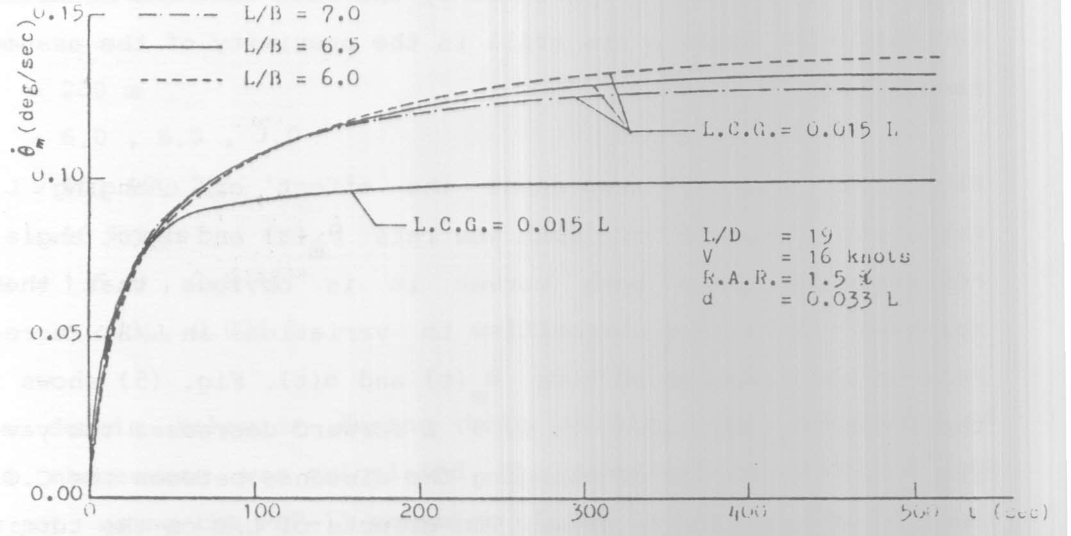

Fig. (5) Effects of L/B and L.C.G. on the yaw rate in turning manoeuvre

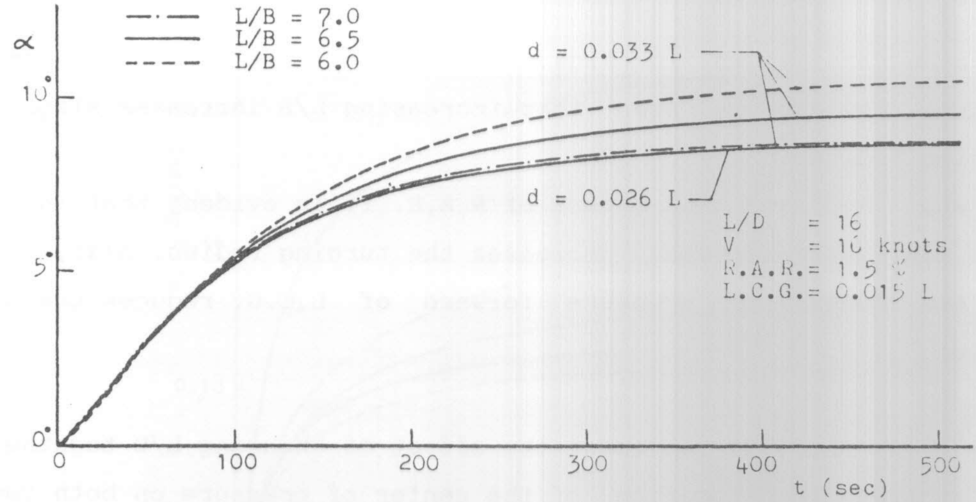

Fig. (6) Effects of L/B and C.P. on the drift angle in turning manoeuvre

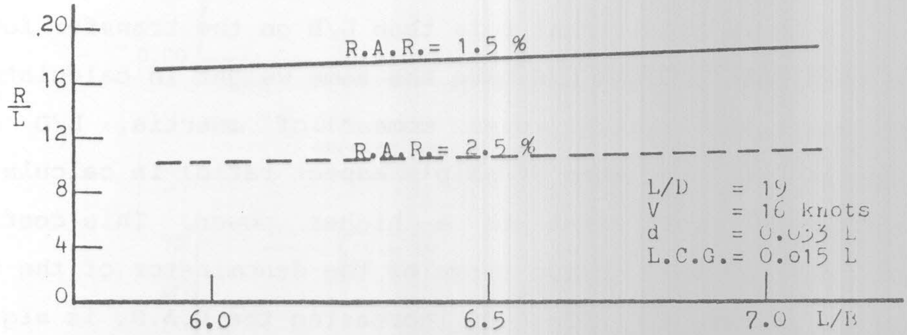

Fig. (7) Effects of L/B and R.A.R. on steady turning radius

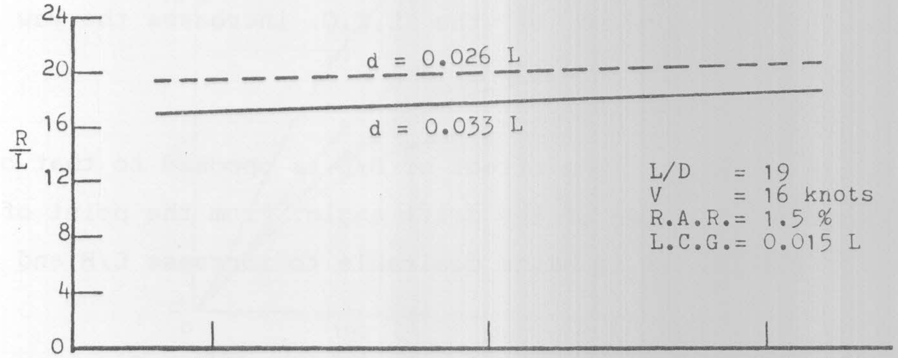

Fig. (8) Effects of L/B and C.P. on steady turning radius pressure are shown in Figs. (7) and (8), respectively. From these figures it can be seen that increasing L/B increases slightly the turning radius.

In what concerns the effect of R.A.R. it is evident that increasing the R.A.R. considerably decreases the turning radius. Also, movement of the center of pressure forward of L.C.G. reduces the turning radius.

Figs. (9) and (10) represent the effect of changing L/D together with the R.A.R. and location of the center of pressure on both yaw rate $\dot{\theta}_m(t)$ and drift angle $\alpha(t)$, respectively. Increasing L/D increases appreciably the yaw rate and drift angle. This is due to the fact that L/D has a more significant role than L/B on the transfer functions. Despite that both L/D and L/B have the same weight in calculating both virtual mass and virtual mass moment of inertia, L/D alone is incorporated (in the form of ship's aspect ratio) in calculating the damping torque coefficient to a higher power. This coefficient, moreover is present in two terms of the denominator of the transfer function. Further, the effect of increasing the R.A.R. is significant on the yaw rate and drift angle. Here again, moving the center of pressure further forward of the L.C.G. increases the yaw rate and drift angle.

Fig. (11) idicates that the effect of L/D is opposed to that of L/B on the rise and delay times of the drift angle. From the point of view of steering dynamics, it is hence desirable to increase L/B and decrease L/D.

Fig. (12) and (13) illustrate the effect of L/D together with the effects of R.A.R. and location of the center of pressure on the

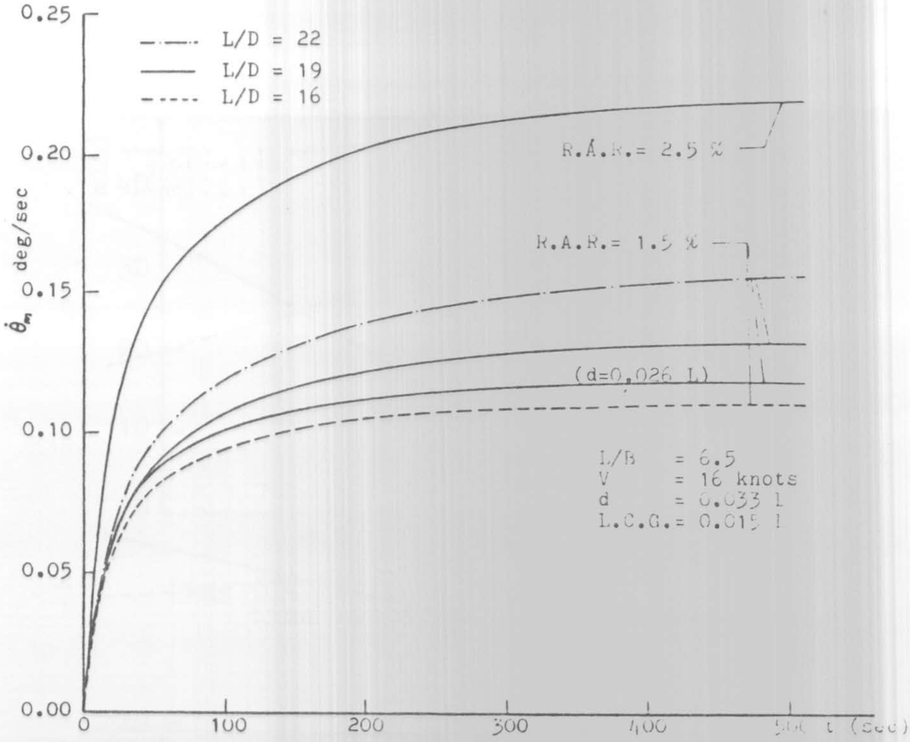

Fig. (9) Effects of L/D, R.A.R. and C.P. on yaw rate for turning manoeuvre

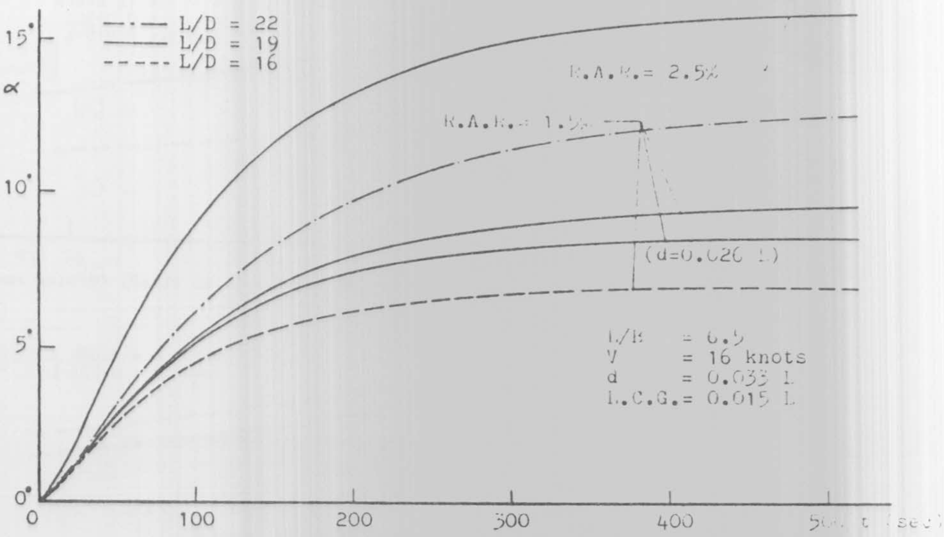

Fig. (10) Effects of L/D, R.A.R. and C.P. on drift angle for turning manoeuvre

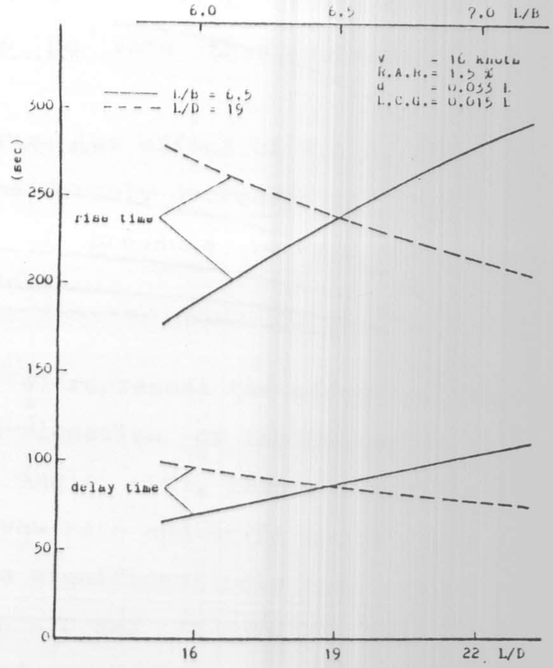

Fig. (11) Effects of L/B and L/D on delay and rise times of drift angle in turning manoeuvre

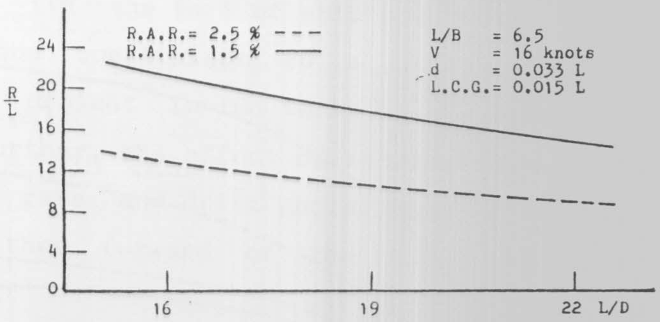

Fig. (12) Effects of L/D and R.A.R. on steady turning radius

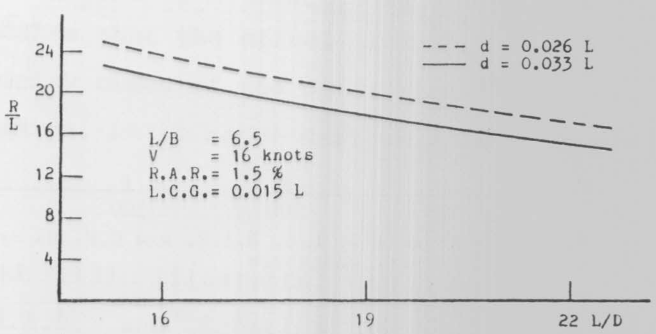

Fig. (13) Effects of L/D and C.P. on steady turning radius

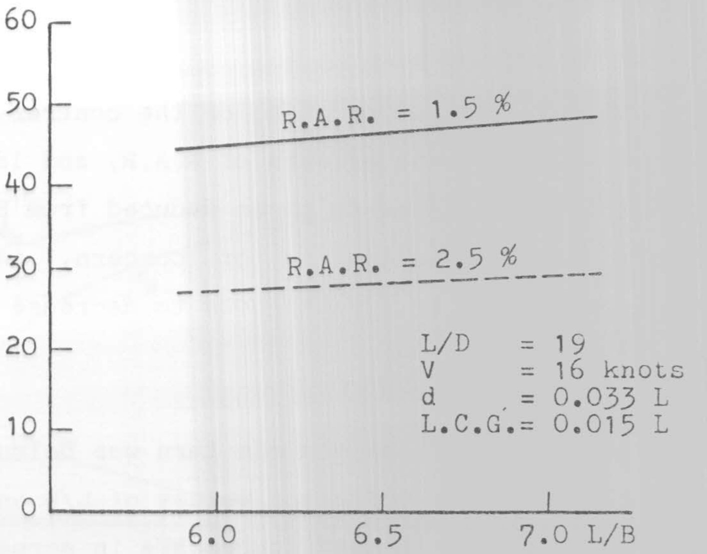

Fig. (14) Effects of L/B and R.A.R. on time required for a complete turn

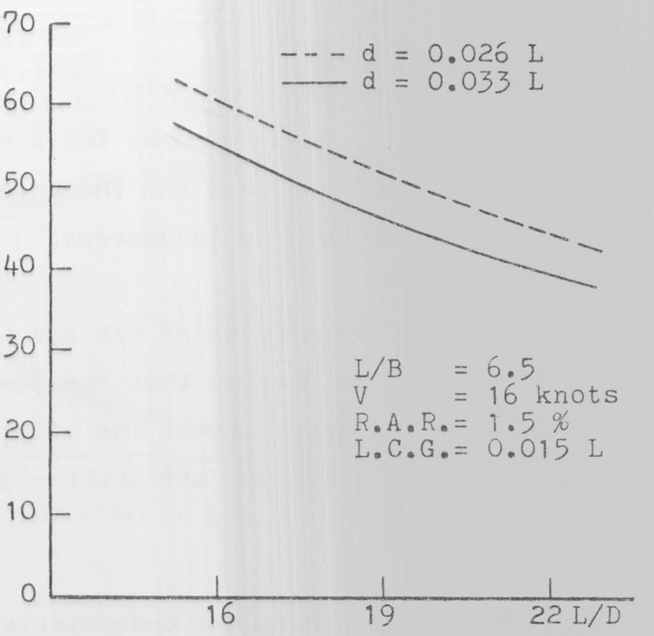

Fig. (15) Effects of L/D and C.P. on time required to complete one turn turning radius. Increasing L/D, on the contrary to L/B, reduces the turning radius, while the effects of R.A.R. and location of the center of pressure are similar to those deduced from Figs. (7) and (8). If the steady turning radius is of concern, on the contrary to the steering dynamics, it is advantageous to decrease L/D and increase L/B in order to reduce it.

The time required to complete one turn was calculated and plotted in Figs. (14) and (15) for different values of L/B and L/D, respectively. Results deduced from these figures are in agreement with those from Figs. (7), (8), (12) and (13), where smaller turning radii correspond to shorter turning times.

4.2 Change of direction manoeuvre

Fig. (16) illustrates a typical response for a rudder pulse input of 10 degrees on $\dot{\theta}_m$, θ_m, θ_p and α , and for the duration necessary to change the course of the ship by 10 degrees.

Figs. (17) and (18) show the effects of L/B and L/D on both yaw & drift angles, respectively. It is obvious that the yaw angle tends to 10^{o} and α tends to zero in steady state. The effect of the position of the center of pressure on the yaw and drift angles is shown in Fig. (19).

To judge the time domain response characteristics, delay and rise times for the yaw angle were calculated and displayed in Fig. (20). As was discussed before, increasing L/D increases both delay and rise times, whereas increasing L/B has the opposite effect.

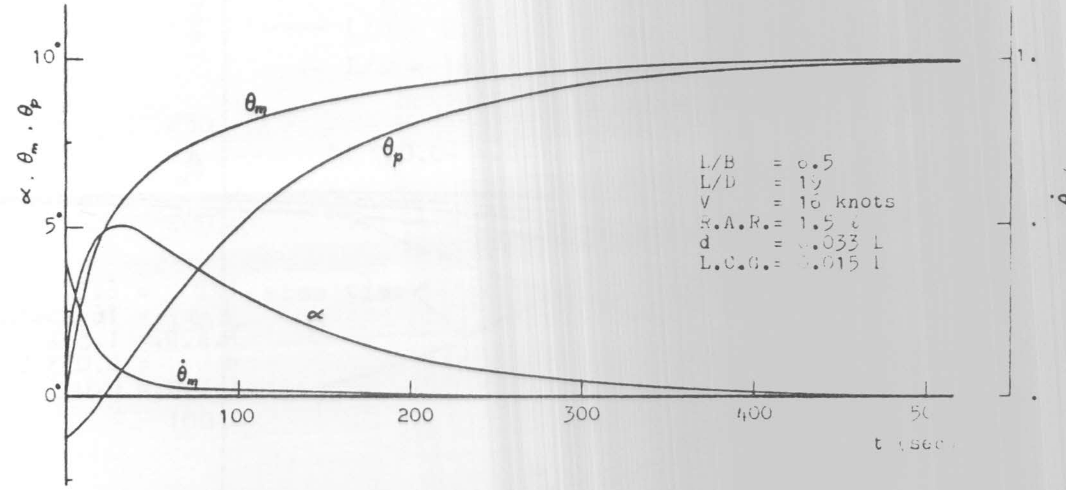

Fig. (16) Typical response for $\dot{\theta}_m$, θ_m, θ_p and α in change of direction manoeuvre

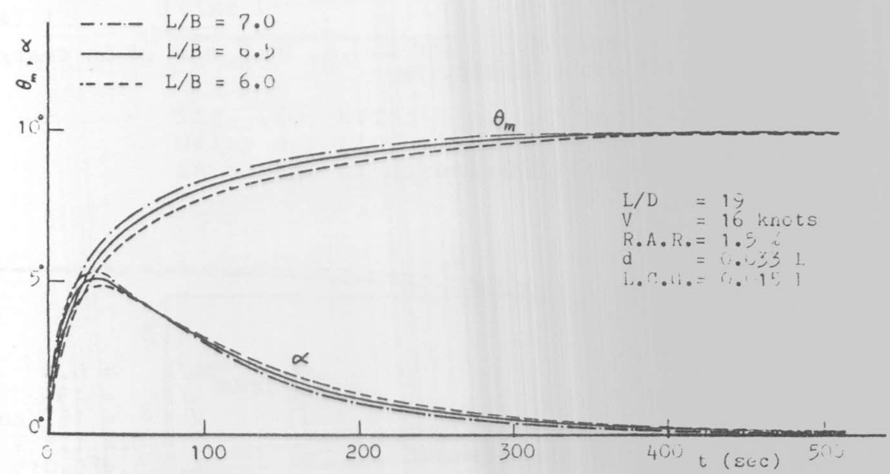

Fig. (17) Effects of L/B on θ_m and α in change of direction manoeuvre

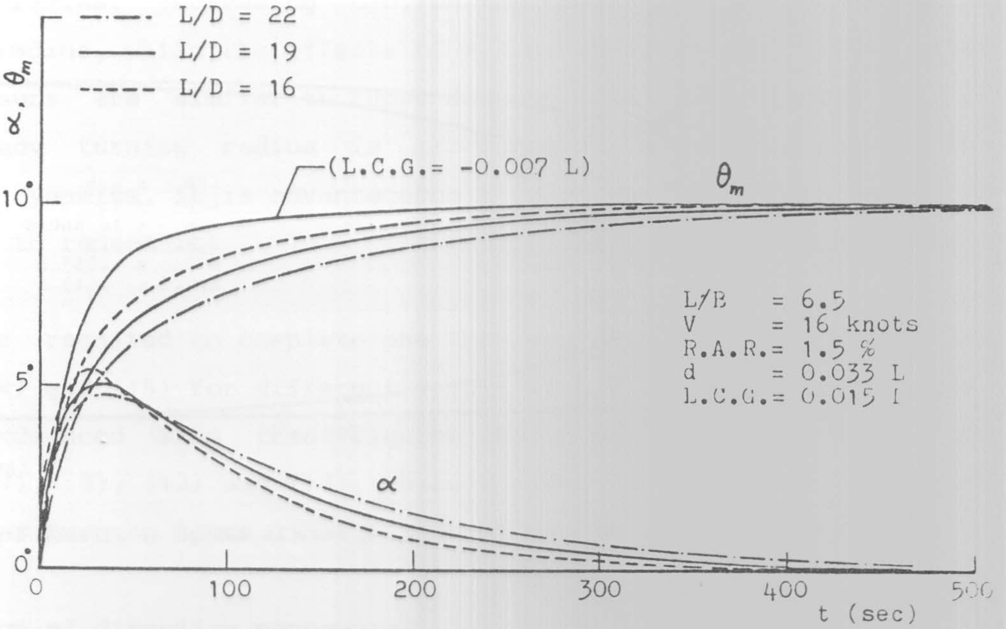

Fig. (18) Effects of L/D and L.C.G. on θ_m and α in change of direction manoeuvre

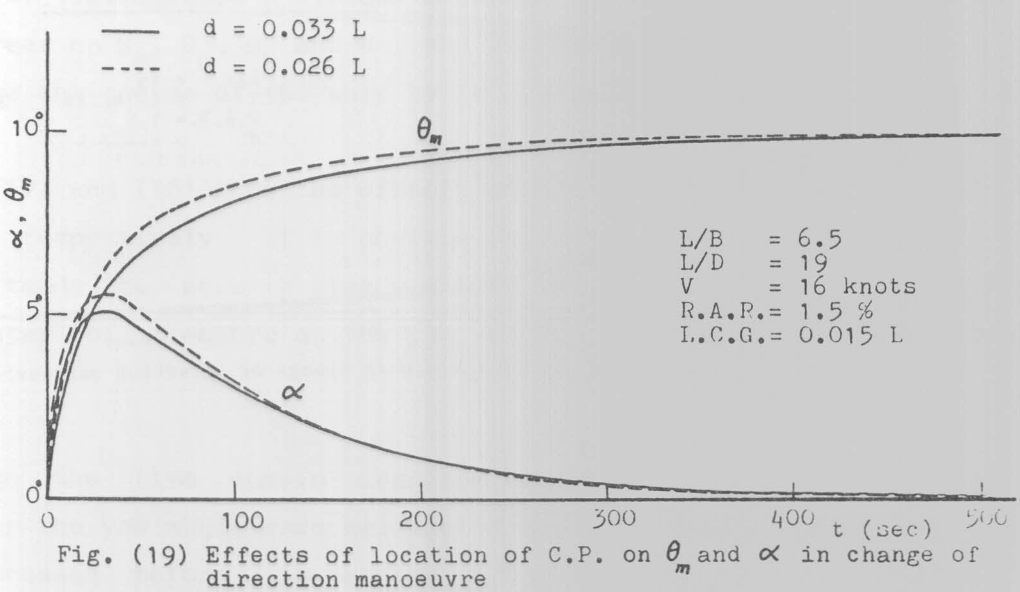

Fig. (19) Effects of location of C.P. on θ_m and α in change of direction manoeuvre

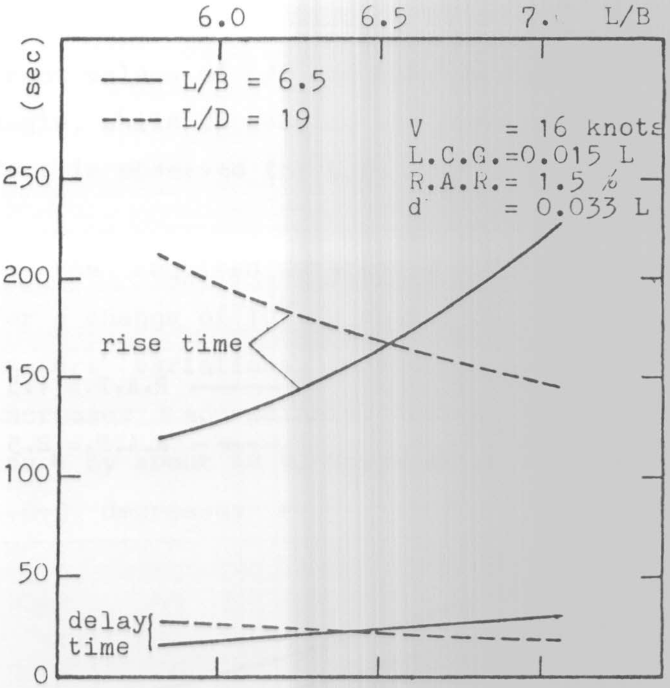

Fig. (20) Effects of L/B and L/D on delay and rise times for yaw angle in change of direction manoeuvre

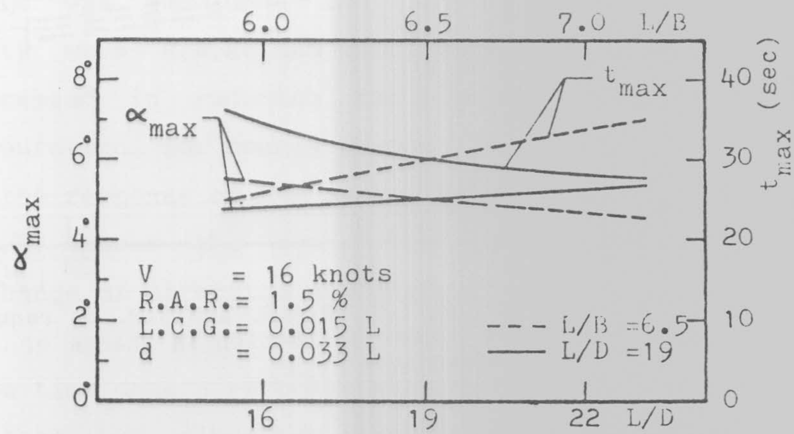

Fig. (21) Effects of L/B and L/D on t_{max} and α_{max}

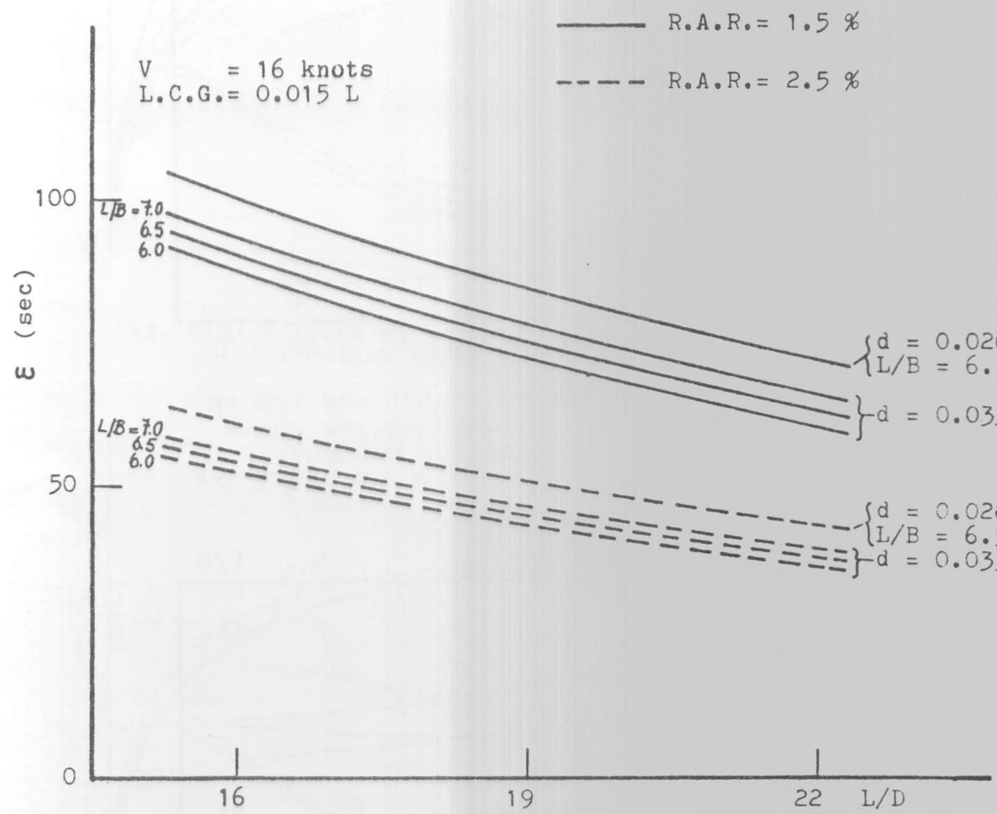

Fig. (22) Effects of L/B, L/D, R.A.R. and C.P. on required duration of rudder deflection in change of direction manoeuvre

The maximum drift angle and its corresponding time are shown in Fig. (21) for different values of L/B and L/D. Increasing L/D reduces the maximum drift angle, while increasing the time necessary to reach it. The opposite effect is observed for L/B.

Fig. (22) shows the required duration ε of the rudder pulse input ($\bar{\delta}_r = 10^o$) for a change of 10^o in ship's course, for all the considered parameters' variations. Increasing L/D reduces ε while increasing L/B increases ε appreciably. Increasing the R.A.R. from 1.5% of 2.5 % reduces ε by about 60 %. Movement of the center of pressure forward of the L.C.G. decreases ε .

5. Conclusion

Extensive computations were carried out for the turning and change of direction manoeurves. A linearized mathematical model for the coupled yaw and sway motions was adopted for determining yaw rate, yaw, course and drift angles. The time domain response characteristics were calculated. It was concluded that the major factors affecting the manoeuvrability were R.A.R. and L/D. The effect of increasing any of them is expressed in reducing the turning radius and the required rudder time duration, for course change. It was shown that increasing L/D causes the response of the system to be slower. On the contrary, increasing L/B causes the system to have faster response in both turning and change of direction manoeuvres.

Generally, the time domain response was found to be more influenced by L/D rather than L/B, since L/D alone is incorporated in the damping torque coefficient, and to a higher power. The effects of changing L/D and L/B on the system response are always opposite in both manoeuvres.

6. References

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