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EFFECT OF OPENING LOCATIONS ON SHELL STRESSES

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Abstract

The openings in cylindrical shell roofs may be made in any location according to the purpose of use of the shell roof.

Four locations of openings, for simply supported single shell were chosen in this study. The stress variations, between the shells with and without openings, were investigated. This was done for both types of shells, with and without edge beams.

Sixteen problems were solved to cover this investigation. One of these lacations was selected to be the best from the stress variations point of view. Sixty different dimension shells were solved twice, once with openings of the selected location, the other without openings.A comparison of the internal forces between each pair was done in a different paper. The analyses was made, by the finite element technique, using a well checked computer programme. A quadrilateral rectangular elment was chosen to represent the shell surface.

Notations

L	Shell span.
S	Shell spacing.
HI	Shell rise.
H2	Edge beam height.
R	Radius of Shell.
t	Shell thickness.
u,v	Inplane displacement components .
w, θ_x, θ_y	Out of plane displacement components.
N_x, N_y, N_{xy}	Inplane internal force components.
M_x, M_y, M_{xy}	Out of plane internal force components.
φ_o	Half of shell central angle.
φ_1, φ_2	Shell central angle limiting the openings.
A_1-A_{12}	Coefficients of the inplane displacement functions.
C_1-C_9	Coefficients of the bending displacement functions.
f	Normal stress component.
A	Area of the section.
Z	Section modulus.

1. Introduction

The openings in cylindrical shell roofs are introduced, in most cases, for day lighting and aeriation purposes. It is usually made in one side only from the shell axis of symmetry.

The analysis of cylindrical shell roofs has been covered in a wide range. The analysis of shell with openings, specially that unsymmetrically opened, still needs more investigation.

The shape, area and location of the openings, of course, are the main parameters in the analysis. The location of openings, with rectangular shape and fixed area, was chosen to investigate its effect on the internal forces variation.

The optimum location, from the stress variation point of view, is defined.

The purpose of this work is to show the effect of the openings upon the internal forces variations.

2. The Analysis Technique

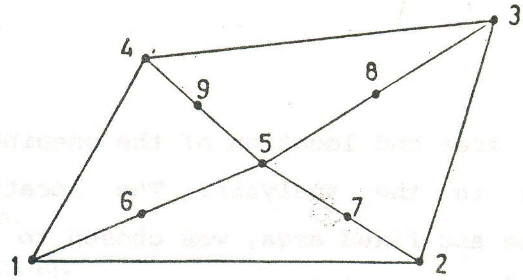
The finite element method was used to analye the shells, either with or without openings, in both cases with or without edge beams.

2.1 Shell Idealization

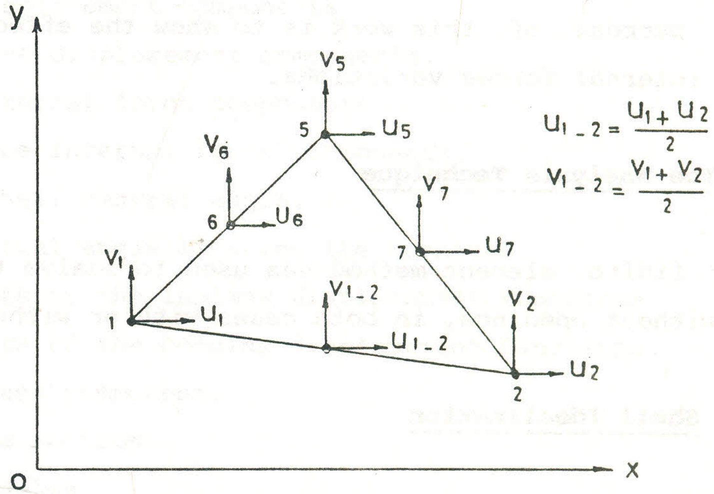
The shell surface was considered as an assemblage of rectangular elements. Quadrilateral element with four external and five internal nodes is used. The element is the assemblage of four triangular elements as shown in Fig (1). The common node of the four triangles is the centre of the rectangular element.

2.2 The Displacement Functions

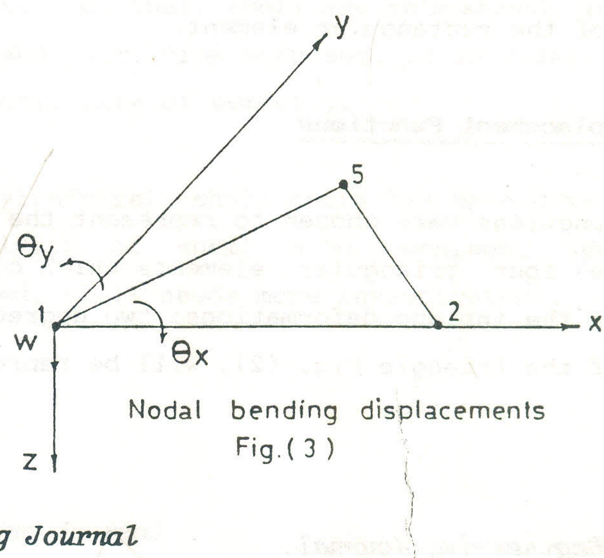
Quadratic functions were chosen to represent the displacements in each one of the four triangular elements which compose the rectangular element. In the inplane deformations; two degrees of freedom (u,v) at each node of the triangle Fig. (2), will be represented as:



Quadrilateral element
Fig.(1)



In_plane displacements
Fig.(2)



Nodal bending displacements
Fig.(3)

$$u=A_1+A_2x+A_3y+A_4x^2+A_5xy+A_6y^2 \quad (1)$$

$$v=A_7+A_8x+A_9y+A_{10}x^2+A_{11}xy+A_{12}y^2 \quad (2)$$

For the bending deformation, three degrees of freedom (w, θ_x, θ_y), Fig. (3) were assumed at each node of the triangular element. They are represented as:

$$w=C_1+C_2x+C_3y+C_4x^2+C_5xy+C_6y^2+C_7x^3+C_8xy^2+C_9y^3 \quad (3)$$

$$\theta_x = - \partial w / \partial y \quad (4)$$

$$\theta_y = \partial w / \partial x \quad (5)$$

2.3 The Stiffness Matrix

The derivation of the stiffness matrix for the element could be obtained easily from the above mentioned displacement functions. The coefficients of the element stiffness matrix are presented in many references. The internal forces were considered as:

N_x, N_y, N_{xy} for the inplane components.

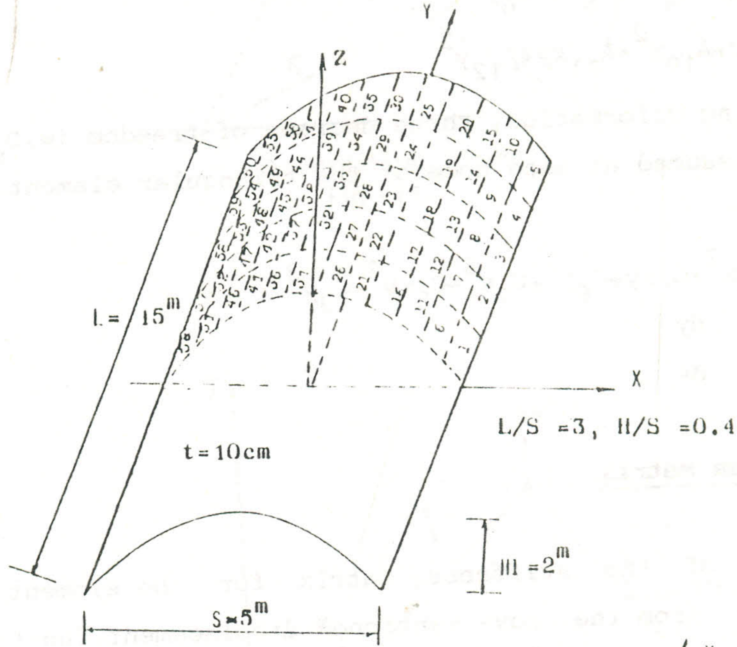
M_x, M_y, M_{xy} for the bending components.

The determination of such internal forces is very easy. There is no need, here, to proceed in a well known derivation.

3. The Computer Programme

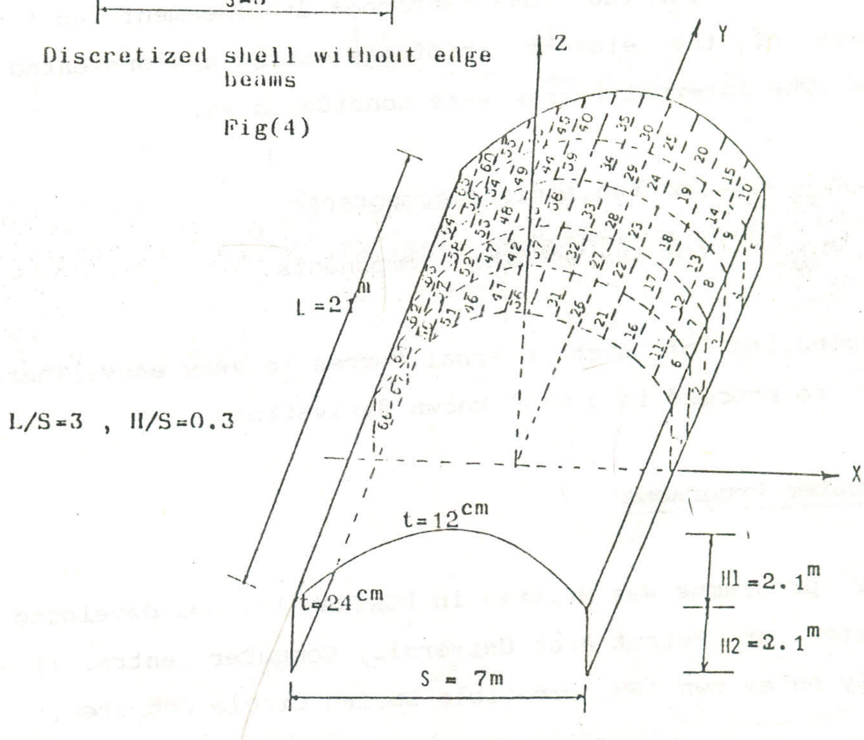
A computer programme was written in FORTRAN IV, and developed on IBM PS-2 computer at Beirut Arab University Computer Centre. It was run successfully on my own IBM Compatible Spring Circle Computer.

The programme has been tested and run several times for solving different types of shells and other plane and space structures.



Discretized shell without edge beams

Fig(4)



Discretized Shell with Edge Beams

Fig(5)

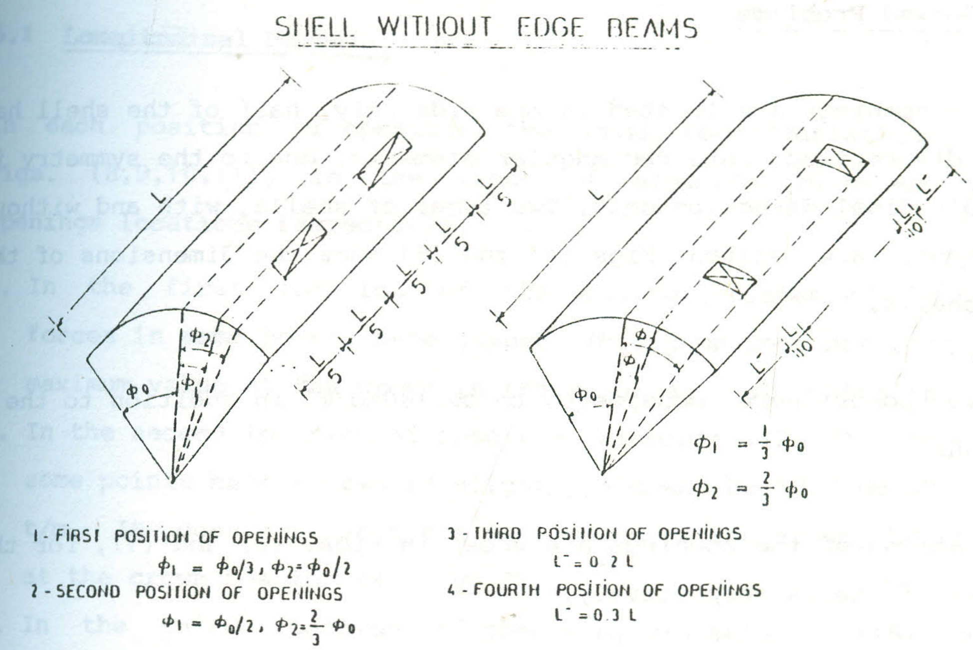

SHELL WITHOUT EDGE BEAMS
1- FIRST POSITION OF OPENINGS
φ1 = φ0/3 , φ2 = φ0/2
2 - SECOND POSITION OF OPENINGS
φ1 = φ0/2 , φ2 = 2/3 φ0
3 - THIRD POSITION OF OPENINGS
L⁻ = 0.2 L
4 - FOURTH POSITION OF OPENINGS
L⁻ = 0.3 L
φ1 = 1/3 φ0
φ2 = 2/3 φ0

Fig.(6)

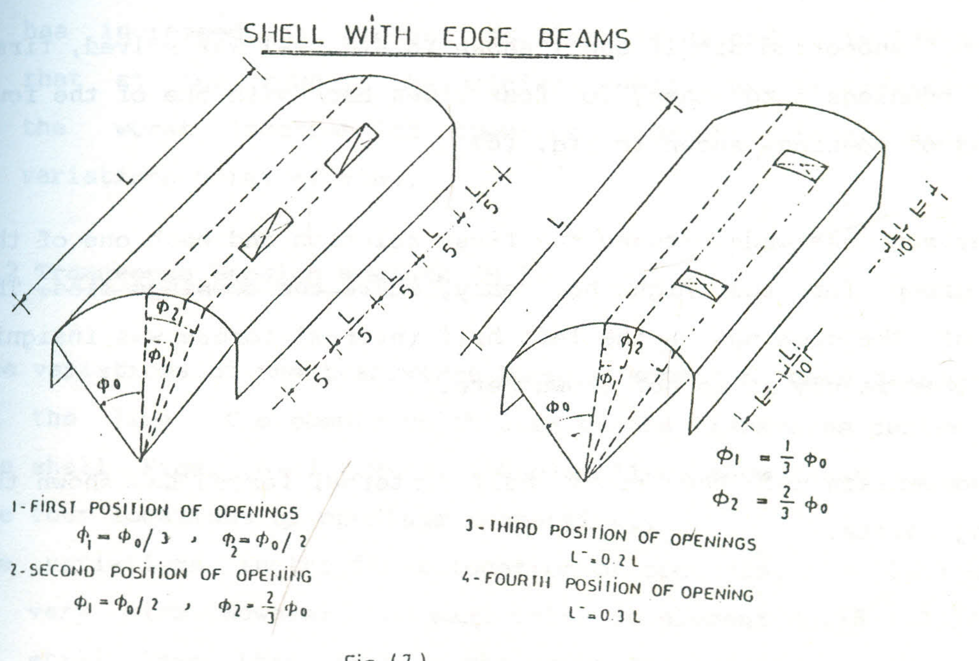

SHELL WITH EDGE BEAMS
1-FIRST POSITION OF OPENINGS
φ1 = φ0/3 , φ2 = φ0/2
2-SECOND POSITION OF OPENING
φ1 = φ0/2 , φ2 = 2/3 φ0
3-THIRD POSITION OF OPENINGS
L⁻ = 0.2 L
4-FOURTH POSITION OF OPENING
L⁻ = 0.3 L
φ1 = 1/3 φ0
φ2 = 2/3 φ0

Fig.(7)

4. The Solved Problems

Since the openings are located in one side only, half of the shell has to be discretized into rectangular elements, due to the symmetry in the longitudinal direction only. Two types of shells, with and without edge beams, are solved. Figs (4) and (5) show the dimensions of the solved shells.

The Live Loads were considered to be 100kg/m^2 in addition to the dead loads.

The locations of the openings are shown in Figs. (6) and (7), for the two types of shells respectively.

5. Analysis of Results of The Shell Without Edge Beams

The simply supported single shell shown in Fig. (4) was solved, first without openings and then, for four times each with one of the four locations of openings shown in Fig. (6)

A comparison was made between the first solution and each one of the second group for the right half only, where the openings lied. The effect of the openings on the left half internal forces was insignificant, that is why it is not shown here.

The comparison of the right half internal forces has shown the following points.

5.1 Longitudinal Force(N_y) :

In each position of openings, the normal force variations are shown Figs. (8,9,10,11), in the lines of elements below and obove the openings locations respectivly.

1. In the first location of the openings, elements (17& 18), the forces in some points have jumped. This jump does not reach the maximum value at the crown in the original shell which is -16.7 t/m
2. In the second location of openings, elements (12&13), the forces in some points have increased slightly,element No.18 from -8.1 to-12.8 t/m. It does not reach the max. value at the crown, but the force at the crown has increased by 2% .
3. In the third location of openings, elements (13 &18), the max. increase of the force is 2% as in the previous case.
4. In the fourth location of openings, elements (12 & 17), the force has increased at element No.22 by 44.8% and it is 18% more than that at the crown in the original shell. This is considered to be the worst location of openings from the longitudinal force variations point of view.

5.2 Transverse Bending Moments (M_x)

The variations of the transverse Bending Moment M_x have been plotted at the line of elements which lies on the transverse centre line of the shell. Figs. (8-c), (9-c), (10-c)& (11-c) show these variations in the four locations of openings respectively.

The variations in the first location of openings, Fig (8-c) seems to be very high. However, the max. value at element No.16 (-0.354 tm/m) is still less than that at the crown (-0.367 tm/m) in the original shell without openings.

SHELL WITHOUT EDGE BEAMS

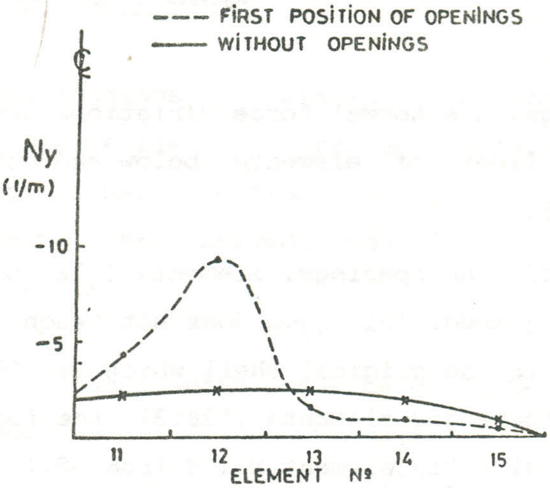


Fig. (8-a)

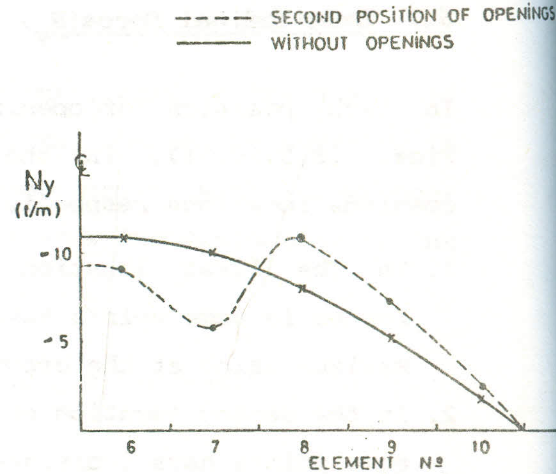


Fig. (9-a)

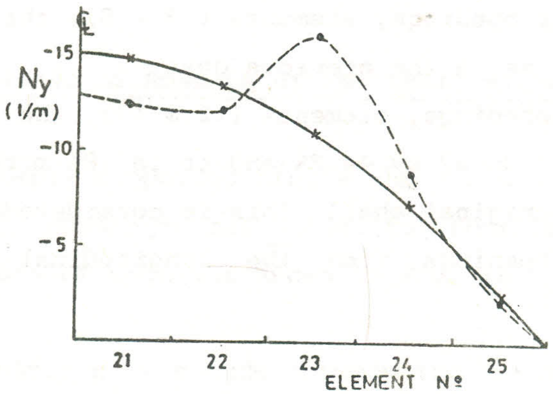


Fig. (8-b)

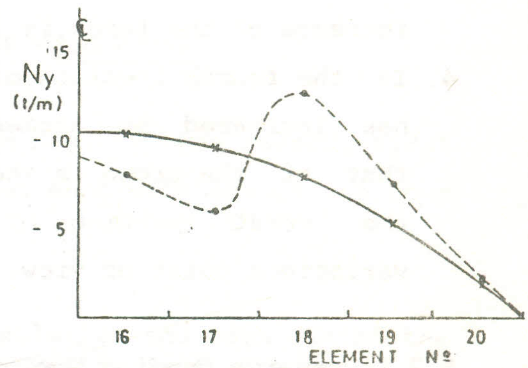


Fig. (9-b)

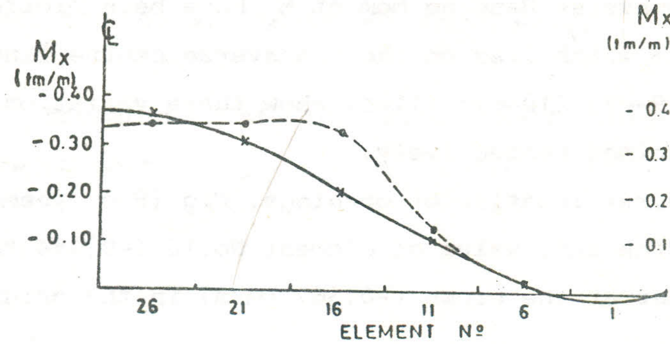


Fig. (8-c)

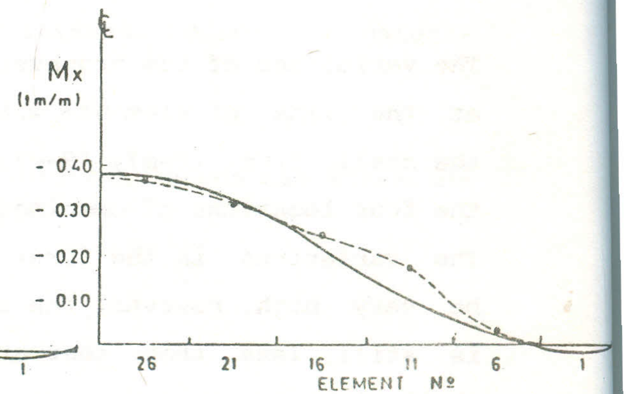


Fig. (9-c)

SHELL WITHOUT EDGE BEAMS

--- THIRD POSITION OF OPENINGS
— WITHOUT OPENINGS

---- FOURTH POSITION OF OPENINGS
— WITHOUT OPENINGS

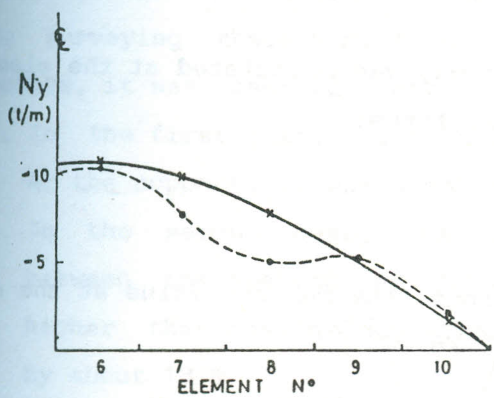


Fig.(10-a)

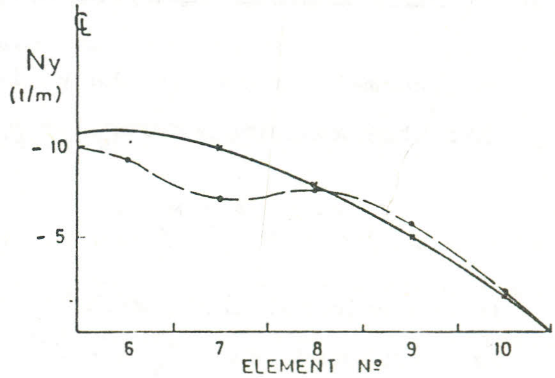


Fig.(11-a)

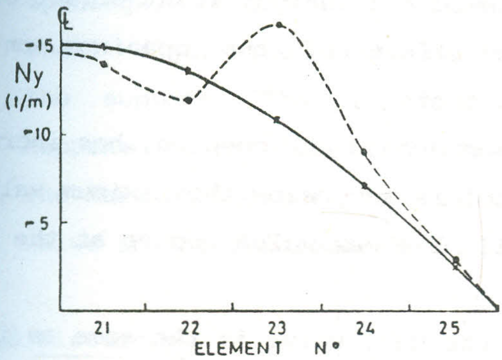


Fig.(10-b)

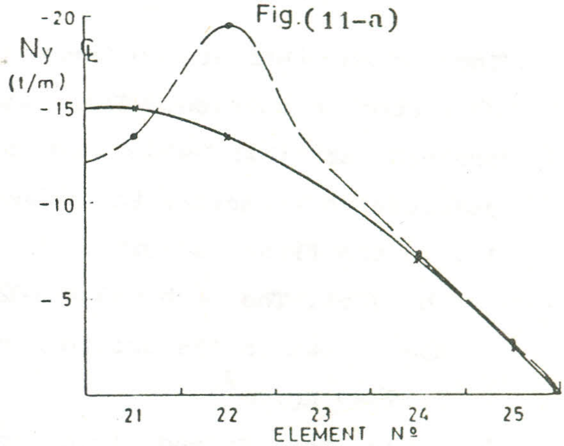


Fig.(11-b)

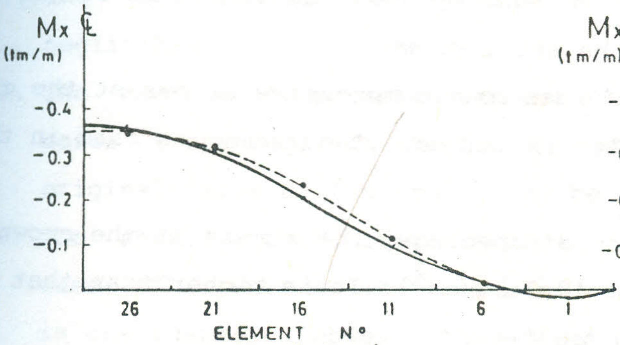


Fig.(10-c)

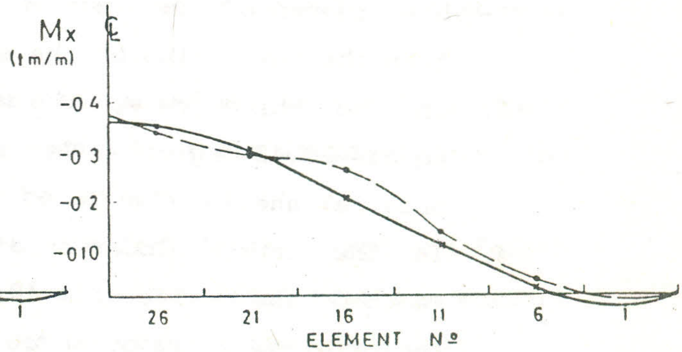


Fig.(11-c)

The variations of the moments in the other three locations of openings could be ignored

5.3 Maximum Normal Compressive Stress

The normal stress, in the y-direction, was calculated at the elements which has maximum compressive normal force.

$$f_y = - N_y/A \pm M_x/Z_x$$

In the original shell without openings, the maximum value at the crown is calculated as: $f_y = - 38.72$ Kg/cm^2

The normal stress in the tension side, at the shell lower edge, has not been discussed. This is due to the insignificant effect of the opnings at this side. For the effect of the openings on the compressive stresses, it was noted that:

1 . In the first location, the stess around the opening, has increased by 7.5%. The high value (-26.56) is far below the maximum value at the crown in the original shell. The max.value occurs at the crown (-37.40 Kg/cm^2).

2 . In the second location, the max. value is the same as in the previous case (-37.10 Kg/cm^2). The stress around the openning has increased in some points by about 20 %, but it is far less than the max. value at the shell crown.

3. In the third case, the maximum compressive stress at the crown equals-38.42 kg/cm^2. It is almost the same as that in the original shell.

4. In the fourth location of openings, the stress at the crown has reached the value of (-40.2 kg/cm^2). It is higher than that of the original shell by about 4 % .

5.4 Shearing Forces N_{xy}

Figs (12 & 13) show the variations of the shearing forces along the lines of elements surrounding the openings in the different locations. By surveying these variations as well as the computer out-put of all points, it was observed that:

1. In the first case, the high increase of the sheraing force occured at the support. It was about 6.5 %.
2. In the second case, the increase was about 41 % at the element between the opening and the support. The value at this element was higher than the maximum value at the support of the original shell by about 18 % .
3. The increase of the shearing force, in the third location of openings, occured at element No. 8, between the opening and the shell edge. The increase was about 4.8 % of the maximum value at the support. The drawback of this shearing force is that it happened at an element relatively far from the support .
4. The max. shearing forces, in the fourth case was less than the max. value in the original shell.

5.5 Discussion

1. The increase of the tensile stress, at the shell lower edge is negligible.
2. The maximum compressive stress has no effect in the first three cases of openings. In the fourth case the stress increase, over the original value at the crown, may be accepted.
3. The shear stress increase has no effect on the first and fourth case of openings. In the second case, the effect near the support is considerable. However the effect is less in the third case.

SHELL WITHOUT EDGE BEAMS

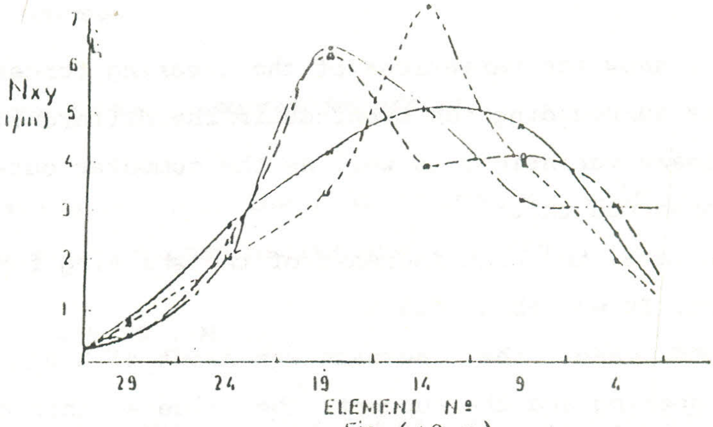


Fig (12-a)

——— WITHOUT OPENINGS
– – – – FIRST POSITION OF OPENINGS
- - - - - - SECOND POSITION OF OPENINGS
—·—·— THIRD POSITION OF OPENINGS

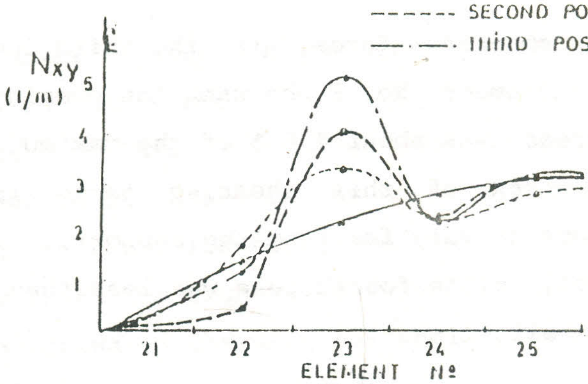


Fig. (12-b)

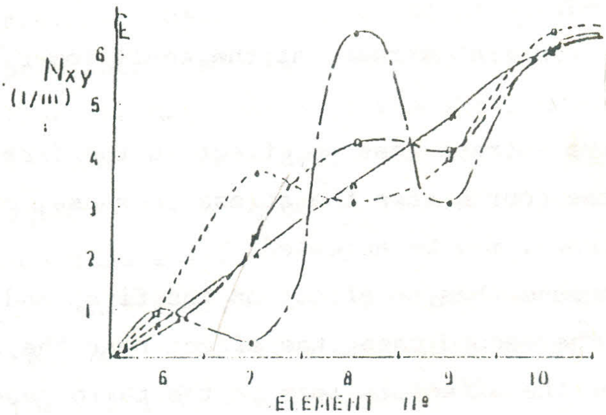


Fig. (12-c)

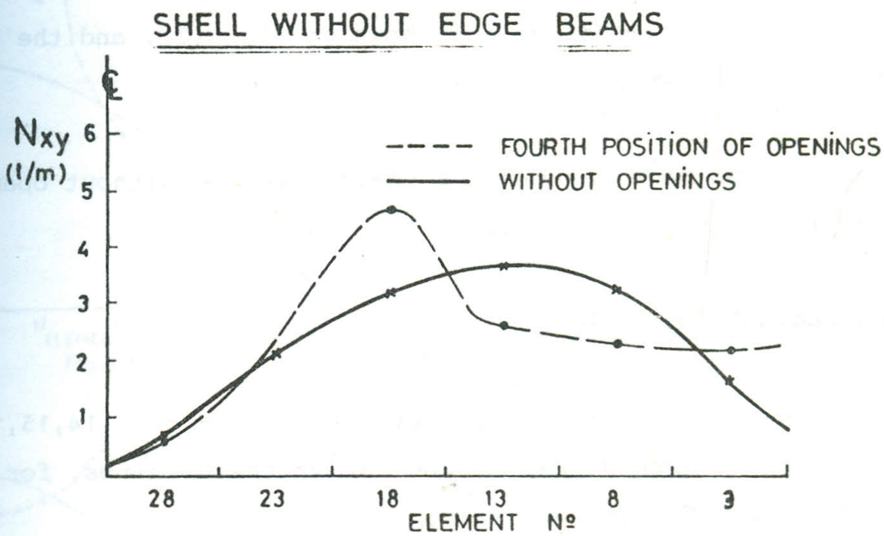


Fig. (13-a)

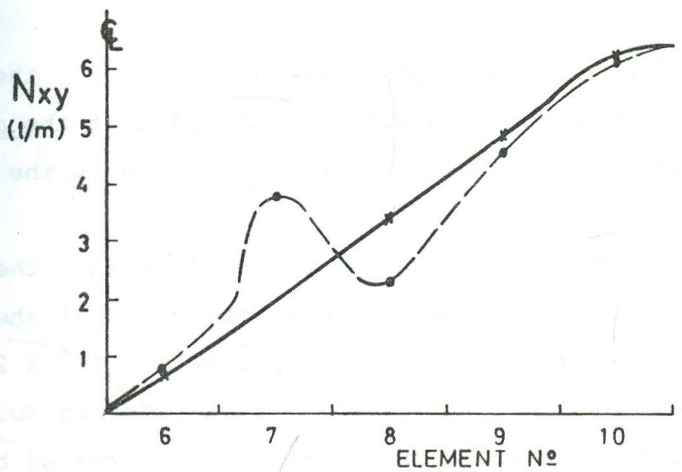


Fig. (13-b)

6. Analysis of Results of the Shell with Edge Beams

As in the first type of shell, the second type with edge beams was solved. The shell dimensions are shown in Fig. (5), and the locations of openings are shown in Fig. (7).

A comparison was made between the shell with and without openings for each one of the four locations.

6.1 Longitudinal force (N_y)

The variations of the normal force are shown in Figs. (14,15,16,17) in the lines of elements below and above the openings, for the four positions of openifgs respectively.

1. In the first position of openings, elements (22 & 23), the maximum values are more than the value at the crown in the original shell by 15.6 %.
2. In the second position of openings, elements (17 & 18), the maximum increase in the normal force is 8.6 % . The value at the crown has increased by 1.6 % over the corresponding value in the original shell.
3. For the third position of openings, elements (18 & 23), the maximum increase is 17.8 % over the crown value in the original shell.
4. In the fourth position of openings, elements (17 & 22), the maximum increases are 4.9 % & 40.5 % at elements No. 12 & 27 respectively. However the the crown value has increased by 11,4 % only.
5. In all cases, the tensile forces, at the lower part of the edge beams, have increased by only 0.7 %. There is no need to show here such variations.

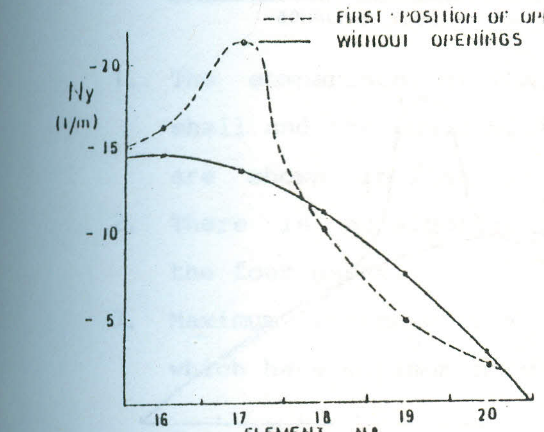


Fig. (14-a)

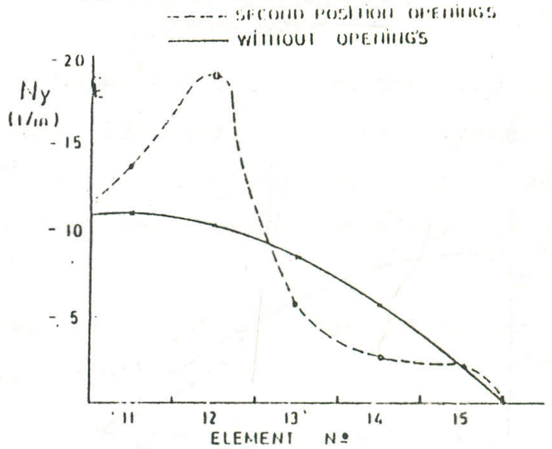


Fig. (15-a)

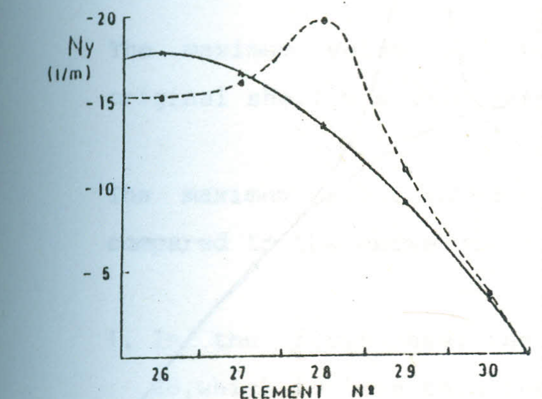


Fig. (14-b)

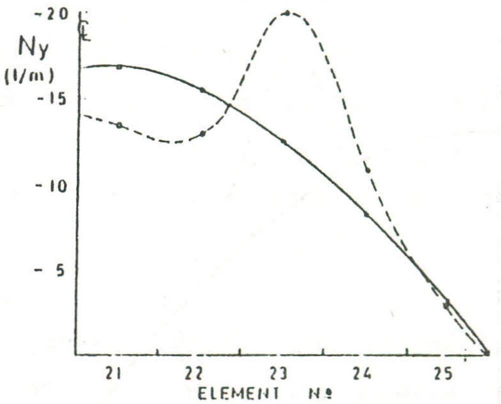


Fig. (15-b)

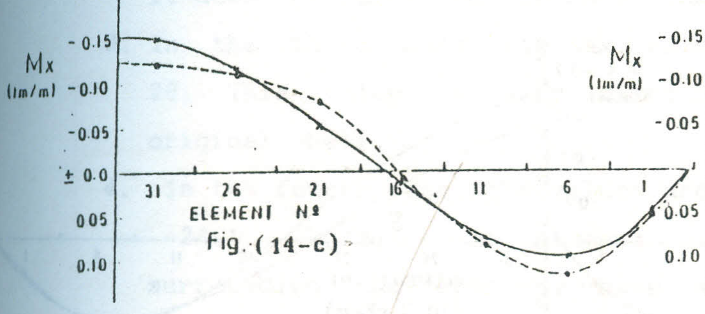


Fig. (14-c)

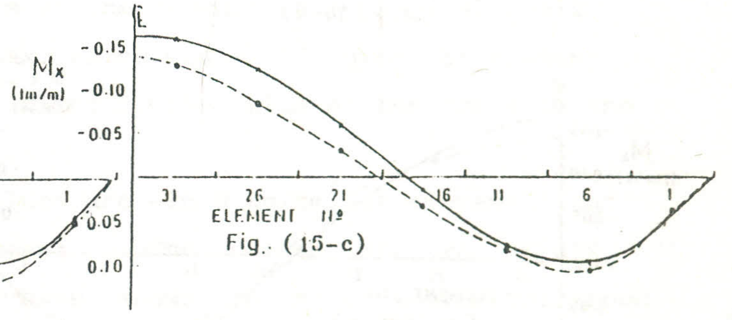


Fig. (15-c)

SHELL WITH EDGE BEAMS

- - - - THIRD POSITION OF OPENINGS
——— WITHOUT OPENINGS

— — — FOURTH POSITION OF OPENINGS
——— WITHOUT OPENINGS

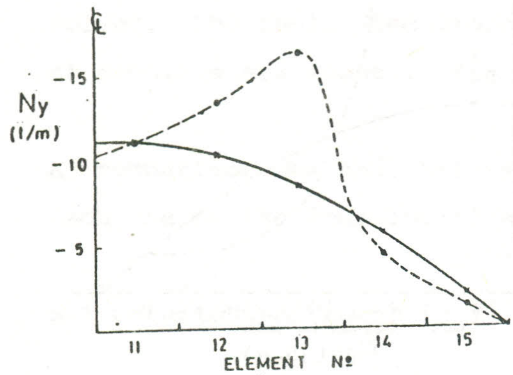


Fig. (16-a)

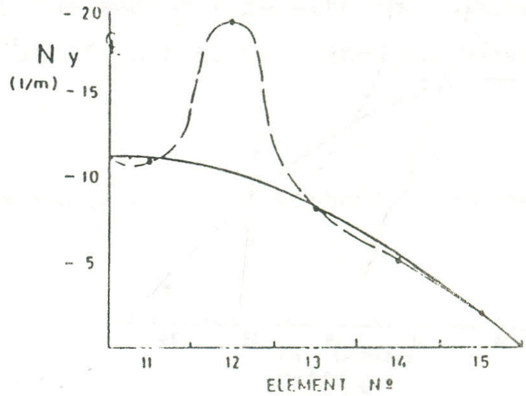


Fig (17-a)

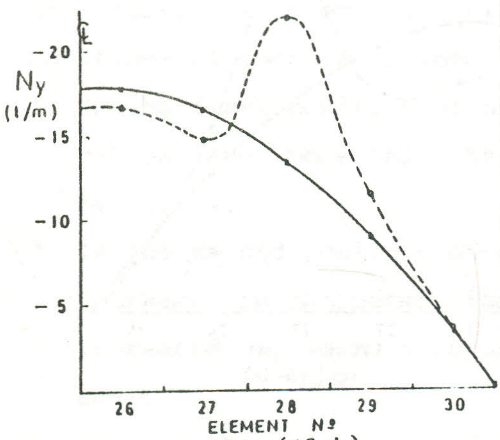


Fig .(16-b)

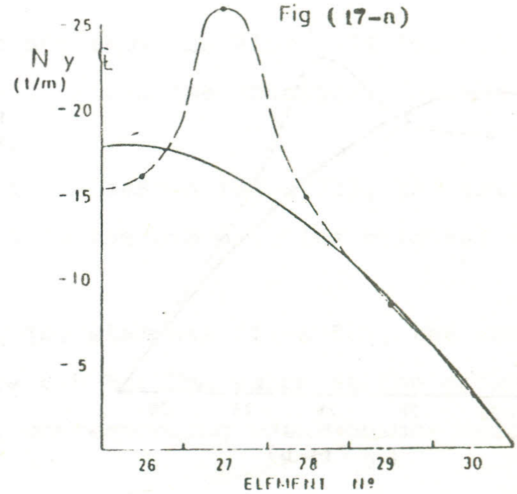


Fig.(17-b)

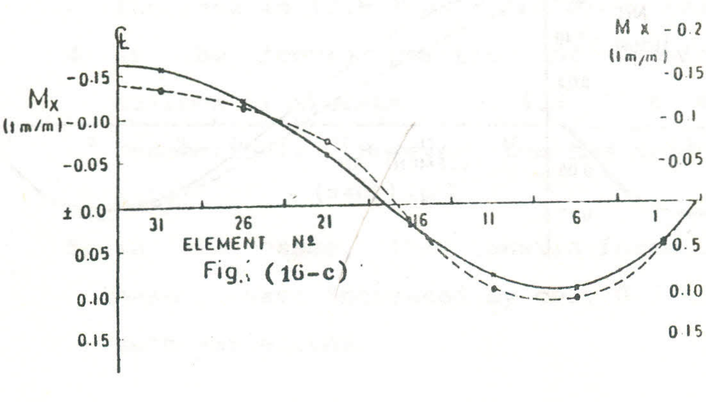


Fig. (16-c)

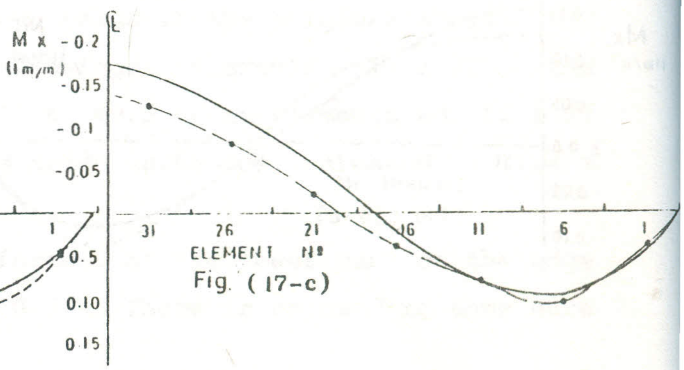


Fig. (17-c)

6.2 Transverse Bending Moments (M_x)

1. The comparison, of the Bending Moments M_x, between the original shell and the shell with the four different positions of openings, are shown in figs. (14-c), (15-c), (16-c) & (17-c) respectively.
2. There is no significant effect of the increase of M_x in any of the four cases.
3. Maximum increase of M_x is about 10 %, but it occurs at elements which have minimum normal longitudinal force.

6.3 Maximum normal compressive stress

The maximum value of the compressive stress, at the crown, in the original shell was calculated as before: (f_y = -22.08 kg/cm^2)

The maximum value in each case of the four locations of openings was compared to the above value.

1. In the first case, the max. value was -20.23 kg/cm^2 at element 28,which is less than the value at the crown in the original shell.
2. In the second case, the max. value was -20.92 kg/cm^2 at the crown, it does not reach the stress at the crown in the original ahell.
3. In the third case, the max. value was -21.7 kg/cm^2 at element 28. This value is very close to the value at the crown in the original shell.
4. In the fourth case, the values are -26.0 kg/cm^2 at element27 and -24.1 kg/cm^2 at element 32.The values at two points, surrounding the opening, have shown an increase over the crown value. The max. increase reached 18 % of the value at the crown in the original shell

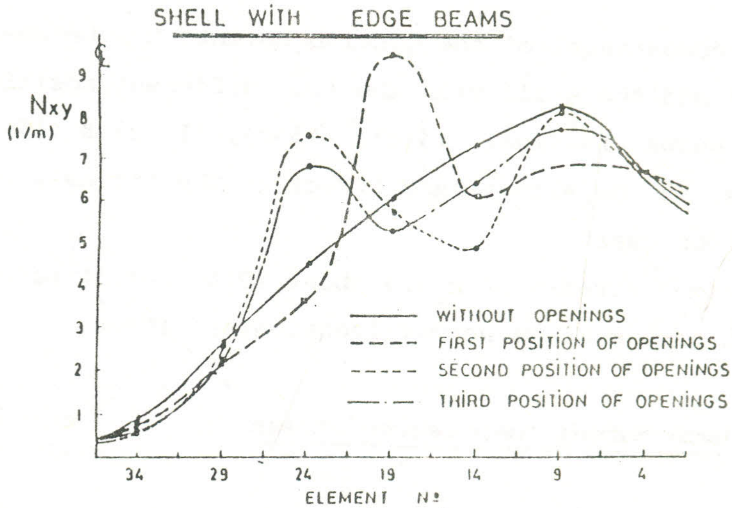


Fig.(18-a)

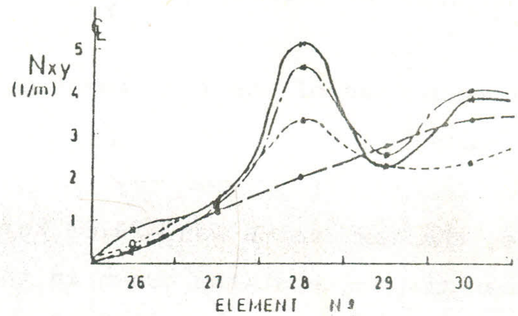


Fig.(18-b)

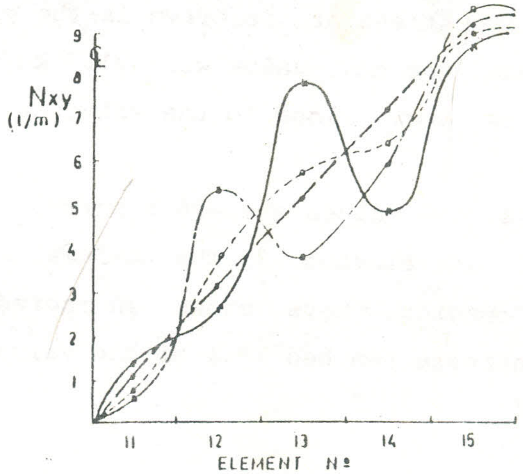


Fig.(18-c)

6.4 Shearing Forces N_{xy}

The variations of the shearing forces are shown in figs. (18 & 19), along the lines of elements surrounding the openings. The increase around the openings was as follows:

1. For the first case, the high increase around the opening was still less than the maximum value of the shearing force at the support by 34%.
2. In the second case, the high value was less than the maximum value at the support by 9.5%. However, some other points at the support have almost reached the maximum value in the original shell.

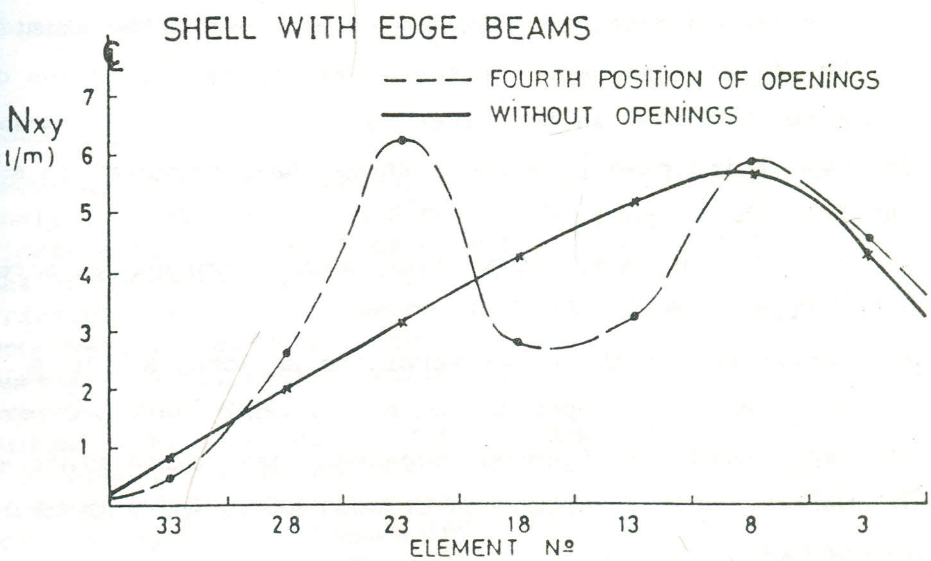


Fig.(19)

3. In the third case, the max. increase was 52% over the original value at the same point. This value is less than the maximum value at the support in the original shell by 25%, but it took place in a point half way between the support and the shell centre

4. In the fourth case, there is a high Jump of the shearing force around the opening. However it is far less than the maximum value at the support in the original shell. Some points might be critical because they lie far from the support.

6.5 Discussion

1. The increase of the tensile stress at the ends of the edge beams, could be ignored.
2. In the first case, both the maximum normal compressive and maximum shear stresses have retreated. The increase of such stresses around the openings has no effect.
3. In the second case, the maximum stresses have the main feature as in the first case. However the shear stress around the opening is 38% more than that in the first case.
4. In the third case, the shear stress has increased at some points far from the support.
5. In the fourth case, the maximum normal compressive stress is the only critical one.
6. All variations of the other straining actions, N_x, M_y, M_{xy}, in all cases of openings were thoroughly surveyed. Most of the changes could be ignored because they cause minor effect. The discussion of the variations of such straining actions has little importance.

7. CONCLUSION

For both types of shells, the four locations of openings are accepted, some of them with reservations, as follows:

1. The first position is considered to be the optimum one. It gives the best stress distribution over the other three locations. The normal stress is accepted and the increase of stear stress, near the support, is to be covered by the ordinary diagonal reinforcement of the shell.
2. In the other three positions of openings, the ordinary diagonal reinforcement of the shell has to be extended beyond the openings.
3. If the fourth position has to be chosen, check of the compressive stress at the crown is needed. An increase of the shell thickness, by 10% for the first type and 20% for the second type, will cover the compressive stress increase.
4. In all cases, it is recommended to provide rips around the openings. The rip thickness and width may be taken as twice as the shell thickness. This will decrease the shear stress around the openings considerably.

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