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CONTRIBUTION OF THE Z-DISK TO THE INTRACELLULAR RESISTIVITY OF A MYOFIBER

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Abstract

The effect of the Z-disk as a nonconductive obstacle on the intracellular resistivity of a myofiber has been studied the in this work. A hypothetical fiber model is proposed as a long homogeneous conductive cylinder with one longitudinal array of Z-disks located at the cylinder axis at equal spaces. A constant current is assumed to flow along the fiber axis. Solving Laplace's equation for the proposed model, the calculated equipotential surfaces are used to determine the effective interacllular resistivty of the fiber.

The model has been simplified by assuming plane equipotential surfaces. To reduce the error in resistance calculations arising from this assumption, the Z-disks are assumed to have an effective thickness "t". From the obtained results, it is concluded that the simplified model can be used for resistance calculations with a marginal error of + 5% compared with the original model.

Introduction

A myofiber may be regarded as a long, cylindrical, single cell. It is surrounded by a membrane called sarcolemma. Each myofiber contains sveral hundred to several thousand myofibrils (1), (2) laying in parallel along the fiber axis. The myofibrils are regularly interrupted by the so-called Z-disks (Fig. 1). These disks are equally spaced by the unit sarcomere length S. Thin filaments (actin) are attached to each Z-disk from both sides and are interdigitated with thick filaments (myosin). Actin and myosin interaction is responsible for muscle contraction. Resistivity of the intracellular compartment is greater than that of the intracellular fluid (3) - (6) due to the non-conductive constituents suspended in the sarcoplasm.

Several factors contribute to the measured value of ρ_i so that it ranges from 130- to-470 Ω cm for different muscle fibers (4)-(6). Also, it is not constant even for the same muscle type (5), (6). In this work, we have limited ourselves to the effect of the Z-disks on the intracellular resistivity of the myofiber.

The Z-disk, being a non-conductive obstacle, causes a change in the axial current flow pattern. It does not allow the axial current to flow through it and forces the current to flow within the space between the myofibrils. This twisted flow pattern and the associated equipotential surfaces will be calculated in this work.

The assumption of parallel plane equipotential surfaces is very tempting in resistance calculations even knowing that it envolves a considerable source of error. To take benefit of this simplification

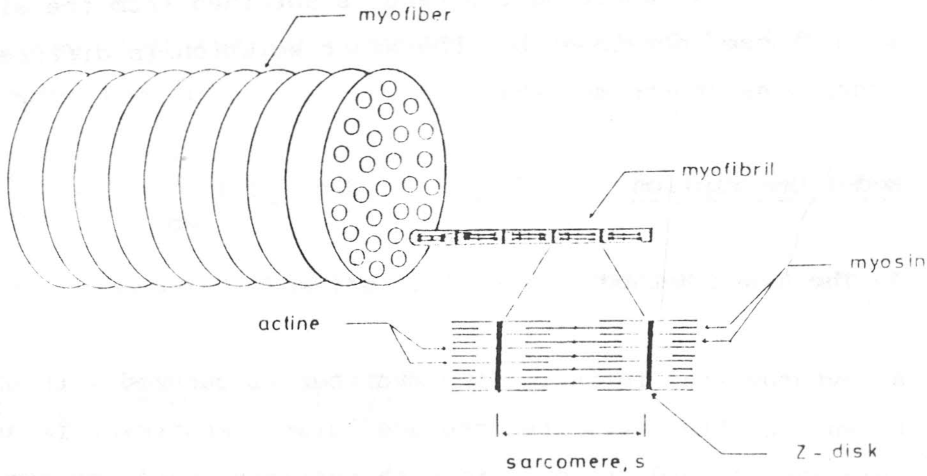

fig. 1 : The ultrastruture of a myofiber

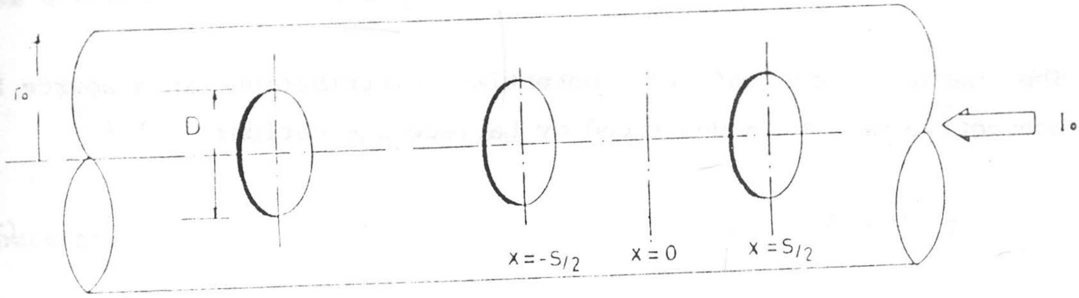

fig 2 . Myofiber model with one-dimension array of Z - disks .

with an acceptable margin of error, a thick Z-disk is assumed to account for the effect of the current evacuated region behined the disk. The effective thickness "t" of the Z-disk will be calculated in this work by comparing the results obtained from the simplified model with those obtained by the exact solution in different geometrical conditions of the myofiber.

Model Description

A- The Exact Method

A hypothetical infinity long myofiber is assumed with a radius r_o as shown in Fig. (2). The interacellular compartment is assumed to have homogeneous conductive fluid with resistivity ρ A one dimensional array of the Z-disks is located at the fiber axis. The disk diameter is D, and the distance between the adjacent disks equals to the sarcomere length S. A constant current I is assumed to flow along the fiber axis. The resultant potential distribution V(r,z) is to be calculated.

The general form of the potential distribution in a source free conductive medium is described by Laplace's equation:

$$\nabla^2 V = 0 \tag{1}$$

Considering our model described before, the analytical solution of Eq. 1 in cylindrical coordinates takes the following form:

$$V = Z + \sum_{m=1}^{\infty} B_m \sinh (K_m Z) J_o(K_m r) \tag{2}$$

where, B_m & K_m are constants. Eqn. 2 is valied for :

$$- S/2 < Z \leq S/2 \quad \text{and} \quad r_o \geq r \geq 0$$

Accordingly, the longitudinal component of the electric field E_z has the following form :

$$E_z = - \frac{\partial V}{\partial Z} = - 1 - \sum_{m=1}^{\infty} B_m K_m \cosh (K_m Z) J_o (K_m r) \tag{3}$$

and the radial component E_r has the following form:

$$E_r = - \frac{\partial V}{\partial r} = \sum_{m=1}^{\infty} B_m K_m \text{ Sinh } (K_m Z) J_1 (K_m r) \tag{4}$$

Boundary conditions:

i) Radial boundary consitions

At the outer surface of the cylinder $r = r_o$, the radial component of the electric field is zero,

$$E_r \Big|_{r=r_o} = 0 \tag{5.a}$$

therefore:

$$0 = \sum_{m=0}^{\infty} B_m K_m \text{ Sinh } (K_m Z) J_1 (k_m r_o) \tag{5.b}$$

Since Eq 5-b is valid for all values of $S/2 \geq Z > - S/2$, therefore , $K_m r_o$ are zeros of $J_1 (K_m r_o)$. This means that:

$$K_m r_o = 3.83,\ 7.02,\ 10.17,\ 13.32\ ,\ \ldots \quad (6)$$

From Eq. 6, one can obtain all values of K_m

ii) Longitudinal boundary conditions

At the location of the Z-disk, $Z = \pm S/2$, the longitudinal field component is zero for $r = 0$ to $r = D/2$.

$$E_z \Big|_{z = \pm S/2} = 0 \qquad (7.a)$$

therefore:

$$0 = -1 - \sum_{m=1}^{\infty} B_m K_m \text{ Cosh } (K_m S/2) J_o(K_m r)$$

substituting number of values for r in the above equation, such that $0 \leq r \leq D/2$, we get a set of simultaneous equations in B_m.

While for $Z = S/2$ and $r_o \geq r > D/2$, the potential is constant, therefore:

$$V = \text{cosnt} = S/2 + \sum_{m=1}^{\infty} B_m \text{ Sinh } (K_m S/2) J_o(K_m r) \qquad (8)$$

substituting number of values for r such that $r_o \geq r > D/2$ in eq.8 we get another set of equations.

B_m's constants are calculated by using the two sets of equations 7 and 8. The fiber radius r_o is devided into n sections so that Eqns 7 & 8 give (n+1) simultaneous equations. The solution of these equations gives B_1, B_2, ..., B_n, and the potential V at Z = S/2. In this work the solution is limited to n=10.

Nine cases were considered with different values of the disk radius D/2 in relation to the fiber radius r_o. Potential distribution inside the fibre (Fig. 3) as well as intracellular resistivity $_i$ (Table 1) are calculated for each case.

B. The Simplified Method

The results obtained (Fig. 3) show almost a current evaccuated region created behind the disk. It is obvious that this region depends on the disk dimension relative to the fiber dimensions. To take benifit of the plane equipotential assumption, with an acceptable margin of error, a thick Z-disk is assumed to account for the current evacuated region behind the disk (Fig. 4). The intracellular resistivity in this case is simply given by:

$$\rho_i = \rho_{i0} \left[1 + \frac{t}{S} \left(\frac{1}{1 - D^2/4r_o^2} - 1 \right) \right] \qquad (9)$$

where t is the effective thickness of the Z-disk, is the resistivity of the homogineous intracellular fluid. The values of ρ_i obtained from the exact solution were used in equation (9) to give the value of the effective thickness "t".

Results and Conclusion

Table (1) shows values of ρ_i calculated from the exact solution at different conditions of S, r, D. Also, it shows the calculated value of the effective thickness "t" of the Z-disk according to Eq. (9). Using the average value of t/r_o which is 0.488, the error is limited

D/2 = 0.7 r_0

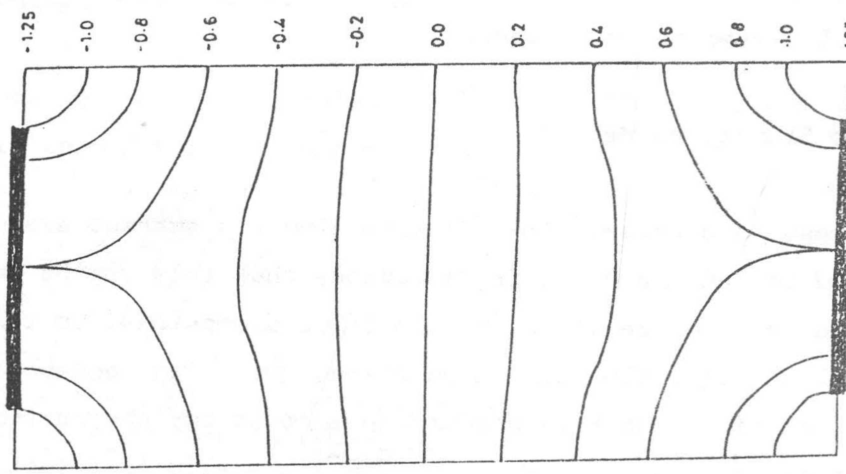

fig. 3-a : Potential distribution inside the myofiber.

$\rho = 1.25\rho_0$

D/2 = 0.9 r_0

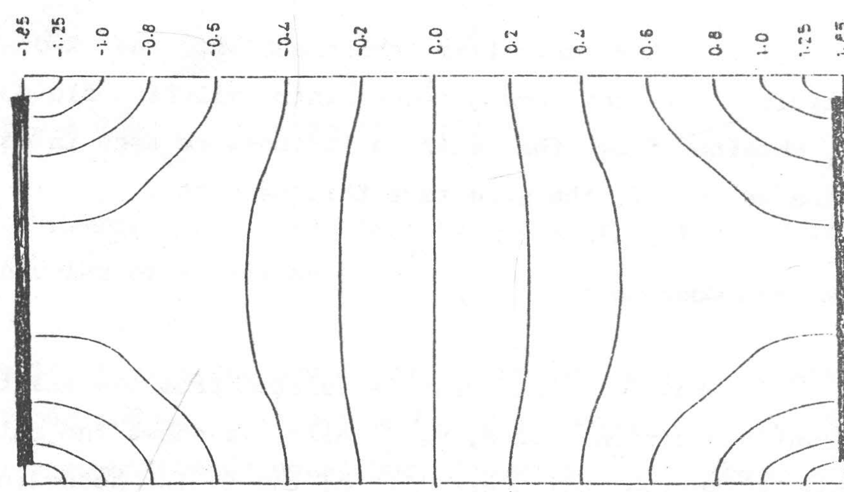

fig. 3-b: Potential distribution inside the myofiber.

$\rho = 1.85\rho_0$

to within $_{max}$ = 3. 54 %.

From the above results we may conclude the following:

1. The Z-disks of a myofiber are responsible in part for the observed increase of intrucellular resistivity of the fiber and their effect may increase the resistivity up to 50 % above the average resistivity of the cytoplasm .
2. The simplified model with plane equipotential surfaces may be used with an acceptable error in the calculations of myofiber resistance with the assumption of thick Z-disks (t = 0.488 * radius of the hypothetical fiber) in the model of one-array of Z-disks.
3. For a real myofiber the effective thickness (t) of the Z-disks may be calculated from the following formula

$$t = 0.488\sqrt{\frac{\text{Crossectional area of the myofiber}}{* \text{ no. of Z-disks in this section}}}$$

Table : 1

$S\,\mu m$	$D/2r_o$	r_o	ρ_i	t	t/r_o	ρ'_i	$\varepsilon = \frac{\rho'_i - \rho_i}{\rho_i} x100$
2	0.4	1	1.042	0.44	0.44	1.04	+ 0.43
2	0.5	1	1.086	0.516	0.516	1.081	- 0.46
2	0.6	1	1.139	0.492	0.492	1.137	- 0.17
2	0.7	1	1.245	0.51	0.51	1.234	- 0.85
2	0.8	1	1.385	0.434	0.434	1.434	+ 3.54
2	0.6	0.833	1.115	0.4088	0.451	1.114	- 0.09
2	0.7	0.833	1.205	0.425	0.510	1.195	- 0.83
2	0.6	1.25	1.172	0.604	0.483	1.171	- 0.08
2	0.7	1.25	1.305	0.645	0.516	1.293	- 0.92

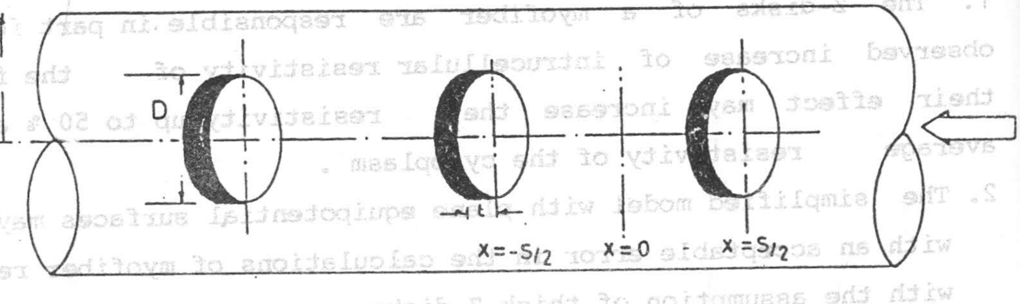

Fig. 4: A hypothetical model of a myofiber with thick Z-disks and plane equipotential surfaces. Effective thickness of each Z-disks equals "t".

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