# A SOLUTION TO THE HUNTING PROBLEM IN REFRIGERANT EVAPORATOR USING A MECHANICALLY CONTROLLED THERMOSTATIC EXPANSION VALVE

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# Abstract

The dynamic model of the refrigerant evaporator and its connected devices was developed. The control of the evaporator capacity using the conventional thermostatic expansion valve was considered, and its hunting problem was demonstrated. To prevent this undesirable behavior, a mechanical PI controller was used to control the thermostatic expansion valve. The optimum controller parameters were synthesized. The dynamic behavior of the controlled system was examined. The introduced system has been proven to be capable of damping the oscillatory response and adjusting the degree of superheat to the desired value with minimum settling time.

## Nomenclature

- A Area
- c Specific heat
- C Capacitance
- d Valve diameter
- e Internal energy
- er Error signal
- h Heat transfer coefficient
- i Specific enthalpy
- k Spring stiffness
- k. Valve constant
- K Gain
- l Length
- L Valve opening at design condition
- M Mass flow rate of refrigerant
- P Pressure
- R Resistance
- t Time
- T Temperature
- u Fluid velocity
- v Specific volume
- V Volume
- x Length coordinate of evaporator tube
- y Valve displacement from design condition
- ρ Fluid density
- $\boldsymbol{\theta}_{\text{S}} \quad \text{Degree of superheat}$

#### Subscripts

a Air

- aw Air-wall
- b Bulb
- c Compressor

cond Condenser

- d Design condition
- ev Evaporator
- exp Expansion valve
- i Integral
- R Refrigerant
- Rw Refrigerant-Wall
- Sat Saturation
- Suc Suction
- w Wall

#### Introduction

Dry evaporators are often used in the field of refrigeration and air conditioning. There are two contrary demands for the evaporator charge. First, to maximize the evaporator surface, the liquid-dry-out point should be near the end of the evaporator, i.e. the superheat is minimum. On the other hand, the superheat must be maximum to protect the compressor against the liquid refrigerant. A superheat of 4-7 K is commonly selected as an efficient compromise. For a good evaporator utilization, it is important to control the mass flow rate of refrigerant (consequently, the superheat) with maximum cooling capacity under changing operating conditions.

In practice, the liquid flow is controlled by a thermostatic expansion valve, which uses the superheat of the vapor at the end of the evaporator as the control signal. This control has the advantage of low assembling and operating costs. However, an undesirable

oscillatory behavior, known as hunting, is often exhibited because there are difficulties to adapt the valve characteristics to the static and dynamic behavior of the evaporator. Unfortunately, most refrigeration systems require higher superheat for stable operation which reduces the evaporator efficiency. The hunting problem has not been solved yet . Wedekind et al. [1,2] described random fluctuations the liquid-dry-out point due to the nature of two phase flow in evaporators without considering a controlled evaporator. Najork [3], Huelle [4] gave statical explanations for the unstable behavior. Broersen and van der Jagt [5] described the hunting using a modified mean void fraction model. The equations can be characterized by an open-loop transfer function in which the coefficients are the physical parameters of a refrigerant system. It has been concluded that all methods that diminish the influence of the bulb without diminishing the influence of the evaporator pressure may improve the system stability. However, they demonstrate that new valve construction is needed to solve the practical problem. Gruhle and Isermann [6] developed a theoretical model of a refrigerant evaporator and the connected expansion valve, superheater and compressor. The oscillating behavior of the plant was explained by the strong non-linear course of heat transfer coefficient. The disadvantages of the conventional thermostatic control were demonstrated, at which the effect of hunting explained by too high a gain of the controller. The results obtained were used to improve the control of the real process with a process computer.

In this work, a theoretical model is developed for the evaporator plant, consisting of evaporator, expansion valve, and compressor suction point. A new controller is introduced to prevent the undesirable oscillatory performance. The control behavior of the

evaporator plant with a conventional thermostatic expansion valve and with the suggested controller are simulated.

## Modeling of Evaporator Plant

The refrigerant evaporator and the connected devices, i.e. expansion valve and compressor, are shown in Figure 1.

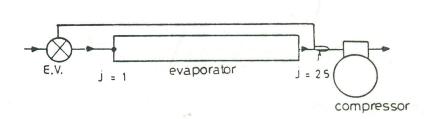


Figure 1: The refrigerant evaporator and the connected devices .

The flow through the evaporator, when it is subjected to step load variation, is unsteady one-dimensional which can be described by the following balancing equations:

#### Continuity equation

$$\frac{\partial \rho}{\partial t} = \frac{\partial (\rho u)}{\partial x}$$

$$\frac{\partial \rho}{\partial t} = 0$$
(1)

# Momentum equation

$$\frac{\partial (\rho u)}{\partial t} = \frac{\partial (\rho u^2)}{\partial x} = \frac{\partial P}{\partial x}$$
(2)

# Energy equation

$$\frac{\partial [\rho(e+u^{2}/2)]}{\partial t} = \frac{\partial [\rho u(i+u^{2}/2)]}{\partial x} = \frac{h_{RW}}{A_{RW}} - (T_{W} - T_{R})$$
(3-a)

or

$$\frac{\rho \partial i}{\partial t} + \rho u \xrightarrow{\partial x} \frac{\partial P}{\partial x} + u \xrightarrow{\partial P} \frac{\partial P}{\partial x} + u \xrightarrow{\partial RW} \frac{A_{RW}}{A_{R}} (T_{W} - T_{R})$$
(3-b)

#### Heat balance of the tube walls

$$\rho_{w} \quad c_{w} \quad A_{w} \quad \ell_{w} \quad \frac{\partial T_{w}}{\partial t} = h_{aw} \quad A_{aw} \quad (T_{a} - T_{w}) - h_{Rw} \quad A_{Rw} \quad (T_{w} - T_{R})$$
(4)

#### Refrigerant Properties

The thermal properties of the refrigerant either, in wet or superheat region, are computed using the state computational equations of Reynolds (1979). It is believed that the manipulation of these equations needs several recasts to be suitable for the iterative routines required for the present numerical solution.

i) Saturated pressure vs saturated temperature:

$$\ln P_{\text{sat}} = F_1 + F_2 / T_{\text{sat}} + F_3 \ln T_{\text{sat}} + F_4 T_{\text{sat}}$$

+ 
$$F_s$$
.  $\frac{(\gamma-T_{sat})}{-1}$  ln  $(\gamma-T_{sat})$ 

ii) Saturated liquid density vs saturated temperature

$$\rho_{\text{f sat}} = \sum_{j=1}^{5} D_{j} x^{(j-1)/3} + D_{6} x^{\frac{1}{2}} + D_{7} x^{2}$$

where 
$$X = 1 - T_{sat} / T_{crit}$$

iii) Specific heat at constant volume vs temperature:

$$c_{v} = \sum_{j=1}^{4} G_{j} T^{j-1} + G_{5}/T^{2}$$

iv) Refrigerant gas state (Saturated or Superheated)

$$P = \frac{RT}{v-b} \int_{j=2}^{5} \frac{1}{(v-b)} * (A_{j} + B_{j}T + C_{j} e^{-KT/T} crit + \frac{(A_{6} + B_{6}T + C_{6}e^{\alpha V})}{e^{\alpha V}(1 + C_{6}e^{\alpha V})}$$

v) Latent heat of Evaporation

$$i_{fg} = T_{sat} (v_g - v_f) \frac{d P_{sat}}{d T_{sat}}$$

vi) Internal energy and enthalpy (saturated or superheated)

$$e = \int_{C}^{T} c_{V}(T)dT + \int_{\rho}^{\rho} \frac{1}{\rho} \left[P-T(\frac{-\rho}{-\rho})\right] d\rho + e_{\rho}$$

and

$$i = Pv + e$$

The constants  $A_j$ ,  $B_j$ ,  $C_j$ ,  $D_j$ ,  $F_j$ ,  $G_j$ , K,  $\alpha$ , b,  $T_{crit}$ ,  $T_o$  and  $e_o$  are listed for Refrigerant R-12 in appendix (A)

#### Compressor capacity

The refrigerant mass flow rate through the compressor is directly proportional to the compressor speed and the volumetric efficiency while it is inversely proportional to the suction specific volume. In the normal operation of the refrigerating system, the compressor speed is fixed while the volumetric efficiency is considered constant. Since the present study is restricted in a narrow range of evaporating pressure variation, therefore the refrigerant mass flow rate through the compressor relates to the one at the design condition by:

$$M_{c} = M_{c,d} * \frac{\rho_{suc}}{\rho_{suc}}$$
(5)

# Expansion valve

The evaporator is fed with the wet vapor refrigerant from the high pressure liquid refrigerant incoming from the condenser through the expansion valve. For linear valve characteristics, the mass flow rate through the expansion valve is:

$$M_{\text{exp}} = k_{\text{v}} ( \pi \text{ d}) (L-y) \sqrt{\rho (P_{\text{cond}} - P_{\text{ev}})}$$
 (6)

## a) Conventional Thermostatic Valve

For a thermostatic expansion valve, the degree of superheat is measured by an equivalent pressure difference which actuates on the needle directly. The valve lift, which represents the output signal, is proportional to the pressure difference signal according to the relation:

$$(L - y) = \frac{a}{k_s} (P_b - P_{ev})$$
 (7)

where k is the equivalent stiffness of the valve spring and diaphragm, and a is the effective diaphragm area.

The gain of the thermostatic controller ( $k_V^{-\pi} da/k_S^{-}$ ) is a design variable for the mass flow rate of the refrigeration system. This means that the gain can not be changed of the valve is selected.

Considering the pressure lag in the bulb measurement, the pressure at the end of the bulb can be determined according to the following differential equation:

$$RC \frac{dP_b}{dt} + P_b = P_{ev}$$
 (8)

where C is the line and bulb capacitance =  $Vg \frac{d}{dp}$ V is the volume of the line and bulb

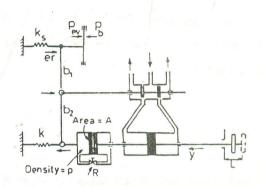
R is the line and bulb resistance.

It is obvious that the thermostatic expansion valve is a proportional controller which usually produces a steady state error. Increasing the gain of the valve, reduces the steady state error but it leads to instability and oscillatory dynamic response.

#### b) Proposed controller

To improve the plant response, the valve needle is driven by a PI-Controller actuated by the pressure difference signal. The proportional-plus-integral action is obtained by implementing a

servomotor and a feed back line consisting of a dashpot and a proportional mechanism. The servomotor type can be hydraulic, pneumatic or operated on a hot refrigerant under pressure from a pilot line. Figure 2 shows a schematic diagram of the proposed controller and its block diagram. Under normal operation  $|Kb_1t_i/[(b_1+b_2)(t_1+1)]|\gg 1$ , therefore the controller transfer function is given by:



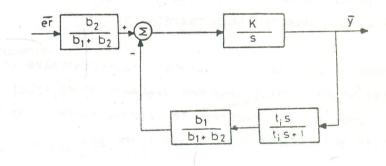


Figure 2: Control system schematic diagram and its olockdiagram.

$$\frac{\overline{y}(s)}{-----} = K_{i} (1 + ---)$$

$$er(s) t_{i} s$$
(9)

where  $K_i = b_2/b_1$ ,  $t_i = RA^2 \rho/k$ , R is the valve resistance, A is the dashpot piston area, and  $\rho$  is the dash pot fluid density.

Define  $\Delta P$  as  $(P_b - P_{ev})$ , the valve displacement  $\Delta y$  during an increment  $\Delta t$  is

$$\frac{y}{L} = K_{p} \left( \Delta P_{t} - \Delta P_{(t+\Delta t)} \right) + \frac{\Delta t}{t_{i}} \left[ P - \frac{1}{2} \left( \Delta P_{(t+\Delta t)} \right) \right]$$
 (10)

where  $K_p = aK_i/k_s L$ 

#### Solution Technique

The differential equations (1,2,3 and 4) of the continuity, momentum, energy and heat balance of the tube wall together with the compressor capacity equation (5) are solved either with equation 6 and 7 for the thermostatic expansion valve or with equations 6 and 10 for the proposed PI-controller. The evaporator tube including the suction line is divided into 25 nodal points (j = 1 to 25) where the first point (j=1) represents the refrigerant condition at the outlet of the expansion valve while the last point (j = 25) represents the evaporator outlet, the location of the expansion valve remote bulb and the compressor suction. The finite difference equations with respect to a time step  $\Delta$ t are formulated in appendix (B).

The boundary values T,i and v or  $\rho$  at the evaporator enterance are determined from the throttling process from a predetermined condensing pressure to an evaporator pressure  $P_{j=1} = P_{ev}$ . The enternace

velocity 
$$u_{j=1}$$
 is directly obtained from  $u_{j=1} = \frac{M_{exp}}{(\rho_1 * A_F)}$ 

The initial values of the refrigerant parameters for all nodal point are considered the steady state values which satisfy  $M_{C} = M_{exp}$ .

#### Heat Transfer Coefficients

The heat transfer coefficient of the refrigerant-wall side  $h_{RW}$  is obtained from Gruhle et al (1985) as a function of the refrigerant dryness fraction ranged from 3000 W/m $^2$ K at saturated liquid to 300 W/m $^2$ K at saturated and superheated vapor while it has a maximum value of 9000 W/m $^2$ K at dryness fraction of 0.7.

The heat transfer coefficient of the air-wall side  $h_{aw}$  is estimated according to an air face velocity of 3 m/s across a horizontal circular tube equal to 16.75 w/m $^2$ K in the proposed operating range of the DX-coil.

According to the previous formulations,  $\rho$  ( $\rho u$ ), and  $\rho$  ( $e+u^2/2$ ) or  $\rho$ ,u, and e are determined for each point from j =2 to as well as the pressure distribution. The obtained values should be iteratively corrected to satisfy  $M_c = (\rho u)_{j=n}^{n} A_R$  for one time step  $\Delta t$ .

#### Results and Discussions

Figures 3 and 4 present the transient response of the refrigerant mass flow rate through the conventional thermostatic expansion valve and the mass flow rate charged to the compressor due to a step change of -2K in the surrounding air temperature, when the desired degree of

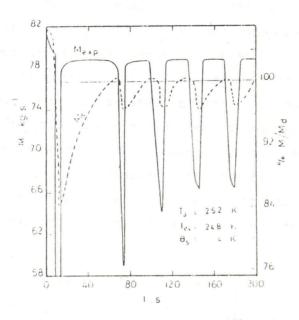


Figure 3: Response at the retrigerant mass flow rate with conventional thermostatic expansion valve, ( $I_3 = 252 \text{ K}$ ,  $I_{ev} = 243 \text{ K}$ ,  $\Delta I_a = 26 \text{ K}$ ,  $\theta_s = 46 \text{ K}$ )

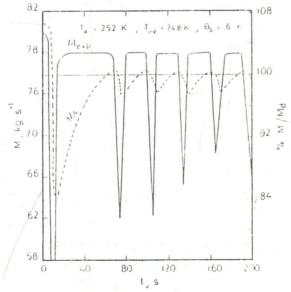


figure 4: Response of the refrigerant mass flow rate with conventional thermostatic expansion valve, ( $I_a = 252 \text{ K}$ ,  $I_{ev} = 248 \text{ K}$ ,  $\Delta I_a = 26 \text{ K}$ ).

superheat is 4K and 6K respectively,  $T_a = 252 \text{ K}$  and  $T_{ev} = 248 \text{ K}$   $(-25^{\circ}\text{C})$ . Similar results are obtained for  $T_a = 270 \text{ K}$  and  $T_{ev} = 266 \text{ K}$   $(-7^{\circ}\text{C})$  as shown in Figures 5 and 6. In both cases, an oscillatory response for  $M_{exp}$  and  $M_{c}$  is observed. The peak amplitude of the mass delivered to the compressor is lagging and smaller than that of the mass fed into the evaporator. The amplitude of oscillation is getting smaller as the degree of superheat increases. It can be noticed that the amplitude of oscillation in  $M_{c}$  is reduced as the evaporator temperature increases because of the increase in the evaporator inertance. As it is stated earlier in the paper, this hunting behavior is attributed to the high gain of the thermostatic expansion valve which is considered as a proportional controller.

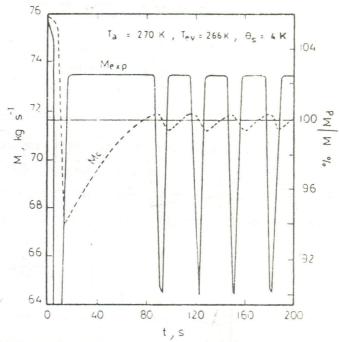


Figure 5: Response of the refrigerant mass flow rate with conventional thermostatic expansion valve, (I = 27 pK, I = 266K, ΔI = -2k, Θ = 4K).

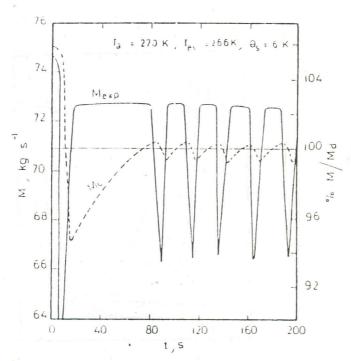


Figure 5: Response of the retrigerant mass flow rate with conventional thermostatic expansion valve, ( $I_a=270K$ ,  $I_{ev}=266K$ ,  $\Delta I_a=-2K$ ,  $\Theta_S=6K$ )

The effect of the integral-proportional controller parameters K and to the settling time is demonstrated in Figure 7. For 1.0 < to <!-- The time is demonstrated in Figure 7. For 1.0 < to <!-- The time depends on the proportional gain K and K and

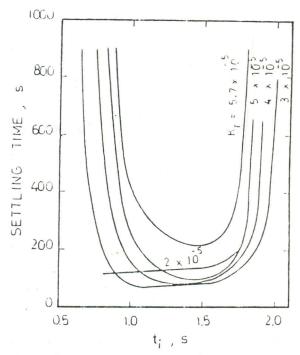


figure 7: Effect of K and t on the settling time.

Upon implementing the controlled thermostatic expansion valve, the oscillation in the refrigerant mass rate in the evaporator is damped, as shown in Figure 8 through 11. One may observe from the figures that the increase in the adjusted degree of superheat increases the settling time. For example, for  $T_a = 252 \text{ K}$  and  $T_{ev} = 248 \text{ K}$ , the settling time increases from 55 S to 75 S as the degree of superheat is raised from 4 K to 6 K. This is because the increase in the refrigerant density, resulted from the decrease in the degree of superheat, reduces the evaporator capacity as well as its time

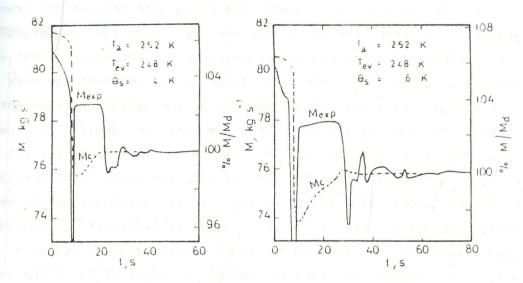


figure 8: Response of the retrigerant mass flow rate with controlled thermostatic expansion valve, ( $I_a = 252 \text{ K}$ ,  $I_{ev} = 248 \text{ N}$ ,  $\Delta I_s = -28 \text{ N}$ ,  $0_s = 4 \text{ K}$ )

figure 9: Response of the refrigerant mass flow rate with controlled thermostatic expansion valve,  $(\frac{1}{4}-252)$  K,  $\frac{1}{4}=248$  K,  $\frac{1}{4}=-26$  K,  $\frac{1}{$ 

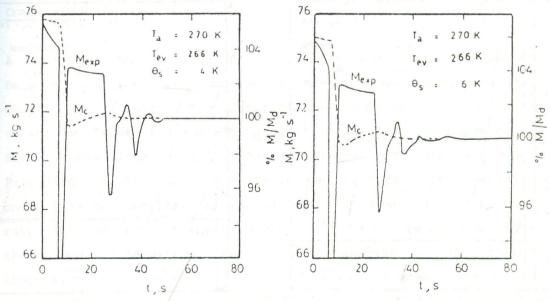


figure 10: Response of the retrigerant mass flow rute with controlled thermostatic expansion valve,  $(I_3=270 \text{ K}, I_{ev}=266 \text{ K}, \Delta I_{a}=-2 \text{K}, \widehat{o}_s=4 \text{K})$ .

Figure 11: Response of the refrigerant mass flow rate with controlled thermostatic expansion valve, (I = 270 K, I = 266K,  $\Delta T_a = -2$  K,  $\theta_c = 4$ K).

Figures 12 though 15 present the comparison between the transient degree of superheat response resulted upon using the conventional and the controlled thermostatic expansion valve. The conventional system renders an oscillatory response. It can be noticed that the peak value of the degree of superheat corresponds to the minimum refrigerant mass charged to the compressor, and the minimum degree of superheat coincides with the maximum  $M_{\rm C}$ . On the other hand, for all the considered operational conditions and when the controlled thermostatic expansion valve is used, no oscillation is traced during the steady-state operation. Meanwhile the amount of overshoot is reduced more considerably than the conventional thermostatic expansion valve operation, during the transient period. An error of 2% in the desired degree of superheat is observed because the setting point is based on the change in  $(P_{\rm b}-P_{\rm ev})$  rather than the degree of superheat.

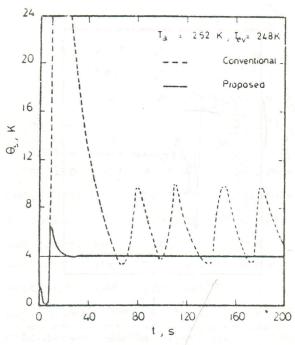


figure 12: Iransient response of the degree of superheat, ( $I_a$  = 252 K,  $I_{ev}$  = 248 K,  $\Delta I_a$  = -2K,  $U_{sd}$  = 4 K).

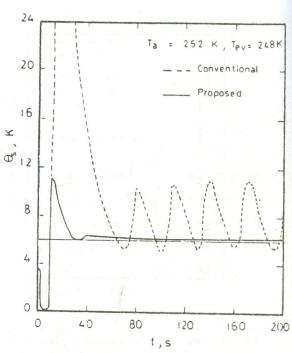
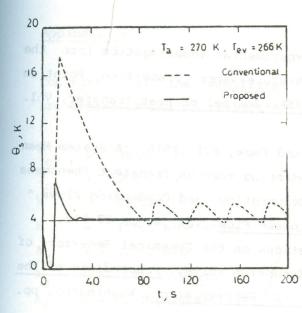


Figure 13: Iransient response of the degree of superheat, (I=252~K,  $I_{ev}=243~K$ ,  $\Delta I_{e}=24.5~K$ ,  $\Delta I_{e}=24.5~K$ ).



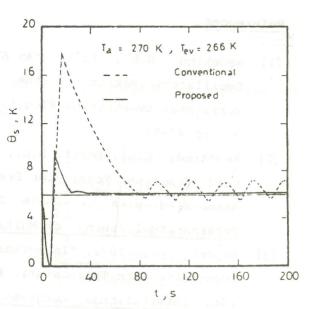


Figure 14: Transient response of the degree of superheat,  $(T_a = 270 \text{K})$   $T_{ev} = 266 \text{ K}$ ,  $\Delta T_a = -2 \text{ K}$ ,  $C_{sd} = 4 \text{ K}$ 

Figure 15: Transient response of the degree of superheat, ( $I_a$  = 270K,  $I_e$  = 266 K,  $\Delta I_e$  = -2K,  $\Theta_{so}$  = 6 K).

## Conclusions

A model of the refrigerant evaporator was developed based on the balance equations for continuity, momentum and energy. The model is coupled with equations that described the compressor and the expansion valve. Two types of evaporator capacity control were considered:

the conventional thermostatic expansion, and a mechanical PI-controller for the thermostatic expansion valve. The conventional control causes oscillations in the degree of superheat and in the mass rate charged to both the evaporator and the compressor. According to the model developed, the dynamic behavior of the controlled thermostatic expansion valve is stable and the steady-state error in the degree of superheat is 2%. It is believed that the proposed mechanically controlled expansion valve is feasible in the field of refrigeration and air conditioning because the servomotor type can be hydraulic, pneumatic, or operated on hot refrigerant under pressure from a pilot line.

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# Appendix (A)

Constants of the equations used for computing the refrigerant properties [Reynolds (1979)]:

Tcr.t	=	385.17 K	F <sub>1</sub>	= - 93.3438056
		200 K	F <sub>2</sub>	$= 4.39618785 \times 10^3$
	_=	68.743	F <sub>3</sub>	= - 12.4715223
b	=	$4.06356926 \times 10^{-4}$	F <sub>4</sub>	$= 1.96060432x10^{-2}$
A <sub>2</sub>	= -	91.6210126	D <sub>1</sub>	$= 5.5808454 \times 10^{2}$
A <sub>3</sub>	=	1.01049598×10 <sup>-1</sup>	D <sub>2</sub>	$= 8.5444580 \times 10^{2}$
A <sub>4</sub>	= -	5.74640225×10 <sup>-5</sup>	D <sub>3</sub>	= 0.0
A <sub>5</sub>	=	_	D <sub>4</sub>	$= 2.994077103 \times 10^{2}$
B <sub>2</sub>		$7.71136428 \times 10^{-2}$	D <sub>5</sub>	= 0.0
B <sub>3</sub>	= -	$5.67539138 \times 10^{-5}$	D <sub>6</sub>	$= 3.521500633 \times 10^{2}$
B <sub>4</sub>		0.0	D <sub>7</sub>	=-50.47419739
B <sub>5</sub>		$4.08193371 \times 10^{-11}$	G <sub>1</sub>	= 33.89005260
C <sub>2</sub>	= -	1.52524293x10 <sup>3</sup>	4	= 2.507020671
c <sub>3</sub>	=	2.19982681	G <sub>3</sub>	$=-3.274505926 \times 10^{-3}$
C <sub>4</sub>	=	0.0	$G_4$	$= 1.641736815 \times 10^{-6}$
C <sub>5</sub>	= -	1.66307226×10 <sup>-7</sup>		$= 1.6970187 \times 10^{5}$
K	=	5.475		

<sup>\*</sup> The remaining constants are zeros

# Appendix (B)

Finite difference formulations of the balancing equations:

# Continuity

$$\rho_{j}^{*} = \rho_{j} - \Delta t. [(\rho u)_{j} - (\rho u)_{j-1}] / \Delta x$$

#### Momentum

$$(\rho u)_{j}^{*} = (\rho u)_{j} - \Delta t. [(\rho u^{2} + P)_{j} - (\rho u^{2} + P)_{j-1}] / \Delta x$$

# Energy

$$[\rho(e+u^{2}/2)]^{*}_{j} = [\rho(e+u^{2}/2)]_{j} -\Delta t. [\rho u(i+u^{2}/2)]_{j}$$
$$- [\rho u(i+u^{2}/2)]_{j-1} \Delta / x - h_{Rw}^{A}_{Rw} (T_{wj}^{-1} - T_{Rj}^{-1}) / A_{R}]$$

# Wall Heat Capacity

$$T^*_{wj} = T_{wj} - \Delta t. [h_{aw}A_{aw}(T_a - T_{wj}) - h_{Rw}A_{Rw}(T_{wj} - T_{Rj})]$$

$$/(\rho_w C_w A_w \ell_w)$$

$$\rho_{j}^{*}$$
,  $(\rho u)_{j}^{*}$ ,  $[\rho(e+u^{2}/2)]_{j}^{*}$ , and  $T_{wj}^{*}$  are the

system properties after time step  $\Delta t$ .