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AN ALTERNATIVE METHOD FOR CALCULATING THE RESPONSE OF CARDIAC TISSUE TO STIMULTING CURRENTS

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ABSTRACT

Response of a one dimensional myocardial fiber to a constant stimulating current was analyised bv R. Plonsey and R.C. Barr [1], [2]. An alternative simple method is presented in this communication to solve the problem. The fiber is regarded as cascaded sections. Results obtained are almost identical to those obtained from the exact method by R. Plonsey and R.C. Barr.

INTRODUCTION

The effect of a deffibrillating stimulus current when applied to a one dimensional fiber with a junction resistance joining the individual cells was analysed by R.Plonsey & R.C. Barr [1], [2]. They have solved the problem by three methods, the first two are approximate and, the third one is exact. In the exact method they considered the solution to be a combination of two more easily solved problems.

a) The primary source solution was obtained by setting the coupling resistance to zero. Thus the cells would constitute a single continuous fiber.

b) The secondary source solution took into account the effect of junction resistance. This was obtained by considering the effect of each junction resistance as a dipole source placed at the junction. This method involves solution of 29 simultaneous equations and does not give the exact solution for the transmembrane potentials in a direct way. Therefore, in the present work a simple approach is proposed to give a direct solution for currents and voltages at the junction and at any point along each cell.

THEORETICAL ANALYSIS

The proposed method is based on representing the fiber as a cascaded sections (Fig. 1). Each section represents one cell as a uniform T.L. with its junction resistance connected to its receieving end.

Condidering the T.L. equations for one of the cells (Fig. 2) we get:

$$\frac{dVm}{dX} = - i(x)\,(r_e + r_i) + Io.\, r_e \qquad (1)$$

$$Vm(x) = - r_m \frac{di(x)}{dX} \qquad (2)$$

where :

$Vm(x)$ = transmembrane voltage.

$i(x)$ = induced loop current.

Io = injécted current

r_m = transmembrane resistance per unit meter

r_i = intercellular resistance per unit meter

r_e = extracellular resistance per unit meter

Using equations (1) & (2)

$$\frac{d^2Vm}{dX^2} = \frac{r_e + r_i}{r_m} . Vm \qquad (3)$$

The general solution for the above equation for cell number "k" is given by:

$$Vm = A_k . \exp(\gamma x) + B_k . \exp(-\gamma x) \qquad (4\text{-a})$$

$$i(x) = \frac{Io.r_e}{r_e+r_i} - \frac{\gamma}{r_e+r_i} \left(A_k \exp(\gamma x) - B_k \exp(-\gamma x)\right) \qquad (4\text{-b})$$

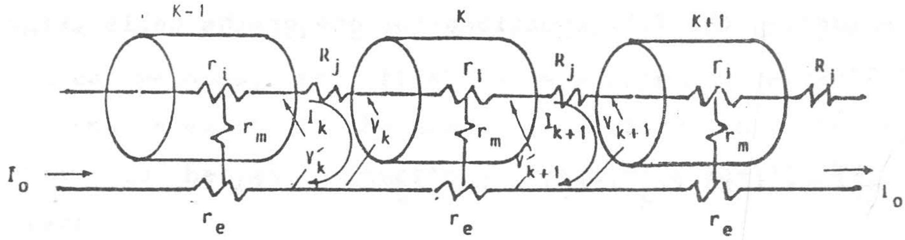

Fig. 1: Currents and voltages at the input and output of cell no. (k).

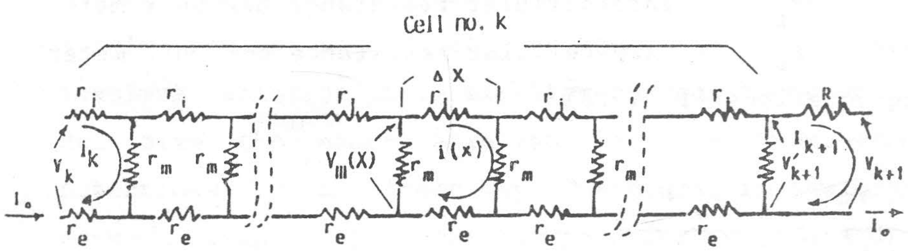

Fig. 2: Current and voltage distribution inside cell no. (K).

where , $\gamma = \sqrt{\dfrac{r_e + r_i}{r_m}}$

Applying the boundary conditions (Fig. 2) at x = 0

$Vm(x) = V_k$ and $i(x) = i_k$

and at $x = \ell$: (where ℓ is the cell length.)

$Vm(x) = V'_{k+1}$ and $i(x) = i_{k+1}$

we get :

$$A_k = 0.5\ (V_k + Zo.\ (I\acute{o} - i_k)) \qquad (5.a)$$

$$B_k = 0.5\ \ (V_k - Zo.\ (I\acute{O} - i_k)) \qquad (5.b)$$

where,

$$I\acute{o} = Io.\ \frac{r_e}{r_e + r_i} \qquad \& \ Zo = \sqrt{r_m.(r_e + r_i)}$$

From equations (4) & (5) we get :

$$\begin{bmatrix} V'_{k+1} \\ i_{k+1} \\ I'_o \end{bmatrix} = \begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix} \begin{bmatrix} V_k \\ i_k \\ I'_o \end{bmatrix} \qquad (6)$$

where:

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} \cosh\gamma\ell & -Z_o \sinh\gamma\ell & Z_o \sinh\gamma\ell \\ \frac{-\ \text{shin}\gamma\ell}{Z_o} & \cosh\gamma\ell & 1-\cosh\gamma\ell \\ 0 & 0 & 1 \end{bmatrix}$$

and considering the effect of the junction resistance Rj (Fig.1) we get:

$$V_{k+1} = V'_{k+1} - Rj.\ i_{k+1} \qquad (7)$$

Using equations (6) & (7)

$$\begin{bmatrix} V_{k+1} \\ i_{k+1} \\ I'_o \end{bmatrix} = \begin{bmatrix} G \end{bmatrix} \begin{bmatrix} V_k \\ i_k \\ I'_o \end{bmatrix} \qquad (8)$$

where:

$$\begin{bmatrix} G \end{bmatrix} = \begin{bmatrix} \cosh\gamma\ell + \frac{R_j}{Z_o}\sinh\gamma\ell & Z_o\sinh\gamma\ell + R_j\cosh\gamma\ell & Z_o\sinh\gamma\ell + R_j(1-\cosh\gamma\ell) \\ \frac{-\ \sinh\gamma\ell}{Z_o} & \cosh\gamma\ell & 1-\cosh\gamma\ell \\ 0 & 0 & 1 \end{bmatrix}$$

Using equations (6) & (8) and evaluating the cascade of 30 cells we get :

$$\begin{bmatrix} V_{31} \\ I_{31} \\ I\acute{o} \end{bmatrix} = \begin{bmatrix} G \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \\ I\acute{o} \end{bmatrix} \qquad (9)$$

Using the boundary conditions, $I_1 = 0$ & $I_{31} = 0$, we get V_1 & V_{31} in terms of Io.

Starting with V_k, I_k and Io and using equation (8) we get V'_k, V_k & I_k for all cells. Using equation (5) and the values of V'_k , V_k & I_k , we get A_k & B_k for each cell and using equation (4) we get V(x), i(x) at any point inside each cell.

RESULTS AND DISCUSSION

Results obtained in this work are tabulated in tables (1) and (2). Table (1) shows almost identical values of cellular currents compared with those obtained by the exact method of Plonsey and Barr [1,2]. Table (2) shows the exact values of the transmembrane potantials V_m as obtained by the present work compared with those obtained by the approximate method of Poloney and Barr [1], [2]. The values of V_m obtained by their exact method are not published.

Membrane potential $V_m(x)$ along the cell is also obtained in this work (Equation 4).

The proposed method includes less calculation steps. Running the program in an IBM PC with double precision took less than 15 sec. This indicates less probable calculation error.

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Node Number	a) R_j=500 kΩ I_k	I_k^*	b) R_j=849 kΩ I_k	I_k^*	R_j= 50 MΩ I_k	I_k^*
1	582.83	583	554.57	555	115.16	115
2	1056.13	1056	1000.69	1001	168.26	168
3	1440.23	1440	1359.37	1359	192.75	193
4	1751.63	1752	1647.49	1647	204.04	204
5	2003.70	2004	1878.64	1979	209.24	209
6	2207.28	2207	2063.69	2064	211.64	212
7	2371.11	2371	2211.37	2211	212.75	213
8	2502.24	2502	2328.62	2329	213.26	213
9	2606.29	2606	2420.97	2421	213.50	213
10	2687.74	2688	2492.77	2493	213.61	214
11	2750.07	2750	2547.39	2547	213.66	214
12	2795.98	2796	2587.42	2587	213.68	214
13	2827.43	2827	2614.74	2615	213.69	214
14	2845.78	2846	2630.63	2631	213.69	214
15	2851.81	2852	2635.85	2636	213.70	214

Table 1: Node currents I_k calculated by the method described in this paper and the values I_k obtained by POLONSEY and BARR [2].

Node number	a) R_j=500 k V_k	V_k^*(appx.)	b) R_j=849 k V_k	V_k^*(appx.)	R_j= 50 M V_k	V_k^*(appx.)
1	-223.47	--	-212.22	-213	-49.88	-48.9
2	-182.15	--	-171.86	-172	-37.4	-36.8
3	-148.48	--	-139.30	-140	-31.64	-31.4
4	-121.03	--	-113.01	-113	-28.98	-28.9
5	-98.60	--	-91.76	-92.2	-27.76	-27.7
6	-80.25	--	-74.55	-74.9	-27.2	-27.2
7	-65.17	--	-60.57	-60.9	-26.94	-27.0
8	-52.73	--	-49.15	-49.4	-26.82	-26.9
9	-42.38	--	-39.76	-40.0	-26.76	-26.9
10	-33.68	--	-31.95	-32.1	-26.73	-26.8
11	-26.27	--	-25.37	-25.5	-26.72	-26.8
12	-19.81	--	-19.7	-19.8	-26.72	-26.8
13	-14.04	--	-14.66	-14.8	-26.71	-26.8
14	-8.7	--	-10.04	-10.1	-26.71	-26.8
15	-3.56	--	-5.59	-5.6	-26.71	-26.8

Table 2: The transmenibrane potential at the (K th) node V_k obtained by the method described in this paper and the values V_k^* (appx.) obtained by the approximate method described by POLONSEY and BARR [2]. V_k^* (appx) in case of R_j = 500 k is not published

REFERENCES

[1] R.Plonsey & R.C. Barr "Effect of Microscopic and macroscopic discontinuities on the response of cardiac tissue to defibrilating (stimulating) currents", Med. & Biol. Eng. & Comput., 1986, 24, 130-136.

[2] R. Plonsey & R.C. Barr "Inclusion of junction elements in a linear cardiac model through secondary sources: - application to defibrallation", Med. & Biol. Eng. & Compu., 1986, 24, 137-144.

List of Symbols

$V_m(x)$	=	the transmembrane voltage
$i(x)$	=	the induced loop current
I_o	=	the injected current
r_m	=	the transmembrane resistance times length (Ω . m)
r_i	=	the intracellular resistance per unit meter(Ω /m)
r_e	=	the extracellular resistance per unit meter(Ω/m)
R_j	=	the junction resistance between two cells
ℓ	=	the cell length
V_k	=	the value of $V_m(x)$ at x = 0 for cell no. k
V'_{k+1}	=	the value of $V_m(x)$ at x = ℓ for cell no. k