Alex. Eng. J., Alex. Univ., Volum, 28 No. 2, PP. 79-91 (1989)

STUDY ON PRESSURE WAVES IN LIQUID PIPELINES EXPOSED TO HIGH PRESSURE

M.Khalil Ibrahim, Ph.D. and Adel A.A. Ramadan, M.Sc.
Mechanical Engineering Dept., Faculty of Engineering,
Alexandria University, Egypt.

Abstract

The problem of pressure waves in liquid filled pipes is still receiving attention especially in the design of pipelines and the fuel injection system of desel engines. The present study deals with an analytical model including the governing equations, the method of characteristics, and the finite difference solution to simulate a piping system under unsteady flow conditions. The model results, by computer programming, are presented for several factors affecting pressure wave propagation in a pipeline. The study shows that the pressure wave amplitude is affected by increasing the reservoir head or decreasing the pipeline diameter. The time of wave travel is directly proportional to the length of pipeline, but the change in friction factor has no appreciable effect on the wave peak pressures. The acoustic wave speed depends mainly upon the diameter to thickness ratio and the physical properties of both pipe wall material and the flowing liquid.

Nomenclature

a	= acoustic wavespeed	m/s
A	= cross-sectional area	m^2
B	= constant, Eq. (6)	
C	= coefficient of discharge	
D	= pipe diameter	m
e	= pipe wall thickness	m
E	= Young's modulus of elasticity of pipe material	N/m^2
f	= friction factor	
g	= gravitational acceleration	m/s^2
H	= piezometric head	m
K	= bulk modulus of elasticity of liquid	N/m^2
L	= pipe length	m
N	= number of computational reaches	
Q	= discharge	m^3/s
R	= Parameter, Eq. (7)	
t	= time	s
U_b, U_m	= parameters, Eqs. (8) and (9)	
x	= distance along the pipe	m
ρ	= density	kg/m^3
ν	= Poisson's ratio of pipe material	

Subscripts

d	discharge
i	node in x-direction
o	steady state condition
R	reservoir end
v	valve end

INTRODUCTION

The study of unsteady flow in a pipeline is an important design parameter to avoid maximum pressures and propable separation in fluid flow. The unsteady flow occurs when the operating conditions changes such as valve closure, starting or stopping of a pump, or changing the head in a reservoir . Most of the injection system problems in diesel engines are related to wave phenomena in the high pressure line between pump and injector.

Streeter and Wylie [1] studied the fundamental equations governing unsteady flow in a pipeline and showed how to use the method of characteristics to solve the partial differential equations. The sonic velocity of a pressure wave is reduced in the case of flow inside an elastic pipe than the flow inside a rigid pipe. Parmakian [2]. Stuckenbruck, Wiggert and Otwell [4] investigated the factors affecting acoustic wave propagation such as the physical properties of pipe material and the methods of supporting the pipeline. Streeter [3], Chaudhry and Hussaini [5] studied the numerical methods used to treat the unsteady flow governing equations and the use of digital computers instead of the conventional graphical methods.

The present study deals with constructing an analytical model representing pressure wave propagation in liquid pipelines. The method of characteristics is used to transform the partial differential equations into particular total differential equations. Then the finite difference technique is applied to solve the characteristics equations numerically by the digital computer. The study includes the main factors affecting pressure wave propagation such as reservoir head, pipeline length, diameter, pipe wall thickness, the change in friction factor, and the physical properties of both the flowing liquid and the pipe material.

MODEL ANALYSIS

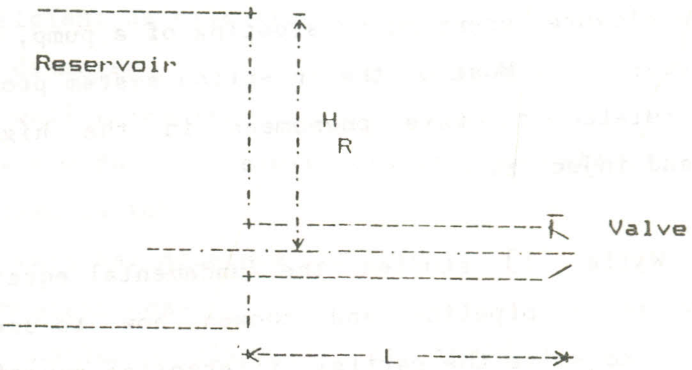

Fig.1 Schematic Diagram of the Piping System .

The change from steady state flow in the piping system shown in Fig. 1 occurs from the change in the valve boundary condition. The governing equations describing waterhammer in closed conduits, which are the momentum equation of motion and the continuity equation, are given as follows, [1, 3]:

$$\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + \left(\frac{f}{2DA}\right) Q\,|Q| = 0 \qquad (1)$$

$$\frac{\partial H}{\partial t} + \left(\frac{a^2}{gA}\right)\frac{\partial Q}{\partial x} = 0 \qquad (2)$$

It is known that the magnitude of the acoustic wavespeed in a pipe flow is influenced by properties of both the fluid and the pipe material. Then, the sonic wavespeed can be written as follows, [3,4]:

$$a = \{(K/\rho)/[1+(D/e)(K/E)\ \phi]\}^{\frac{1}{2}} \qquad (3)$$

where ϕ is a factor depends on the way the pipe is supported and the Poisson's ratio for pipe wall material [2], [3], and [4].

Characteristics Equations

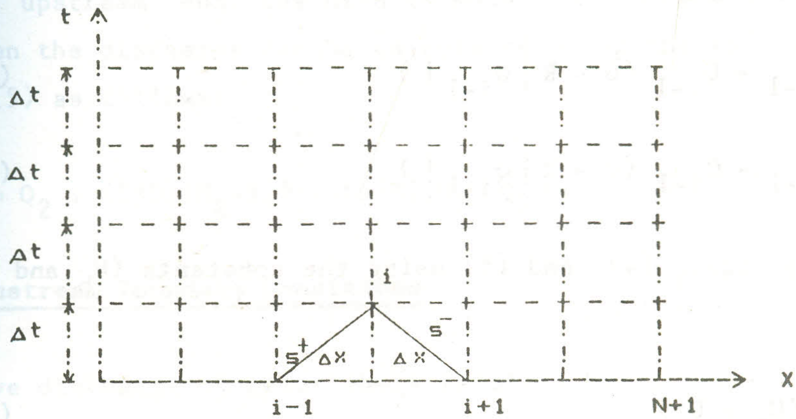

Fig. 2 Rectangular Grid for Solution of Characteristics Equations .

The method of characteristics is used to transform the partial differential equations into particular ordinary differential equations which can be solved by using first-order finite difference technique, [5,6].

for the (S^+) line, where ($adt = +\ dx$):

$$H_i = H_{i-1} - B(Q_i - Q_{i-1}) - RQ_{i-1}\,|Q_{i-1}| \qquad (4)$$

for the (S^-) line, where ($adt = -\ dx$):

$$H_i = H_{i+1} + B(Q_i - Q_{i+1}) + RQ_{i+1}|Q_{i+1}| \quad (5)$$

where,

$$B = (a/gA), \text{ and} \quad (6)$$

$$R = (f.\Delta x)/(2gDA^2) \quad (7)$$

By collecting the known terms of Eqs. (4) and (5) into two constants (U_b and U_m) as follows:

$$U_b = H_{i-1} + Q_{i-1}(B - R|Q_{i-1}|) \quad (8)$$

$$U_m = H_{i+1} - Q_{i+1}(B - R|Q_{i+1}|) \quad (9)$$

Now solving Eqs. (4) and (5) using the constants (U_b and U_m) we get:

$$H_i = \tfrac{1}{2}(U_b + U_m) \quad (10)$$

$$Q_i = (1/B)(U_b - H_i) \quad (11)$$

All enterior sections of the rectangular grid of solution may be calculated in this manner.

Boundary Conditions

(1) Steady State Conditions

The steady state head and discharge at valve end are denoted as (H_o and Q_o) respectively, then we have

$$Q_o = \sqrt{H_R / \left(\frac{fL}{2gDA^2} + \frac{1}{2g(C_d A_v)^2}\right)} \quad (12)$$

$$H_o = (Q_o / C_d A_v)^2 / 2g \quad (13)$$

(2) Upstream Boundary Conditions

At the upstream end, the head is equal to the reservoir head ($H_1 = H_R$), then the discharge can be calculated using the (S^-) line given by Eq. (5) as follows:

$$Q_1 = Q_2 + (1/B)(H_R - H_2) - (R/B).Q_2 \,|\, Q_2 \,| \quad (14)$$

(3) Downstream Boundary Conditions

The valve discharge at point (N+1) is given by:

$$Q_{N+1} = C_d \; A_v \sqrt{2g \; H_{N+1}} \quad (15)$$

refering to the (S^+) line appling Eqs. (4) and (8) we get:

$$H_{N+1} = U_b - BQ_{N+1} \quad (16)$$

solving Eqs. (15) and (16) we get

$$Q_{N+1} = -gB(C_d A_v)^2 + \sqrt{[gB(C_d A_v)^2]^2 + (C_d A_v)^2 \;\; 2gu_b} \quad (17)$$

where U_b as defined by Eq. (8) is given at point (N) as follows:

$$U_b = H_N + Q_N(B - R \,|Q_N|) \quad (18)$$

RESULTS AND DISCUSSION

Fig. 3 shows a computer plotting of pressure wave head at different

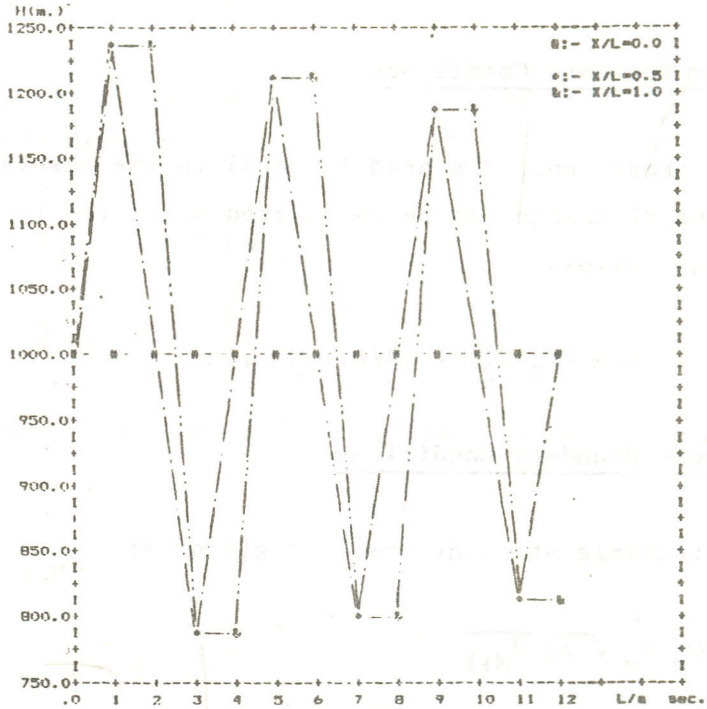

Fig. 3 Complete Waves at Points X/L = (0.0, 0.5, 1.0)
For: L=2000.0 m., D=2.032 m., HR=1000.0 m.

positions of the pipeline. It is clear that the head at the tank end (x/L = 0.0) is constant, but the duration of peak pressures is remarkable at the valve end (x/L = 1.0). It is also noticed that the peak pressure decreases with time (damping effect). Fig. 4 shows the variation of discharge with time at different pipeline sections. It is clear that the oscillations of discharge are large near the tank (x/L = 0.0) and are small at the valve end (x/L = 1.0). Detailed tables of results and computer programming are given by Adel A.A. Ramadan [6].

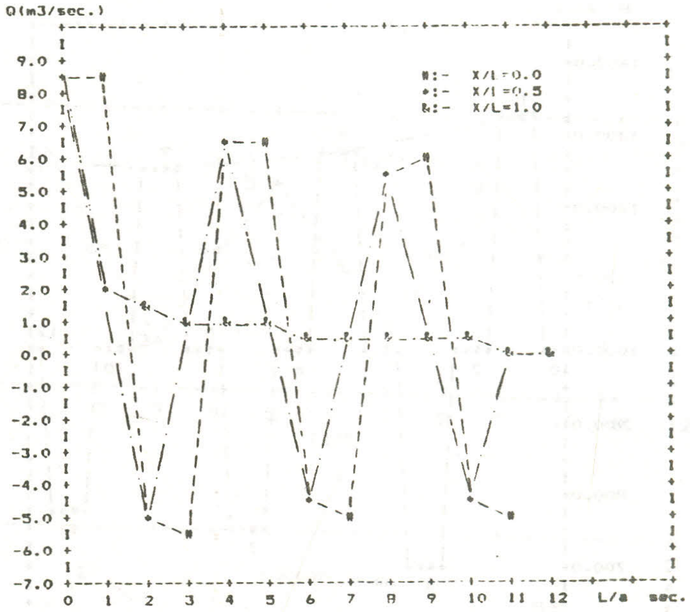

Fig. 4. Dicharge Against Time at Points X/L=(.0.,0.5,1.0)

For:- L=2000.0 m., D=2.032 m., HR=1000.0 m.

In order to show the effect of pipe diameter on the peak pressure, a complete pressure wave is ploted in Fig. 5. A curve connecting peak pressures at points (1,5,9) is drawn to show the wave damping with time. Several cases of changing pipe diameter with the same length and reservoir head are shown in Fig. 6. It is clear that the reduction in pipe diameter from 1.727 m to 1.168 m results in rapid increase in peak pressures.

A case study was examined to show the effect of changing the friction factor on the wave pressure. Several cases of pipes of the same dimensions but different surface roughness resulting in different values of friction factor ranging from 0.016 to 0.029 were examined. It was found that the change in friction factor has no appreciable effect on the wave peak pressures. The effect of pipe length is shown in Fig.7 It is shown that the complete wave period is directly proportional to the pipe length. Thus, decreasing the length of the pipe leads to a decrease in the time of wave travel.

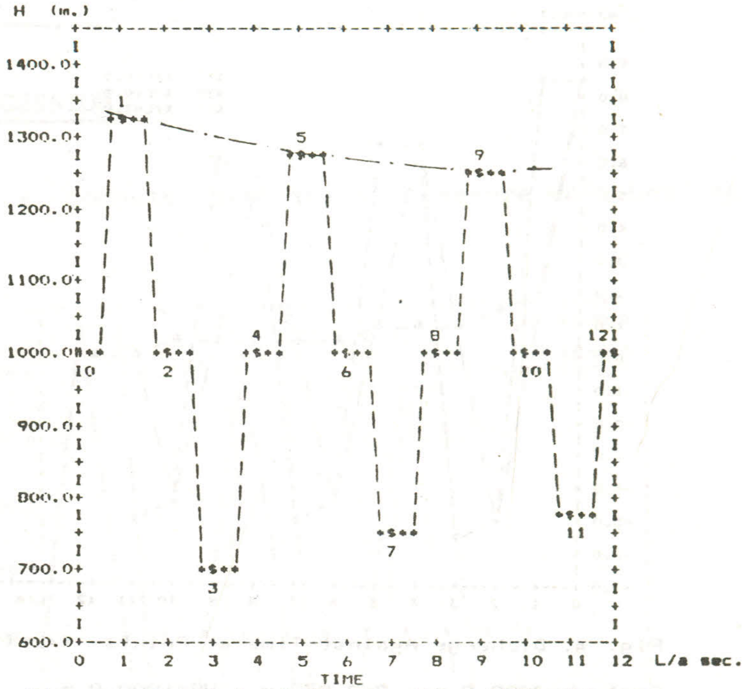

Fig. 5 Complete Pressure Wave With Time
At Point X/L= 0.5
For:- L=1000.0 m.,HR=1000.0 m.,D=1.727 M.

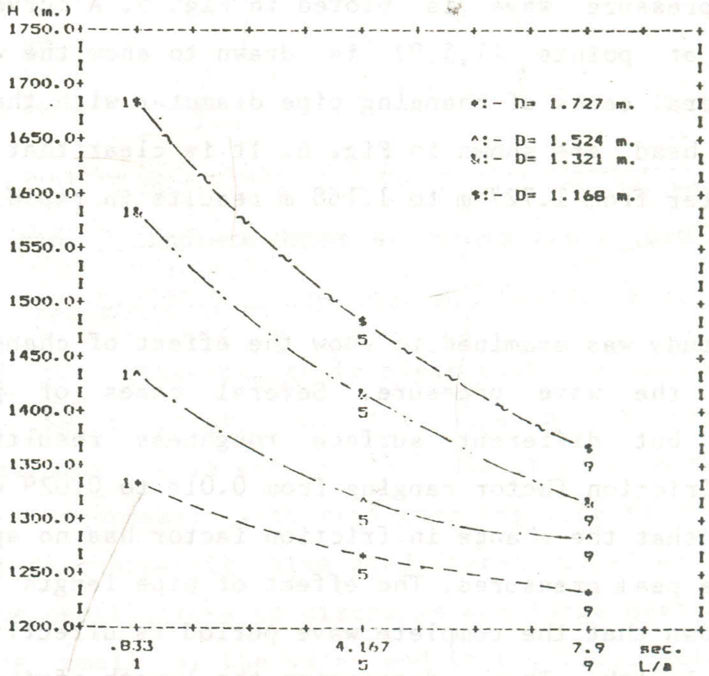

Fig. 6 Change of Diameter for Canstant L,HR
For:- L=1000.0 m. ,HR=1000.0 m.

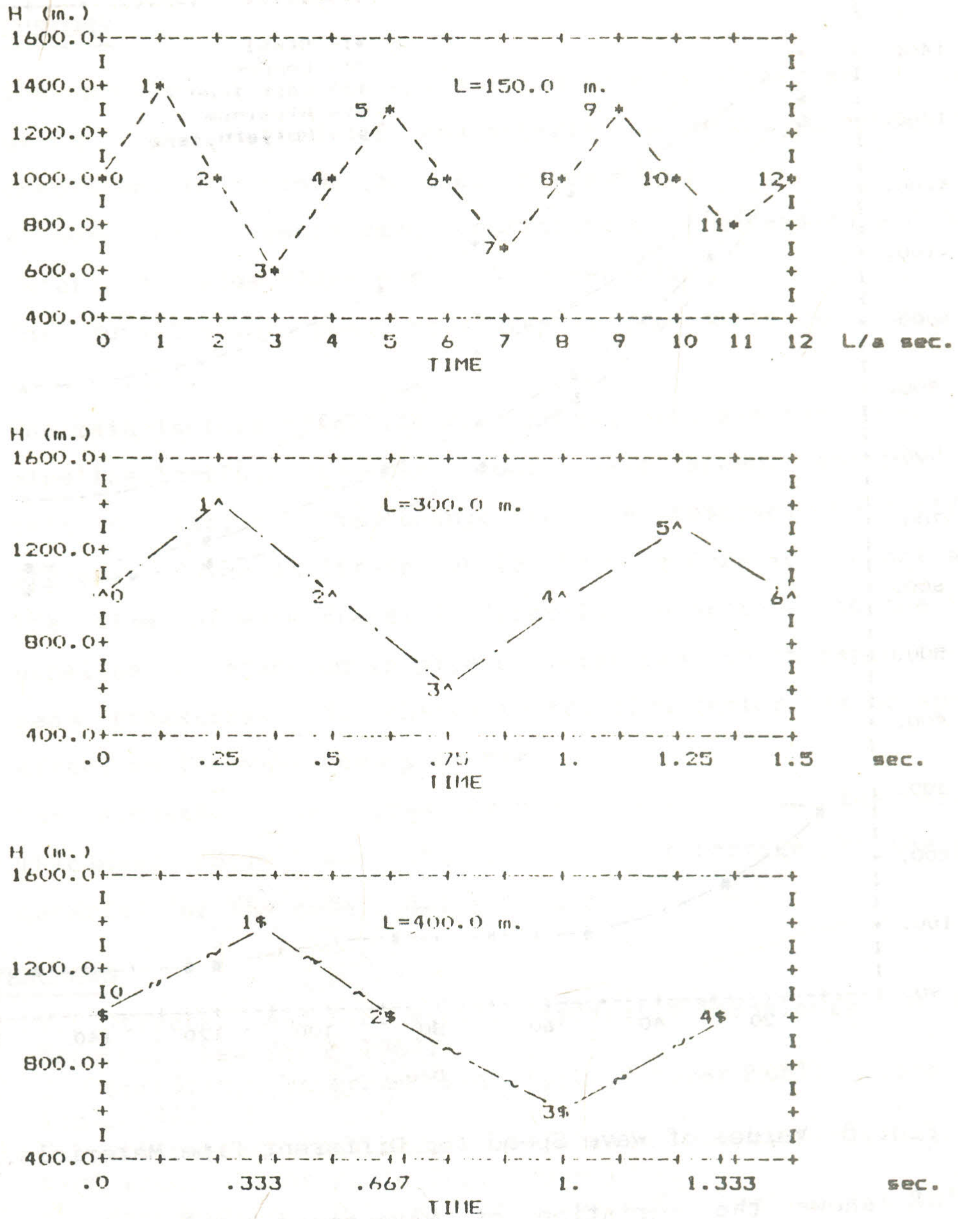

Fig. 7 Effect of Length on Pressure Wave.

At Point X/L= 0.5

For:- D=1.524 m., HR=1000.0 m.

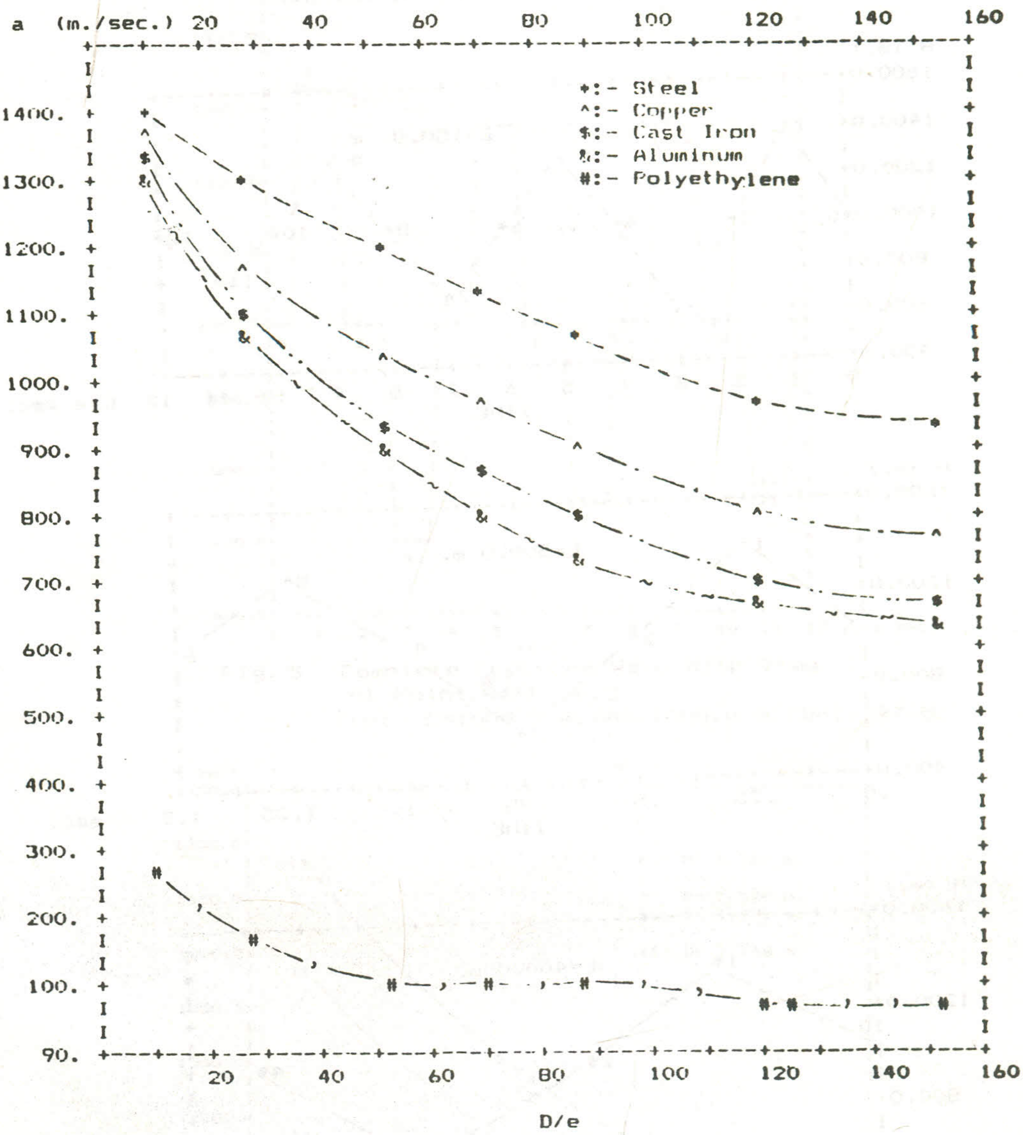

Fig. 8 Values of Wave Speed for Different Pipe Materials.

Fig. 8 shows the variation of wave speed with pipeline (diameter/thickness) ratio and pipe wall material. It is shown that the sonic wave speed decreases as the ratio (D/e) increases. The pipe materials of high values of Young's modulus of elasticity (E) such as steel have much higher values of sonic wave speed than materials of low values of modulus of elasticity such as polyethylene.

CONCLUSIONS

1. The problem of wave propagation in a pipeline can be treated by using the method of characteristics with suitable boundary conditions to perform computer programming.
2. A complete pressure wave, consisting of compression and expansion intervals, takes place every (4L/a)seconds.
3. The duration of peak pressures is remarkable near the valve end (x/L = 1.0).
4. The main factors affecting wave propagation are the reservoir head, pipeline length, diameter, sonic wave speed, and the change in friction factor. The change in the reservoir head affects the pressure wave amplitude while the wave duration is not affected. The time of wave travel is directly proportional to the length of pipeline. A reduction in pipe diameter results in rapid increase in peak pressures. The change in friction factor has no appreciable effect on the wave peak pressures.
5. The acoustic wave speed depends mainly upon the diameter to thickness ratio, and the physical properties of the pipe wall material for the same flowing liquid.

REFERENCES

[1] V.L.Streeter and E.B. Wylie, "Hydraulic Transients," Mc Graw-Hill Book Co., New York, 1967.

[2] J. Parmakian, "Waterhammer Analysis," Dover Publications Inc., New York, 1963.

[3] V.L. Streeter, "Unsteady Flow Calculations by Numerical Methods," ASME trans. J. of Basic Eng., June 1972.

[4] S. Stuckenbruck, D.C. Wiggert, and R.S. Otwell, "The Influence of Pipe Motion on Acoustic Wave Propagation, ASME trans., J. of Fluids Eng., vol. 107, Dec. 1985.

[5] M.H. Chaudhry and M.Y. Hussaini, "Second-Order Accurate Explicit Finite-Difference Schemes for Waterhammer Analysis, "ASME trans., J. of Fluids Eng., vol. 107, Dec. 1985.

[6] Adel A.A. Ramadan, "Advanced Studies on Pressure Waves in Liquid Pipes Exposed to High Pressure," M.Sc. Thesis, Fac. of Engineering, Alexandria Univ., Egypt, 1987.