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SOIL SLOP STABILITY ANALYSIS USING A SEMI-ANALYTICAL METHOD

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Abstract

The location of the critical center of slip circle to give minimum factor of safety for homogenous soil slope is studied. The actual safety factor for a slope failure through the toe point is determined with no approximations or simplification. The equation of minimum factor of safety is presented in a closed form with respect to the radius of the most critical slip circle. However, the direction of the center of such circle, measured from the toe of the slope, is given in the form of data charts obtained with the help of numerical analysis.

Available Methods

Based on numerical analysis, Spencer (1969) suggests that the circular arc is more critical than the logarithmic spiral arc for the rupture surface. The majority of the methods currently available for performing slope stability analyses may be categorized as limit equilibrium methods.

Methods that consider only the whole free body include the culman method with straight rupture surface (Taylor, 1948) and the friction circle method (Taylor, 1948). Slice methods may be categorized into the well known Swedish Circle method (Fellenius, 1927) and Bishop method (1955). Refinements of the limit equilibrium method have been undertaken by Bishop, Morgenstern, Spencer, Hunter and Schuster (1955 - 1971). These refinements have been concerned with defining a more acceptable failure surface or with modifying the method by which the forces acting on the failure surface are handled.

Only limited attention has been given by the investigators to the precise determination of the critical rupture surface. This could be the main objective of this research. In this method a more rigorous semianalytical solution is given for the determination of the most critical slip circle for the soil slope.

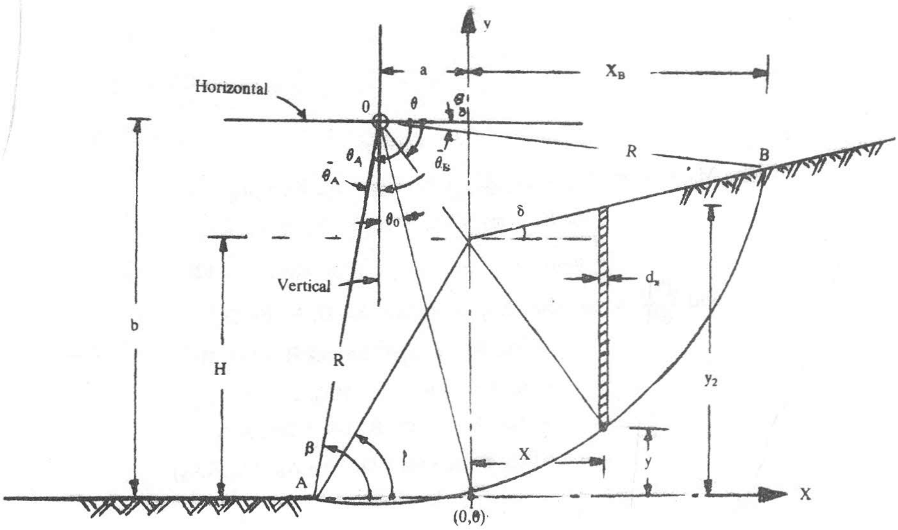
Factor of Safety for a Slip Circle Failurs Surface

With the following assumptions the analysis is carried out;

1. soil is horogeneous and isotropic.
2. Tension cracks are neglected.
3. No free water surface intercepts the slope
4. Slip circle passing through the toe is considered as the failure surface.

The factor of safety may be calculated as the sum of the resisting moments M_r divided by the sum of the moments tending to cause failure M_d (Fellenius, 1927) as;

Equations of the rupture failure, Figure (1); is given by



$a = H \cot \beta + R \cos \theta_A$, $b = R \sin \theta_A$

$X_B = R (\cos \theta_B - \cos \theta_A) - \cot \beta$

$Y_2 = H + X \tan S$ & $X_A = - \cot \beta H$

Figure (1)

In terms of two independent unknowns, mamely: the radus R and the horizontal angle to the toe point θ_A. Figure (1), one may set the formulae for M_d and M_r as follows;

$$M_d = (1.0)\,\gamma \left[\int_0^{X_B} (x+a)(y_2-y)\,dx + \int_{X_A}^{0} (x+a)(H+x \tan\beta - y)\,dx \right]$$

$$\begin{aligned} M_d = \gamma(1.0) \int_0^{X_B} & [X^2 \tan\delta + (H-b)x + ax\tan\delta \\ & + a(H-b) + R\sin\theta\,(x+a)]\,dx + \\ \int_{X_A}^{O} & [x^2\tan\beta + (a\tan\beta + H - b + R\sin\theta) \\ & x + a(H-b) \\ & + a R \sin\theta]\,dx \end{aligned}$$

$$M_d = \gamma\,(m_1 X^3_B + m_2 X^2_B + m_3 X_B + m_4 R^3 + m_5 R^2 + m_6 R + m_7 RX^2_B + m_8 R^2 X_B + m_9 RX_B + m_{10}) \qquad (2)$$

$$\text{and}\quad \frac{dM_d}{dR} = \gamma\,(3m_1 X^2_B D_x + 2m_2 X_B D_x + m_3 D_x + 3m_4 R^2 + D_{m4} R^3 + 2m_5 R + D_{m5} R^2 + m_6 + D_{m6} R + 2m_7 RX_B D_x + m_7 X^2_B + D_{m7} RX^2_B + m_8 R^2 D_X + 2m_8 RX_B + D_{m8} R^2 X_B + m_9 RD_x + m_9 X_B + D_{m9} RX_B)$$

where

$$m_1 = -\tan\delta/6$$

$$,\ m_2 = -H/2$$

$$,\ m_3 = 3\cot\beta\, H^2/2$$

$$,\ m_4 = \sin\theta_A (H + X_B\tan\delta)^2/R^2 - \sin^2\theta_A (H + X_B\tan\delta)/R - \frac{1}{3}(H + X_B\tan\delta)^3/R^3$$

$$,\ D_{m4} = -2\sin\theta_A (H + X_B\tan\delta)^2/R^3 + (2\sin\theta_A)(\tan\delta\, D_x)(H + X_B\tan\delta)/R^2 + \sin^2\theta_A (H + X_B\tan\delta)/R^2 - \sin^2\theta_A \tan\delta\, D_x/R + (H + X_B\tan\delta)^3/R^4 - (H + X_B\tan\delta)^2 \tan\delta\, D_x/R^3$$

$$,\ m_5 = \sin\theta_A \cot\beta\, H[(X_B + H\cot\beta)/R - \cos\theta_A]$$

$$,\ D_{m5} = \sin\theta_A \cot\beta\, H[-(X_B + H\cot\beta)/R^2 + D_X/R]$$

$$,\ m_6 = \frac{\cot\beta\, H^2}{2}[\cot\beta\sin\theta_A + 3\cos\theta_A - 3(X_B + H\cot\beta)/R]$$

$$,\ D_{m6} = -3D_{m5}\, H/(2\sin\theta_A)$$

$$,\ m_7 = \frac{1}{2}[\sin\theta_A - \cos\theta_A\tan\delta + \tan\delta(X_B + H\cot\beta)/R]$$

$$,\ D_{m7} = \frac{(\tan\delta}{2}[D_x/R - (X_B - H\cot\beta)/R^2]$$

$$,\ m_8 = \sin\theta_A\cos\theta_A - \sin\theta_A (X_B + H\cot\beta)/R$$

$$,\ D_{m8} = -D_{m5}/(\cot\beta . H)$$

$$,\ m_9 = H[(X_B + H\cot\beta)/R - (\cos\theta_A + \sin\theta_A\cot\beta)]$$

$$,\ D_{m9} = D_{m5}/(\sin\theta_A\cot\beta)$$

$$,\ m_{10} = -5\cot^2\beta\, H^3/6$$

$$, X_B = C_1R - C_2 + \sqrt{C_5 R^2 + C_6R + C_7}$$

$$, D_x = \frac{dx_B}{dR} = C_1 + \frac{1}{2}(C_5 R^2 + C_6 R + C_7)^{-1/2}(2C_5 R + C_6)$$

With $C_5 = C_1^2$, $C_6 = (C_4 - 2C_1C_2)$ & $C_7 = (C_2^2 - C_3)$

$$, C_1 = (\sin\theta_A \tan\delta - \cos\theta_A)/(1 + \tan^2\delta)$$

$$, C_2 = (\tan\delta + \cot\beta)\ H/(1 + \tan^2\delta)$$

$$, C_3 = (1 + \cot^2\beta)\ H^2/(1 + \tan^2\delta)$$

and, $C_4 = 2(\sin\theta_A - \cos\theta_A \cot\beta)\ H/(1 + \tan^2\delta)$

The resisting moment M_r is computed from Figure (1), by

$$M_r = \gamma \tan\varnothing\,(F_1 R^3 + F_2 R^2 H) \quad \ldots\ldots (4)$$

$$\text{and} \quad \frac{dM_r}{dR} = (\gamma \tan\varnothing[R^3 \frac{dF_1}{dR} + R^2(3F_1 + H\frac{dF_2}{dR}) + 2F_2 RH] \quad \ldots\ldots (5)$$

Where $F_1 = K_1 + K_3\cos^3\theta_0 + K_4(\sin\theta_0 \cos\theta_0 + \theta_0)$

and $F_2 = K_2 + K_5(\sin\theta_0 \cos\theta_0 + \theta_0)$

With
$$K_1 = \tan\beta \cos^3\bar\theta_A/3 - \tan\delta\cos^3\bar\theta_B/3 - \frac{1}{2}\sin\theta_A (\bar\theta_B - \bar\theta_A + \frac{1}{2}\sin 2\bar\theta_B - \frac{1}{2}\sin 2\bar\theta_A) + \frac{1}{2}\cos\theta_A \tan\delta(\bar\theta_B + \frac{1}{2}\sin 2\bar\theta_B) - \frac{1}{2}\cos\theta_A \tan\beta(\bar\theta_A + \frac{1}{2}\sin 2\bar\theta_A) + \sin\bar\theta_B - \sin\bar\theta_A - \sin^3\bar\theta_B/3 - \sin^3\bar\theta_A/3$$

$$, K_2 = \frac{C}{\gamma H \tan\varnothing}(\bar\theta_A - \bar\theta_B) + \frac{1}{2}\bar\theta_B + \frac{1}{4}\sin 2\bar\theta_B - \frac{1}{2}\sin 2\bar\theta_A - \frac{1}{2}\cot\beta \tan\delta(\bar\theta_B + \frac{1}{2}\sin 2\theta_B)$$

$$, K_3 = \frac{1}{3}(\tan\delta - \tan\beta)$$

$$, K_4 = \frac{3}{2}\cos\theta_A\ K_3$$

$$, K_5 = \frac{3}{2} \cot\beta \; K_3$$

$$, \sin\theta_0 = H \cot\beta / R - \cos\theta_A$$

$$, \cos\theta_0 = \sqrt{1 - \sin^2\theta_0}$$

$$, \theta_0 = \sin^{-1}(H \cot\beta / R - \cos\theta_A)$$

$$, \cos\bar{\theta}_B = \sin\theta_A - \frac{1}{R}(H + X_B \tan\delta) = \sin\theta_B$$

$$, \bar{\theta}_B = \frac{\pi}{2} - \sin^{-1}[\sin\theta_A - (H + X_B \tan\delta)/R]$$

$$, \sin\bar{\theta}_B = (X_B + H \cot\beta)/R - \cos\theta_A = \cos\theta_B$$

$$\text{and, } \sin 2\bar{\theta}_B = 2 \sin\bar{\theta}_B \cos\bar{\theta}_B$$

In equation (5), dF_1/dR and dF_2/dR are given by;

$$\frac{dF_1}{dR} = DB_5 - DB_3 + \frac{1}{2}\cos\theta_A \tan\delta\,(DB_2 + DB_4/2) - \frac{1}{2}\sin\theta_A\,(DB_2 + DB_4/2) - \tan\delta\, DB_1/3 + 3 K_3 \cos^2\theta_0 \sin\theta_0 (\cos\theta_A + a/R)/\sqrt{R^2 - a^2} - K_4 [(\cos\theta_A + a/R)/\sqrt{R^2 + a}\,(\cos^2\theta_0 + \sin^2\theta_0 + 1)$$

$$, \frac{dF_2}{dR} = \frac{C}{\gamma H \tan\varnothing} DB_2 + \frac{1}{2}(DB_2 + DB_4/2)(1 - \cot\beta \tan\delta) - K_5 [(\cos\theta_A + a/R)/\sqrt{R^2 - a^2}]\,(\cos^2\theta_o - \sin^2\theta_o + 1)$$

In which

$$DB_1 = \frac{d\cos^3\bar{\theta}_B}{dR} \quad ,DB_2 = \frac{d\,\theta_B}{dR}$$

$$,DB_3 = \frac{d\sin^3\bar{\theta}_B}{dR} \quad ,DB_4 = \frac{d\sin 2\,\theta_B}{dR}.$$

$$\text{and } DB_5 = \frac{d\sin\bar{\theta}_B}{dR}$$

Minimum value of F_s with respect to R, using equation (1) is computing as

$$\frac{dF_s}{dR} = (M_d\frac{dM_r}{dR} - M_r\frac{dM_d}{dR})/(M_d)^2 = 0$$

$$\text{and} \quad M_d\frac{dM_r}{dR} - M_r\frac{dM_d}{dR} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\text{or} \quad \boxed{F_{s_{min}} = \frac{dM_r}{dR} \Big/ \frac{dM_d}{dR}} \quad \ldots\ldots\ldots\ldots\ldots\ldots$$

By solving equation (6) the radus R which correspond to the minimum value Fs_{min} of equation (7) may be determined.

The value of Fs_{min} and the corresponding slip circle depends on the assumed angle θ_A (Figure 1). To arrive at the minimum-minimum value of the factor of safety, and therefore fixe the most critical slip circle; one may proceed as follows;

1. Let a value for θ_A (or a direction angle for slip circle's center).
2. Solve equation (6) (the computer is necessary in this step).

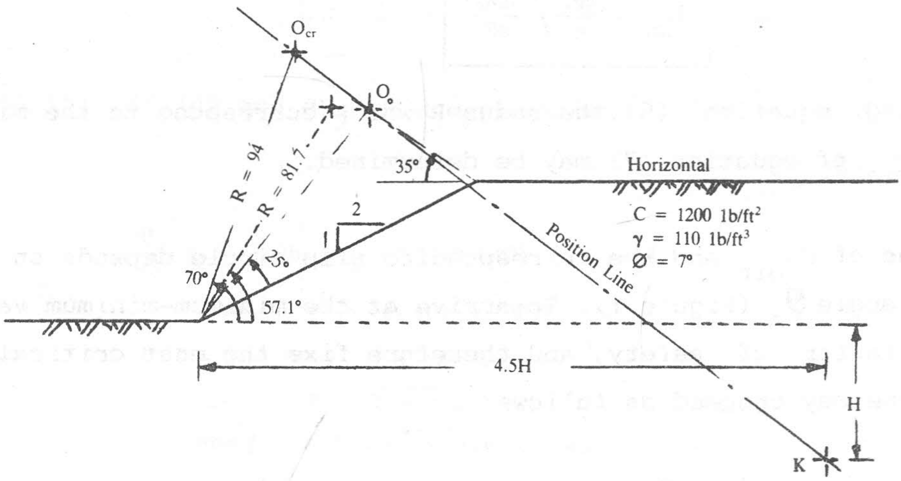
3. Use the radus R_1 of step (2) to obtain F_{smin} from equation (7) for the direction of step (1).
4. Repeat steps (2) & (3) for a new value of $\theta_A = \theta_A + \Delta\theta$, where $\Delta\theta$ may be choosen as 2° to 5°.
5. The critical direction β_{cr} correspond to $F_{min-min}$.

The above steps are set in a computer routine and the results for the analysis using this method are given in the following section.

Results and Discussion

In order to test the equations presented herein, a solved problem carried out by Jumikis (1967) using the Swedish circle method is solved using the above steps. In this problem the data is as follows:

$$\varnothing = 7^\circ,\ \beta = \tan^{-1}(0.5) = 26.6^\circ,\ C/\gamma H = \frac{1200}{110\times 45} = 0.2424$$



Position Line (Swedish) $\beta_{cr} = 70^\circ$, $R_{cr} = 94$, $F_{smin} = 2.06$

Present $\beta_{cr} = 57.1^\circ$, $R_{cr} = 81.7$, $F_{smin} = 1.9038$

Figure (2) solved Problem

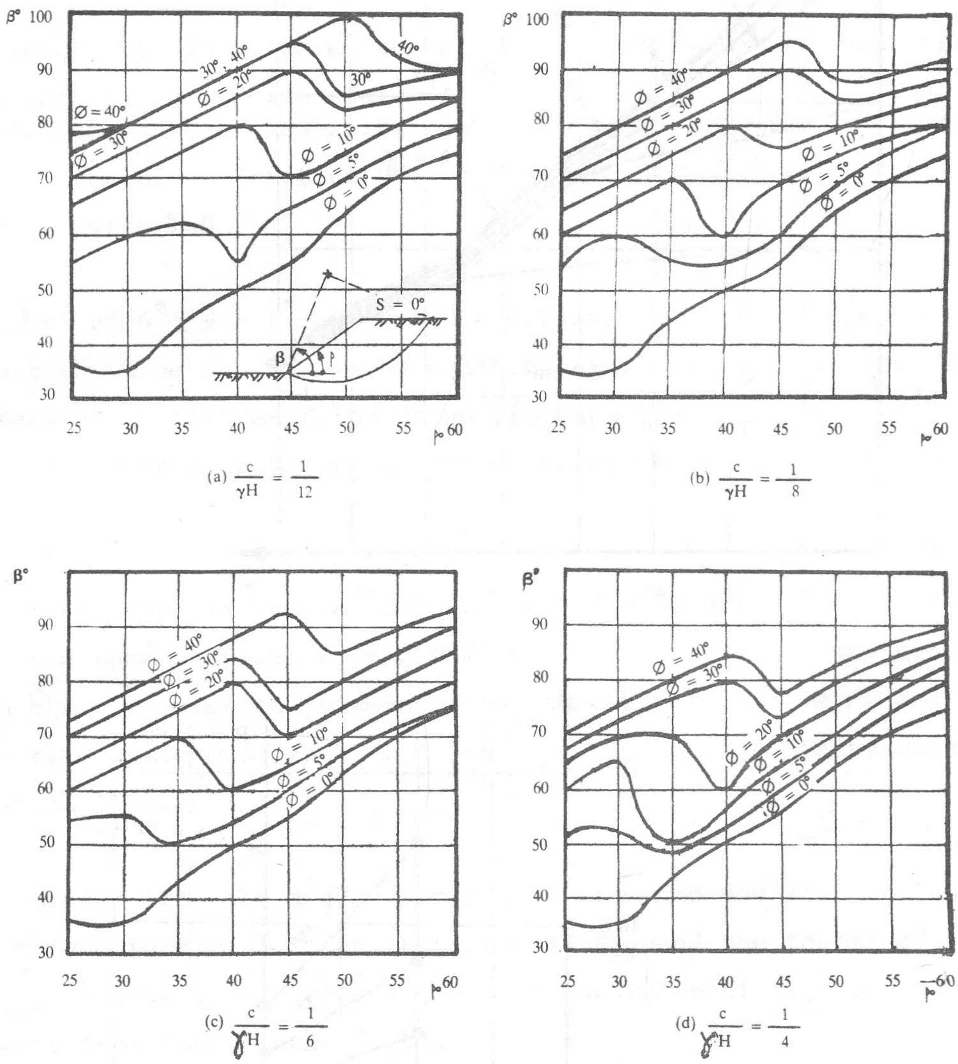


Figure (3) Data charts for $\beta_{critical}$ (δ = 0)

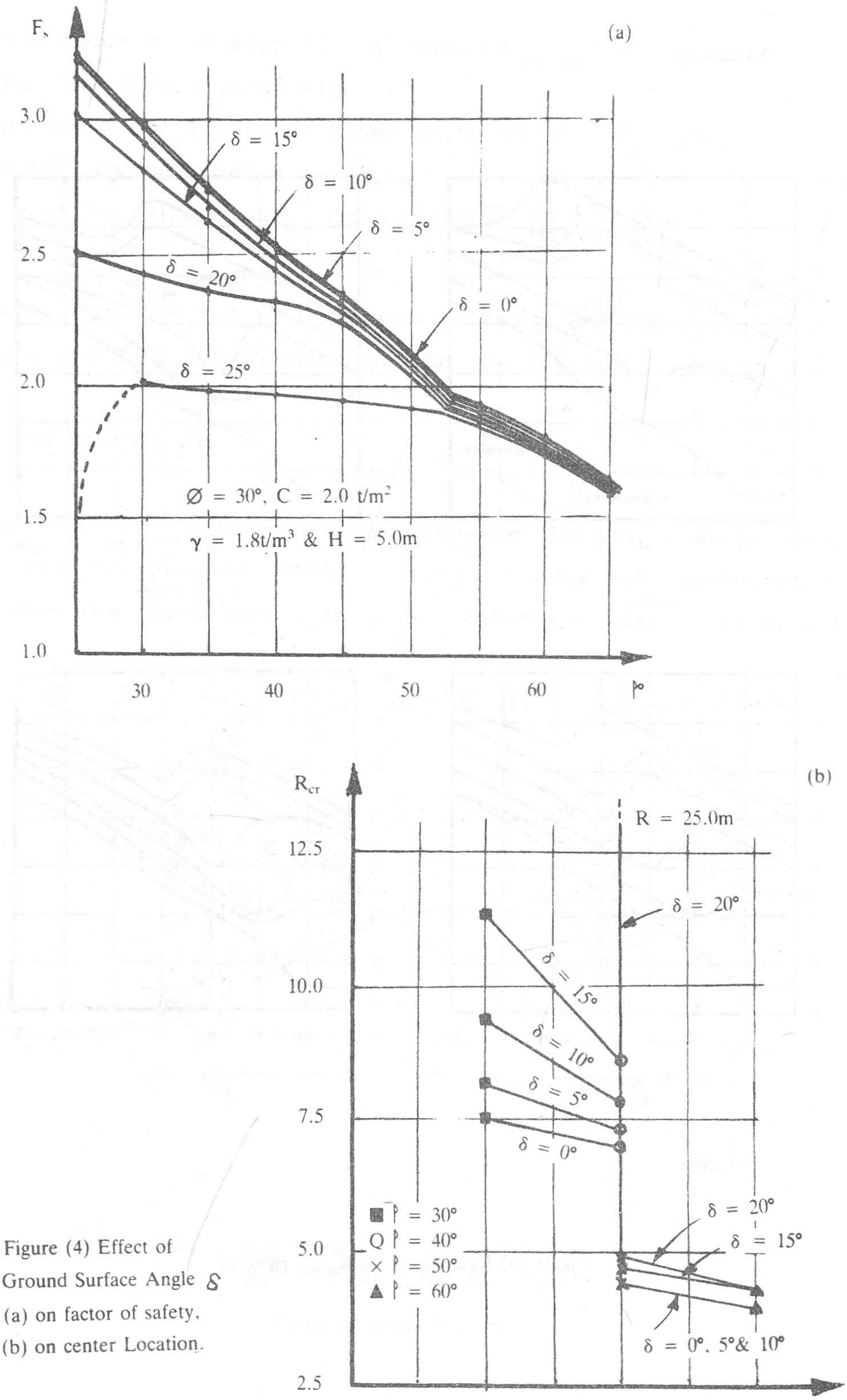


Figure (4) Effect of Ground Surface Angle δ (a) on factor of safety, (b) on center Location.

and the resulting value for minimum factor of safety is 2.06 at an angle β = 70° with a position line for critical centers as shown in Figure (2).

In order to facilitate the solution proposed here, data charts are prepared to locate the angle θ_A (or the angle β) of figure (1) for a practical range of stability number $c/\gamma H$ and deferent values of angle of internal friction ϕ. Figure (3) shows four charts for β vs ρ with δ=0 .

With ρ = 26.6°, ϕ = 7° and $c/\gamma H = 0.2424$; β_{cr} = 57.1° and the resulting value for F_s, using equations (6) and (7), is 1.9038 which is less than the resulting value of the solved example by 7.6 % . Also, there is a deviation of the critical center w.r.t β by about 13°.

One more run is carried out for = 70° produced a factor of afety for the same problem; F_s = 2.0583 which is in a very good agreement with the slice method, however, the corresponding value of R = 89.3 is slightly different than R_{cr} of the solved example by using the position line.

The effect of the ground slope δ is examined and its concluded that the only effect of δ is on the value of R_{cr} and the factor of safety F_{smin}, however, no effect for on the value of β_{cr} as one may observe from from Figure (4.b).

It is intersting to note that the computed value of Fs_{min} for ϕ = ρ (c = 0) comes out to be slightly higher than 1.0 and the slip circle is very shallow (almost a flow type of slide), Figure (5).

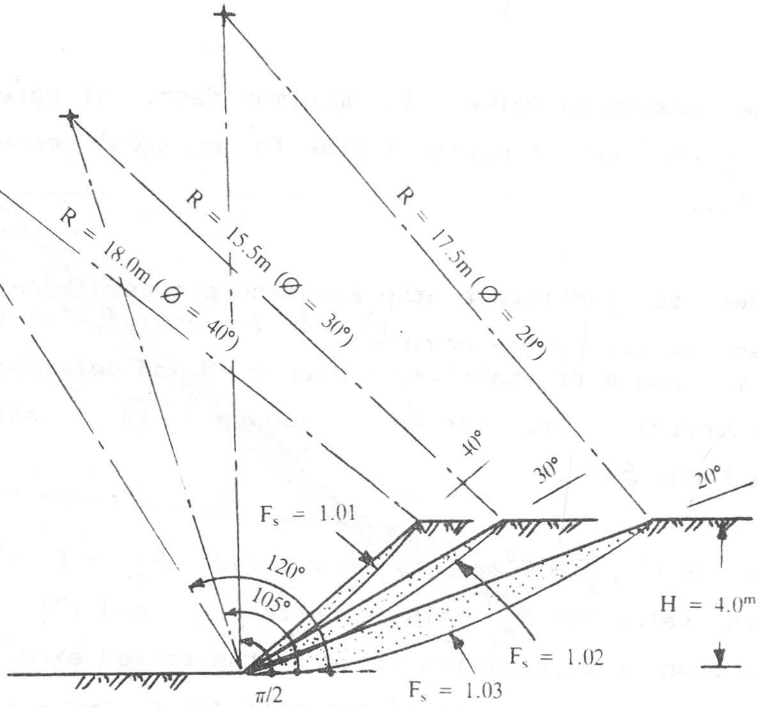


Figure (5) Flow Slide (β = Ø, C = 0)

Summary and Conclusions

A method for stability of slop analysis is introduced which enable the analyst with a quick and accurate mean for determining the most critical slip circle. In this method data charts to locate the direction of the most slip circle's center are provided. These charts cover the different variables involved in the stability of earth slopes, namely: the cohesion, the angle of internal friction, the soil unit weight, the slope height, the ground slope and the slope angle. The most critical raduis of the slip circle is computed from analytical analysis with closed form equations, using the critical direction from the data charts (Figure 3).

The analysis yields the following conclusions

1. The most critical slip circle's center not meccessarily falls on

the position line (Figure 2).

2. Direction angle β_{cr} of the critical center for $\phi = 0$ is independent of the stability number $c/\gamma H$.
3. As the stability number decreases the direction angle β_{cr} for $\phi \neq 0$, increases.
4. A flow type of slide for c = 0 and $\phi = \beta$ is observed, as it should be, with $F_{smin-min}$ very close to unity (Figure 5).
5. The ground surface sloping angle δ has no effect on the direction of the critical center. However, as δ increases the value of $F_{smin-min}$ decreases, and the raduis of the most critical slip circle increases rapidly for $(\delta/\beta) \geq 1/2$ (Figure 4-b).

Acknowledgement

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References

[1] Fellenius, W. (1927) "Calculation of stability of slopes", Berlin: W. Ernst and Sohn, (revied edition, 1939).

[2] Bishop, A.W. (1955), "The use of the slip circle in the stability analysis of slopes", Géotechnique V, No. 1, pp 1-17.

[3] Bishop, A.W. and Morgenstern, N.R. (1960), "Stability coefficients for earth slopes", Geotechnique X, No. 4, pp 129-150.

[4] Chen, W.F. and Giger, M.W. (1971), "Limit analysis of stability

of slopes", Journal of soil Mechanics and Foundations Division ASCE 97, No. SMI, Jan., pp. 19-26.

[5] Spencer, E. (1969), "Circular and logarithmic spiral slip surfaces", Journal of the soil Mechanics and Foundations Division ASCE 95, No. SM1, pp. 227-234.

[6] Taylor, D.W. (1948), "Fundamentals of soil Mechanies", John Wiley and Sons, New York, pp 406-479.

[7] Hunter, J.H. and Schuster, R.L. (1971), "Chart solutions for analysis of earth slopes", Highway Research Record No. 345, pp. 77-89.

[8] Jumikis, A.R. (1967), "Soil Mechanics" D. Van Nostrand Comp. Inc., New Hersey, pp. 699-702.