

## MATHEMATICAL MODELLING FOR THE FORECAST OF NILE BASIN RAINFALL

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### Abstract

Water resources planners are always concerned with the expected future river flows so adequate measures can be taken in due time to balance the water budget. The main objective of the present study is to introduce a mathematical modelling solution for the forecast of annual rainfall values at a rainfall gauge station. The model for any station converts the series of previous continuous annual rainfall values and the corresponding series of average annual sunspot numbers into an optimum moving mean series. The best fit equation combining both series is determined and used for annual rainfall forecast. Time lag of one and two years for the annual rainfall series are considered in the preparation of the one and two years ahead forecast models respectively. Both models are applied to forecast each of the last five years recorded annual rainfall values at 18 rainfall stations located around the equatorial lakes at the upper Nile basin and also for Addis Abbaba station. The comparison between the forecasted and the measured rainfall values confirmed the accuracy of both models for several stations.

Notation

A%E	= average percentage error
CC	= correlation coefficient
CS	= coefficient of skewness
j	= number of years for the calculation of moving mean values
k	= an individual reduction coefficient
l	= time lag in years
n	= number of years for which annual rainfall data is available at a rainfall station
ng	= number of spot groups
ns	= total number of spots on the visible sun disk
R	= annual Rainfall value
S	= average annual sunspot number
SD	= standard deviation

1. Introduction

Sunspots are the most popular phenomenon of solar activity and have been regularly observed since the middle of the 18th century. At photospheric level, sunspots are about 1000 - 2000 degrees Kelvin cooler than the surrounding regions and therefore they look dark. The outstanding physical property of sunspots is their magnetic field which ranges in the central umbrae between about 1000 and 4000 Gauss. This magnetic field can be regarded as a huge accumulation of energy which can reach  $10^{34}$  ergs for a big sunspot group and which controls physical processes not only in the photosphere but also to great heights of the solar atmosphere. The dimensions of sunspots vary greatly, from small sizes up to areas 100,000 km in diameter. Sunspot number (S) have been used as a measure of solar activity and is

defined by:

$$S = k(10 n_g + n_s) \quad (1)$$

in which  $n_s$  is the total number of spots on the visible disk irrespective of their size,  $n_g$  is the number of spot groups and  $k$  is an individual reduction coefficient depending on the observer and observing instrument. The variation in solar activity are represented by the 11 - year cycle of the average annual sunspot numbers [1].

During the past five decades the response of earth weather to solar activity attracted many investigators. As for the Nile basin, Brooks (2) was perhaps the first to claim a connection between the water levels of lakes Victoria and Albert and the sunspots for the period 1896 - 1922. However, Hurst (3) concluded that this connection disappeared for subsequent years 1923 - 1934. Recently El Sayed [4] showed that insignificant relation between water level in lake Victoria and sunspots occurred for the period 1923 - 1950, while for the period 1951 - 1967 the lake level oscillated in an opposite phase with the average annual sunspot number and followed the same pattern for the period 1968 - 1982.

As for other parts of the world, an interesting forecast for years of serious drought in the 1980's in South Africa was given by Dyer [5]. This conclusion was based on a subjective forecasting technique supported by sunspot - rainfall relationship. Vines [6] carried out a comparison with precipitation data for southeast Australia, Tasmania and New Zealand and suggested a correlation with the 10-12 year sunspot cycle. Vines findings are in accord with Dyers [5] estimates of the likelihood of extending drought conditions over much of South

Africa in the mid 1980's. As for New Zealand rainfall record, Tomlinson [7] noticed that both rainfall and sunspots series peak in each decade from 1900 - 1979 and that near a sunspot maximum a year is likely to be wet than dry. He also predicted that during 1983 - 1986 New Zealand may experience a period of generally dry weather. More studies by Klaus [8], Smirnov [9], Golz [10], Jenkinson [11] and Schove [12] supported the correlation between sunspots and rainfall and river flow. However, among other studies which denied such correlation is that given by Rordiguez and Yevjevich [13].

It is expected that more research will be published in the near future studying the anticipated solar - rainfall and solar - river flow and levels relationships in a more precise and quantitative way. The present study is an attempt in this direction. Herein an attempt is carried out to formulate two mathematical models to forecast two years ahead annual rainfall at a rainfall station. The models are primarily based on the relationship between average annual sunspot numbers and annual rainfall series. Time lag of one and two years for rainfall series are considered for the first and second model respectively and moving mean values which will optimise the correlation between both moving mean series will be determined and the best fit equation combining both series will be established and used for two years ahead forecast of annual rainfall values. Both models are applied to forecast annual rainfall on 18 stations located in the equatorial lakes area in the upper Nile basin and at Addis Abbaba.

## 2. Annual Rainfall-Sunspot Number Relationship For the Nile Basin:

The relationship between annual rainfall and average annual sunspot number series for three different rainfall gauge stations located in

the equatorial lakes area is shown by figure (1). These stations are Mbarara in Uganda, Kericho in Kenya and Tarime in Tanzania. The degree of correlation between the two variables for any simultaneous period of time can be detected by studying such plots and can be mathematically measured by estimating the value of the correlation coefficient (CC) given by the following equation:

$$CC = \frac{n \sum R_i S_i - \sum R_i \sum S_i}{\sqrt{(n \sum R_i^2 - (\sum R_i)^2) (n \sum S_i^2 - (\sum S_i)^2)}} \quad (2)$$

in which  $R_i$  and  $S_i$  are the  $i$ -th pair of rainfall and sunspot numbers respectively at year  $i$  and  $n$  is the number of pairs of observation, all summations are considered for  $i=1$  to  $i=n$ . The estimated values of (CC) considering the whole record for the three stations are 0.0275, 0.0917, and 0.2276 respectively. These low values and the plots shown by figure (1) show that low correlation exists if the whole record is considered. However, there are periods within each record where excellent correlation exists. Such periods can be easily identified but eventually it will not serve the purpose of rainfall forecast. To improve this correlation, moving mean values for both annual rainfall and sunspot numbers are used. This will damp down the fluctuation and will smooth out the plotting of both series and will consequently improve the correlation between them.

**3. Formulation of the Mathematical Models at a Rainfall Gauge Station**

The data required to construct the proposed mathematical model at any rainfall gauge station consists of two data series. The first is the past continuous recorded annual rainfall data. The second series is the average annual sunspot numbers corresponding to the period for

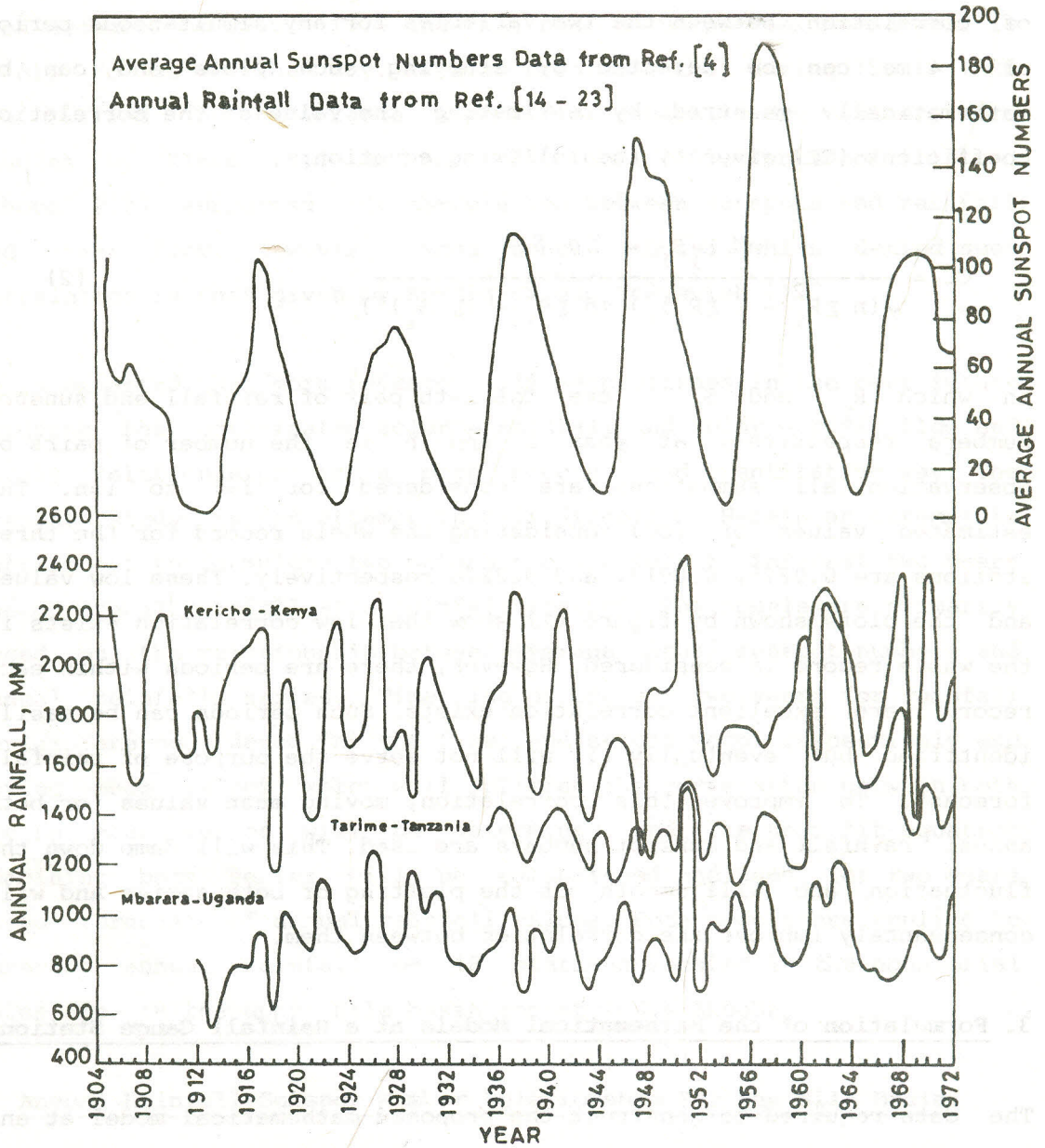


Fig.(1) Annual Rainfall Vs. Average Annual Sunspot Number For 3 Stations .

which annual rainfall are available. The model is based on a search for the value of the number of years (j) to be used for the computation of moving mean values for annual rainfall and average sunspot numbers and which will give the best correlation coefficient for the relationship between both moving mean series. Time lag of one year for annual rainfall series is considered in estimating rainfall moving mean values for the one year ahead annual rainfall forecast model, while for the two years ahead forecast model time lag of two years is considered. The next step is to identify the best fit equation combining both moving mean series and which will be used to forecast annual rainfall values. The following steps delineates how the model is constructed and used for the forecast of the one year ahead annual rainfall  $R_{n+1}$  values at the rainfall station:

1. The previous continuous annual rainfall series of n years at the station is considered and take  $j = 1$
2. Formulate the moving mean sunspot series:

$$S_{i,j} = \frac{1}{j} \sum_{i=i-j+1}^i S_i \quad (i= 1,2,\dots, m) \quad (3)$$

in which:

$S_{i,j}$  = the i th sunspot moving mean value corresponding to the value of j,

$$m = n - j - \ell + 1$$

n = number of recorded annual rainfall data at the station

$\ell = 1$  for the one year ahead forecast model and  $\ell = 2$  for the two years ahead forecast model, thus for given values of  $\ell$  and j there will be m values of  $S_{i,j}$ .

3. Considering the appropriate time lag value  $\ell$  , formulate the moving

mean annual rainfall series given by:

$$R_{i,j} = \frac{1}{j} \sum_{i=i}^{i=i+j-1} R_i \quad (i=1,2,\dots,m) \quad (4)$$

4. Applying equation (2) find the correlation coefficient (CC) between both series obtained by equations (3) and (4).
5. Increase  $j$  by the value of one year and repeat steps 2,3 and 4.
6. Steps 2 to 5 are repeated till  $j = n - \ell - 2$ .
7. Arrange the estimated values of (CC) in descending order.
8. Consider the value of  $j$  which gave the maximum value of (CC) and find the best fit equation for the relationship between both series given by equations (3) and (4). Eleven equations are formulated and are statistically compared to pick up the best fit equation. These equations include a power, exponential and polynomial functions of different degrees up to the ninth degree.
9. The one year ahead forecasted annual rainfall value  $R_{n+1}$  is obtained by estimating the value of  $\frac{1}{j} \sum_{i=n-j+1}^{i=n} S_i$  which is then substituted into the best fit equation determined by step 8 to

obtain the value of  $\frac{1}{j} \sum_{i=n-j+2}^{i=n+1} R_i$  and hence:

$$R_{n+1} = \left( \frac{1}{j} \sum_{i=n-j+2}^{i=n+1} R_i \right) j - \sum_{i=n-j+2}^{i=n} R_i \quad (5)$$

10. The value of the forecasted annual rainfall  $R_{n+1}$  should fulfil the



following condition :  $R_{\max} \geq R_{n+1} \geq R_{\min}$  in which  $R_{\max}$  and  $R_{\min}$  are the maximum and minimum recorded rainfall values respectively. If this condition is not fulfilled, then the value of  $j$  is replaced by the one corresponding to the next maximum correlation coefficient given by step 7 and then steps 8 and 9 are repeated till this condition is fulfilled.

Furthermore, the two years ahead forecast model is formulated by considering the following steps:

1. Steps 1 to 8 of the one year ahead forecast model are followed but with  $\ell = 2$
2. The first year ahead forecasted annual rainfall value  $R_{n+1}$  is obtained by estimating the value of  $\frac{1}{j} \sum_{i=n-j}^{i=n-1} S_i$  which is then substituted into the best fit equation determined by step 8 to obtain the values of  $\frac{1}{j} \sum_{i=n-j+2}^{i=n+1} R_i$  and hence:

$$R_{n+1} = \left( \frac{1}{j} \sum_{i=n-j+2}^{i=n+1} R_i \right) j - \sum_{i=n-j+2}^{i=n} R_i \tag{6}$$

3. The second year ahead forecasted annual rainfall value  $R_{n+2}$  is obtained by estimating the value of  $\frac{1}{j} \sum_{i=n-j+1}^n S_i$  which is then substituted into the best fit equation determined by step 8 to obtain the value of  $\frac{1}{j} \sum_{i=n-j+3}^{i=n+2} R_i$  and hence :

$$R_{n+2} = \left( \frac{1}{j} \sum_{i=n+2}^{i=n} R_i \right) j - \sum_{i=n-j+3}^{i=n-j+3} R_i \quad (7)$$

4. The condition given by step 10 of the one year ahead forecast model is to be applied herein.

Since the model considers the previous rainfall record to forecast the next year rainfall (or the next two years) values, it is obvious that the value of  $j$  and the best fit equation obtained by step 8 above will differ not only from station to another but also from year to another at the same station.

#### 4. Available Annual Rainfall Data On The Upper Nile Basin

To implement the proposed mathematical model for annual rainfall forecast at any station, continuous annual rainfall data are needed for a sufficient period of time and without gaps. All the available rainfall data recorded at stations located around the equatorial lakes are examined and the 18 stations which fulfilled the condition of data continuation are considered. Nine of these stations are located in Uganda, four in Kenya and five are in Tanzania. Figure (2) shows the locations of these stations. All data are obtained from Volume VI of the Nile Basin [14] and from its 9 supplements [15 to 23] and which were published by the Egyptian Ministry of Public Works and Water Resources. Few gaps existed in some months, and rainfall values for these months were estimated by Shahin [24] for Uganda stations and by the author for Kisumu and Kericho stations in Kenya by considering

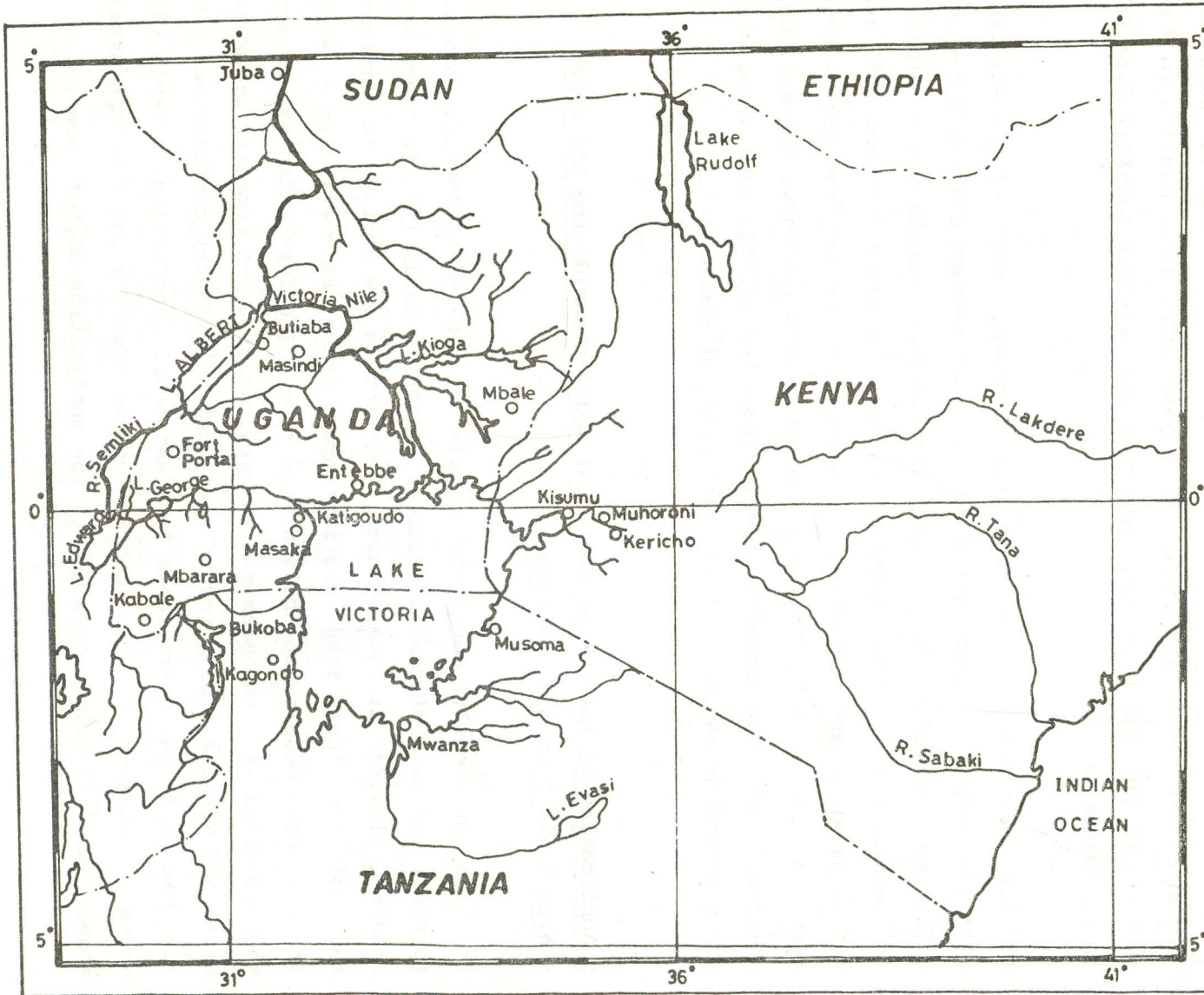


Fig.(2) Locations of Rainfall Gauge Stations .

values of near by stations. Table (1) gives the geographical locations of the stations and the period for which annual rainfall data are available. Unfortunately annual rainfall data for Uganda, Kenya and Tanzania are only available up to year 1972. Supplements 6 to 9 of volume VI of the Nile basin are published without rainfall data for these countries.

As for rainfall on the Abyssinia plateau, none of the existing stations enjoy a lengthy record and the nearest station for which sufficient rainfall record is available is Addis Abbaba where rainfall is measured since 1901. However, rainfall in years 1966 and 1967 are not available and therefore, the 65 years of record from 1901 - 1965 are considered in the present study. Table (2) shows the minimum, maximum, and mean annual rainfall values for all stations along with the standard deviation (SD) and coefficient of skewness (CS).

#### 5 Application of the Two Mathematical Models Considering the Upper Nile Basin Rainfall Stations

The two previously developed models are applied to forecast annual rainfall at each of the 19 stations listed in table (1). To check the models validity at each station, both models are applied to forecast each of the last five years recorded annual rainfall, i.e at each station both models are applied five times. The forecasted values are then compared with the measured rainfall values. Two computer programs Nile1 and Nile2 are prepared to carry out the lengthy computations for the one and two years ahead forecast models respectively. Table (3) shows a summary of the percentage errors (%E) for the forecasted annual rainfall values obtained by the application of both models.

Table (1) Geographical Locations and Period of available continuous annual rainfall data

No	Station	Geographic Location			From - to	
		Latitude	Longitude	Altitude		
A. UGANDA						
1	Butiaba	01	50	31 20	621	1904 - 1967
2	Masindi	01	41	31 43	1146	1910 - 1972
3	Mbale	01	06	34 11	1220	1922 - 1972
4	Fort Portal	00	40	30 17	1539	1906 - 1971
5	Entebbe	00	04	32 29	1182	1900 - 1967
6	Katigond	00	13	31 44	1311	1913 - 1972
7	Masaka	00	20	31 44	1313	1904 - 1970
8	Mbarara	00	37	30 39	1443	1912 - 1972
9	Kabale	01	15	29 59	1871	1921 - 1972
B. KENYA						
10	Kisumu	00	06	34 45	1146	1903 - 1972
11	Muhoroni	00	09	35 12	1300	1904 - 1968
12	Kericho	00	23	35 17	1981	1905 - 1972
13	Kissi	00	41	34 47	1768	1911 - 1969
C. TANZANIA						
14	Bukoba	01	20	31 49	1144	1921 - 1972
15	Tarime	01	22	34 23	1524	1935 - 1971
16	Musoma	01	30	33 48	1147	1930 - 1972
17	Kagondo	01	33	31 42	1296	1926 - 1972
18	Mwanza	02	31	32 54	1131	1922 - 1967
D. ETHIOPIA						
19	Addis Abbaba	09	02	38 44	2440	1901 - 1965

Table (2) Statistical Characteristics of annual rainfall data for 19 stations

No	Station	Min mm	Max mm	Mean mm	SD mm	CS
1	Butiaba	400	1263	753	167.38	0.500
2	Masindi	747	1716	1297	181.02	-0.185
3	Mbale	807	1775	1163	223.36	0.856
4	Fort Portal	1118	2116	1496	205.95	0.723
5	Entebbe	999	2260	1540	248.73	0.155
6	Katigond	738	1522	1067	186.35	0.700
7	Masaka	568	1988	1119	269.47	0.548
8	Mbarara	533	1520	910	170.19	0.876
9	Kabale	636	1486	1002	178.46	0.413
10	Kisumu	493	1808	1113	235.21	0.273
11	Muhoroni	851	2471	1507	392.76	0.654
12	Kericho	1168	2454	1840	277.15	0.035
13	Kissi	1152	2728	1759	311.25	0.579
14	Bukoba	1554	2735	2039	290.13	0.873
15	Tarime	1129	2322	1445	300.35	1.802
16	Musoma	420	1183	808	177.22	-0.148
17	Kagondo	1262	2736	1726	321.21	0.796
18	Mwanza	651	1700	1044	223.84	0.759
19	Addis Abba	898	1900	1203	205.72	1.115

Table (3) Forecast percentage errors for each of the last five Year annual rainfall record obtained by both models

Station	n-4			n-3			n-2			n-1			n			A%E			
	M1		M2	M1		M2	M1		M2	M1		M2	M1		M2	M1	M2	M12	M2
	F	F	S	F	F	S	F	F	S	F	F	S	F	F	S	F	F	F	S
Butiaba	37	34	-32	-3	-4	-38	78	-50	36	21	114	7	38	-18	68	35	44	34	36
Masindi	20	38	39	2	15	5	13	-3	1	-32	18	5	-22	-11	-21	18	17	16	14
Mbale	-1	23	19	36	25	23	-12	-23	3	-21	FA	FA	-27	-19	-13	19	23	20	15
Fort Portal	-13	-27	-7	-12	0	-8	36	-5	-2	-27	-5	-7	-16	-11	15	21	10	11	8
Entebbe	-10	36	6	-14	-18	-14	21	1	24	30	10	22	12	19	29	17	17	14	19
Katigond	18	51	-8	38	17	4	12	27	67	14	27	91	-10	FA	FA	18	30	21	60
Masaka	-16	31	9	31	-19	8	-1	-6	-13	FA	-17	-30	-14	36	-24	15	22	19	17
Mbarara	3	3	8	-16	-15	-18	-11	-18	-5	-7	6	25	-13	5	-14	10	10	12	14
Kabale	24	-18	-24	-21	20	20	-1	-17	-18	-36	11	-39	7	-27	-36	18	19	16	27
Kisumu	-9	-17	-17	78	-2	-2	44	-16	-16	FA	-26	-26	-3	171	171	34	46	13	46
Muhoroni	18	28	-25	-4	-7	72	31	-74	64	143	-7	91	68	182	16	53	59	51	53
Kericho	51	-13	-15	-15	-22	35	3	23	-38	-23	-27	-15	5	-23	-31	20	22	25	27
Kissi	-24	-28	-33	FA	-25	FA	28	-30	-11	-2	19	8	-25	-10	17	20	22	20	17
Bukoba	-28	-12	-28	32	-35	8	-18	10	-6	1	-4	-8	-30	0	13	22	12	20	12
Tarime	-10	FA	FA	-15	FA	FA	13	3	18	-21	31	-5	-1	13	13	12	16	13	12
Musoma	-28	-6	24	-41	-35	25	20	-44	-2	13	8	-27	-18	40	-2	24	27	23	16
Kagondo	-48	-16	-38	-23	-30	-38	-5	FA	FA	-9	FA	FA	-30	9	-6	23	19	23	27
Mwanza	-3	-6	-28	-21	-3	-41	2	-5	-18	0	-25	-37	30	30	-8	11	14	14	26
Addis Abbaba	1	25	-18	30	-4	20	-15	39	4	17	33	-25	-18	6	-21	16	21	11	18

n is the number of years for which annual rainfall is available.

M1 and M2 are the first and second model respectively,

M12 is for values obtained by the average of both models.

F & S are first and second year ahead values, and

FA indicates that model failed to forecast the rainfall value.

**6. Results Given By the One Year Ahead Forecast Model**

The results obtained by the computer program Nile1 shows that for 12 stations the average percentage error (A%E) in forecasting the values of the last five years annual rainfall did not exceed 20 %, and for 16 stations A%E is less than 25%. However, three stations at Butiaba, Kisumu and Muhoroni gave unsatisfactory results as their A%E are larger than 30%. Best results are obtained at Mbarara in Uganda, Tarime and Mwanza in Tanzania for which A%E are 10.18 %, 11.91 % and 11.24 % respectively. It was also found that using moving mean series has drastically improved the correlation coefficient. However, high values of (CC) will not always guarantee accurate forecasted values. It was also noticed that at each station values of (j) differed from one year to another and that the best fit equation was normally a polynomial of the 7 th to the 9 th degree. Meanwhile, the model failed to accurately forecast rainfall values whenever the actual values are beyond the minimum or maximum recorded values. Figures (3) and (4) illustrates the accuracy of the model results where the forecasted values are shown versus the measured annual rainfall values.

**7. Results Given by the Two Year Ahead Forecast Model**

As for the first year ahead forecasted values given by this model, table (3) shows that the A%E for Fort Protal and Mbarara did not exceed 10 %, less than 20 % for 9 stations, less than 25 % for 14 stations, while for 4 stations A%E is bigger than 30%. The worst results are also obtained at Butiaba, Kisumu, Muhoroni in addition to Katigondo.

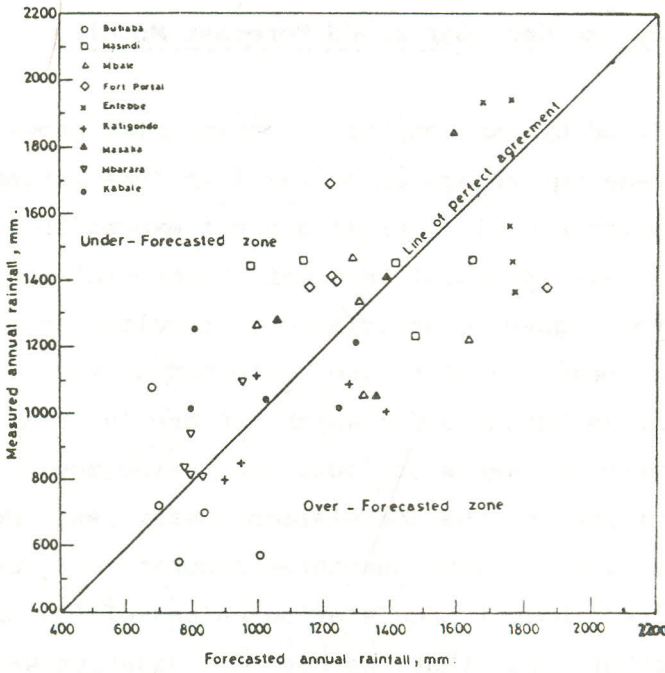


Fig.(3) Accuracy of One Year Ahead Forecasted Values Given by the First Model for Uganda Stations.

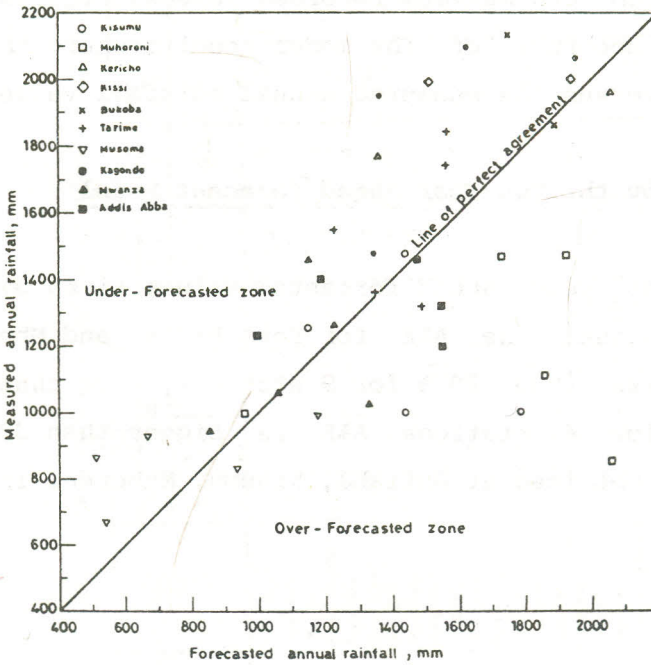


Fig.(4) Accuracy of One Year Ahead Forecasted Values Given by the First Model for Kenya and Tanzania Stations.



Moreover, the comparison of the one year ahead forecasted values obtained by the previous two models reveal that the second model has greatly improved the forecasted values for Fqrt Portal, Bukoba and Kagondo stations. However, similar results are obtained for Masindi, Entebbe, Mbarara and Kabale stations, while for the rest of the stations the first model is more accurate.

Furthermore, the average of the forecasted rainfall results given by the two models for the forecast of the first year ahead rainfall values are compared with the results given by each of the two models. The A%E for the average forecasted values listed in table (3) shows that the consideration of average values has drastically improved the forecasted results of Kisumu and Addis Abbaba satations as their A%E are reduced to 12.59 % and 11.06 % respectively. Figures (5) and (6) show the average forecasted values versus the measured annual rainfall. In summary, for the first year ahead forecasted values it is recommended to apply the first model for Mbale, Katigondo, Masaka, Kericho, Kissi, Tarime and Mwanza stations. The second model is to be used for Fort Portal, Mbarara, Bukoba and Kagondo stations while both models are to be applied and an average value be determined for Masindi, Entebbe, Kabale, Kisumu, Musoma and Addis Abbaba stations. Both models have failed to forecast satisfactory values for Butiaba and Muhoroni stations. The above recommendations will improve the forecasted values as the A%E will not exceed 10 % for 2 stations, less than 15 % for 8 stations and less than 20 % for 16 stations.

Regarding the A%E values for the second year ahead forcasted values, it can be seen that for 6 stations its value is less than 15 %, for 11 stations is less than 20 % and is less than 30 % for 15 stations. However, bad results are obtained at Butiaba, Katigondo, Kisumu and

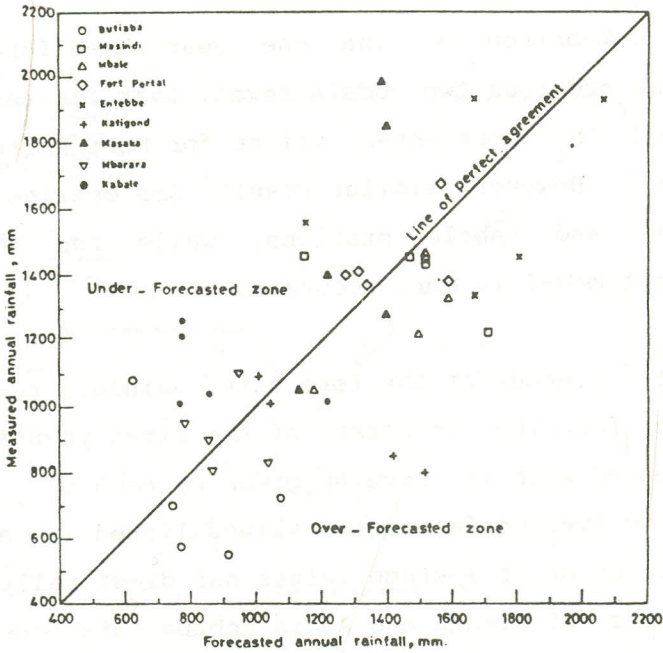


Fig.(15) Accuracy of One Year Ahead Forecasted Values Given by the Average of Both Models for Uganda Stations.

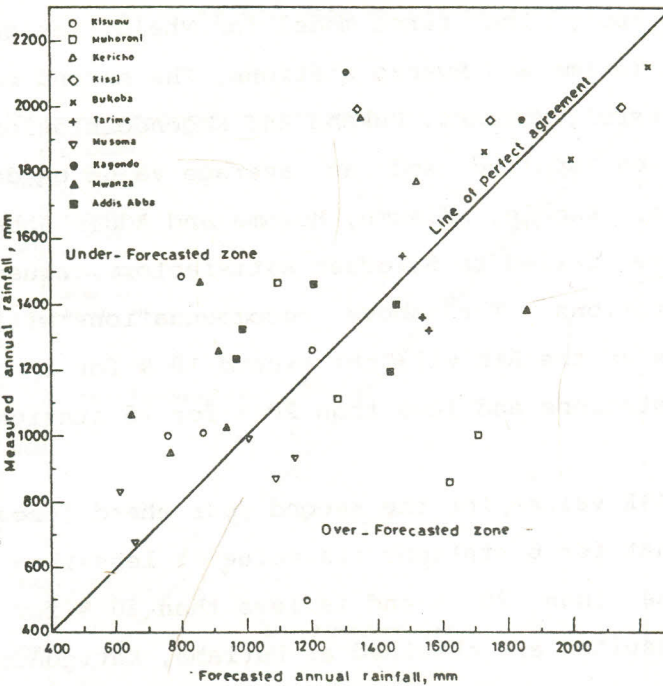


Fig. (6) Accuracy of One Year Ahead Forecasted Values Given by the Average of Both Models for Kenya and Tanzania Stations.

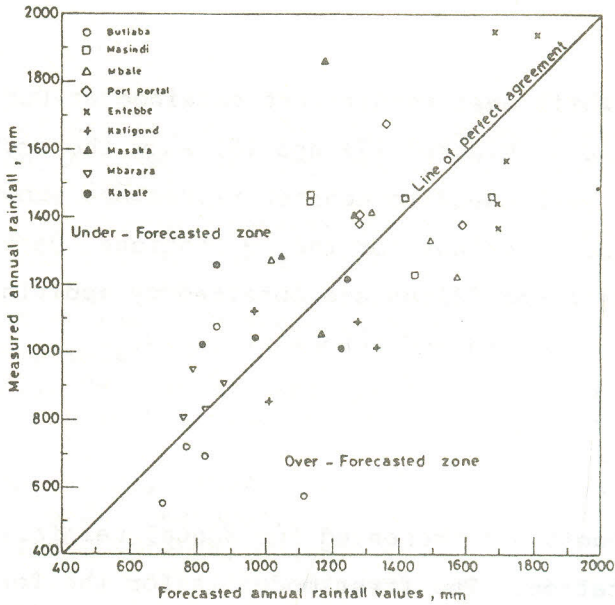


Fig.(7) Accuracy of Second Year Ahead Forecasted Values Given by the Second Model for Uganda Stations .

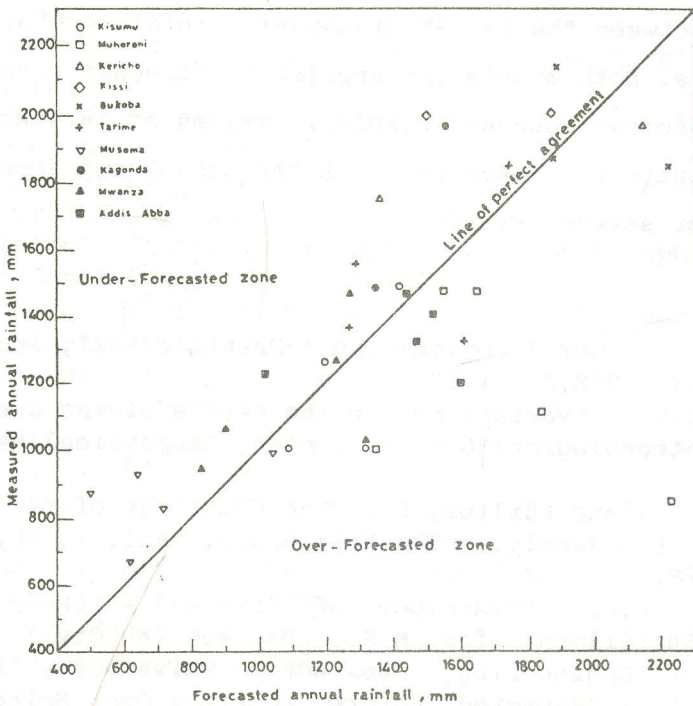


Fig.(8) Accuracy of Second Year Ahead Forecasted Values Given by the Second Model for Kenya and Tanzania Stations .

Muhoroni stations, while best results are obtained at Fort Portal for which A%E is only 7.94 %. Figures (7) and (8) show the accuracy of the model as the two year ahead forecasted values are shown versus the measured annual rainfall values for the 19 stations. Best results for forecasting two years ahead values are obtained by applying the second model at Fort Portal, Mbarara and Eukoba stations.

## 8. Conclusions

Two mathematical models are presented for annual rainfall forecast at a rainfall gauge station. The first model is for the forecast of one year ahead annual rainfall values while the second model is to forecast two years ahead annual rainfall values. The models for a rainfall station are based on an optimum dynamic moving mean values relationship between the series of annual rainfall and average annual sunspot numbers. Both models are applied to forecast each of the last five years recorded annual rainfall values at 18 stations located around the equatorial lakes and at Addis Abbaba and accurate results are obtained for several stations.

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