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DIGITAL MODELLING OF THE DYNAMIC STARTING AND PULL-IN PERFORMANCE OF VARIABLE RELUCTANCE STEP MOTORS

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ABSTRACT

This paper presents a method for estimating the dynamic starting performance and pull-in stepping rates of variables reluctance stepper motors. The model used is based on an equivalent circuit per phase and includes the effect of magnetic saturation, eddy current and multistep and single step operation at different switching modes. In order to be able to evaluate the model, simulation techniques have been used and described and some results are presented. The model is tested with data taken from an experimental prototype motor. The behaviour of the motor during synchronisation and attempted pull-in is examined. This enables the possible combination of inertia, load torque and pulse rate, that any given motor can synchronise, to be simply and reliably predicted. The model can entirely considered for routine design purposes.

NOMENCLATURE

a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4:	flux linkage coefficients
B	friction coefficient
i_k	current in phase k
J	moment of inertia
K	constant 1,2,3,4,...
L_1	total inductance per phase
Q	number of phases
Ri	iron loss resistive component
Rl	resistance per phase
S_r	number of rotor teeth
T_L	load torque
V_{pk}, V_1	applied voltage per phase
W_e	motor speed in elec. rad/sec.
$Y_o(i_k)$	average flux linkage of phase k
$Y_1(i_k)$	peak sinusoidal flux linkage of phase k
Y_k	Total flux linkage per phase
θ	rotor position in mech. rad.

1. INTRODUCTION

In the last decade attention has been focused (1-4) on the dynamic starting and pull-in performance of stepping motors at different inertial loads and high pulse rates. The transient behaviour and pull-in performance are very complex and their precise determination involves the computer solution of a set of electrical and mechanical non-linear differential equations. In addition the magnetic circuit must be treated essentially as 3 dimensional problem since the end effects can not usually be neglected. The presence of magnetic saturation and eddy currents and the dependence of motor inductance on both phase current and rotor position constitute great difficulties in the prediction of such performance.

It would be useful to be able to predict the dynamic response of such type of motors using information obtained from static and simple tests. The present paper shows that simple and direct equations can be established to predict the starting and pull-in capability under different operating conditions, if the effect of magnetic saturation and iron losses are suitably treated. The model employes an electrical equivalent circuit to represent one phase and the motor equations are solved using the Kutta-Merson method with fifth degree accuracy.

The main features of the equivalent circuit are:

- The true variation of the flux linkages with phase current and rotor position.
- The allowance for core eddy currents by including a suitably chosen resistive circuit coupled with the total coil flux.

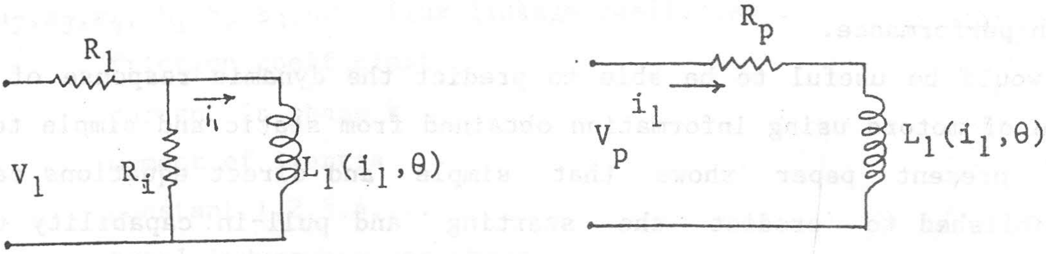
The paper attempts to assess the ability of the model to predict:

1. The multiphase single step and multistep response of the motor.
2. The limit of the response to high frequency pulse trains by considering the pull-in rate versus load torque characteristics at different load inertia.
3. Effect of the switching mode on the dynamic starting and pull-in performance.
4. Effect of the stator resistance on the response.

2. Model Equations

Fig. 1.a shows the equivalent circuit per phase of a variable reluctance step motor in which the eddy current loss is represented by a

resistive component coupled with the total flux per phase. Fig. 1.b gives the Thevenin's equivalent circuit across the inductance L_1.



a. b.

Fig. 1 equivalent circuit per phase

The Thevenin's voltage V_p and equivalent resistance R_p are given by

$$R_p = R_1 R_i / (R_1 + R_i) \quad \text{and} \quad V_p = V_1 R_i / (R_1 + R_i)$$

The voltage equation of phase K may be written in a general form as

$$V_{pk} = i_k R_p + \partial Y_k / \partial i_k . di_k/dt + \partial Y_k / \partial \theta . d\theta/dt \qquad (1)$$

2.1 Calculation of Currents

The 4-phase equations can be formulated in a matrix form as given in equation 2, where w_m is the rotor speed in mechanical rad/sec and equal to $d\theta/dt$. The matrix approach is always a suitable tool for separating the current derivative and for including the effect of mutual inductance between phases at a later stage.

$$\begin{bmatrix} V_{p1}-i_1R_p \\ V_{p2}-i_2R_p \\ V_{p3}-i_3R_p \\ V_{p4}-i_4R_p \end{bmatrix} = \begin{bmatrix} \partial Y_1/\partial i_1 & & & \\ & \partial Y_2/\partial i_2 & & \\ & & \partial Y_3/\partial i_3 & \\ & & & \partial Y_4/\partial i_4 \end{bmatrix} \begin{bmatrix} Pi_1 \\ Pi_2 \\ Pi_3 \\ Pi_4 \end{bmatrix} + \begin{bmatrix} \partial Y_1/\partial \theta & & & \\ & \partial Y_2/\partial \theta & & \\ & & \partial Y_3/\partial \theta & \\ & & & \partial Y_4/\partial \theta \end{bmatrix} \begin{bmatrix} w_m \\ w_m \\ w_m \\ w_m \end{bmatrix} \tag{2}$$

Equation (2) can be written as:

$$[V] = [L]\ [Pi] + [G]\ [W_m] \tag{3}$$

The current derivative can be obtained by matrix manipulation from

$$[Pi] = [L]^{-1}\ \{[V] - [G]\ [w_m]\} \tag{4}$$

The flux linkage variation with rotor position is assumed to be sinusoidal and can be expressed, in a general form, as

$$Y_k = y_o\ (i_k) + Y_1\ (i_k) \cos\ (\xi_k) \tag{5}$$

Where

$$\xi_k = (S_r \theta + 2\pi (k-1)/Q)$$

The parameters R_1, R_i, $Y_o(i_k)$ and $Y_1(i_k)$ are obtained from the static tests carried out on an experimental motor and are given in reference (5). Digital techniques for curve fitting are used to formulate the flux linkage as follows:

$$Y_o(i_k) = a_1\, i_k + a_2\, i_k^2 + a_3\, i_k^3 + a_4\, i_k^4$$
$$Y_1(i_k) = b_1\, i_k + b_2\, i_k^2 + b_3\, i_k^3 + b_4\, i_k^4 \tag{6}$$

The inductance variation with current and rotor position are then evaluated from

$$\partial Y_k / \partial i_k = (a_1 + 2a_2\, i_k + 3a_3\, i_k^2 + 4a_4\, i_k^3) +$$

$$(b_1 + 2b_2\, i_k^2 + 3b_3\, i_k^3 + 4b_4\, i_k^3)\ .\ \cos(\xi_k) \tag{7}$$

The flux linkage variation with rotor position can be obtained by differentiating equation (5) with respect to θ as follows

$$\partial y_k / \partial \theta = -S_r .\ Y_1(i_k).\ \mathrm{Sin}(\xi_k) \tag{8}$$

2.2 Calculation of torque and speed

The coenergy W associated with the motor can be obtained from

$$W = \sum_{k=1}^{Q} \int_0^{i_k} Y_k\,(i_k)\, di_k$$

$$= \sum_{k=1}^{Q} (a_1 i_k^2/2 + a_2 i_k^3/3 + a_3 i_k^4/4 + a_4 i_k^5/5) +$$

$$(b_1 i_k^2/2 + b_2 i_k^3/3 + b_3 i_k^4/4 + b_4 i_k^5/5) . \cos (\xi_k) \quad (9)$$

and the electromagnetic torque T_e is found from

$$T_e = \partial W / \partial \theta \;|_{i_k} = \text{constant}$$

which gives

$$T_e = \sum_{k=1}^{Q} -S_r (b_1 i_k^2/2 + b_2 i_k^3/3 + b_3 i_k^4/4 + b_4 i_k^5/5) \sin (\xi_k) \quad (10)$$

Equations (4) are solved digitally using the Kutta-Merson method with fifth order accuracy to obtained the motor currents i_1, i_2, i_3..., i_Q. The substitution into equation (10) gives the torque. The mechanical system equation is then incorporated to calculate the speed w_m and rotor position θ as follows:

$$T_e = J \, dw_m/dt + B \, w_m + T_L \quad (11)$$

and the speed derivative is found from

$$pw_m = (T_e - Bw_m - T_L)/J \quad (12)$$

Equation 12 is also solved numerically to get the motor speed w_m while the rotor position is obtained from

$$\theta = \int w_m \, dt \tag{13}$$

The load angle in electrical rad can be calculated from

$$\delta = w_e\, t + \theta + \theta_o \tag{14}$$

where w_e is the motor speed in electrical rad/sec or the angular frequency of the phase voltage. Appendix A shows the flow chart of the computer program.

3. Speed and mode of excitation

Fig. 2 gives the voltages applied to the motor phases at 3-different excitation modes and fixed pulse rate P_r. The conduction period T_a is given by $T_a = 1/P_r$. Inspection of Fig. 2 shows that the periodic time T is dependent on the mode of excitation, for instance at mode (1)

$T = Q\, T_a$ (see Fig. 2.a) while at mode (2) and (3) T is equal to $2Q\, T_a/3$ and $0.5Q\, T_a$ respectively. In general, the periodic time T may be expressed in the form

$$T = C\, Q\, T_a$$

where C is a constant dependent on the mode of excitation.

C = 1 in case of single phase excitation of mode (1)

C = 2/3 in case of single and two phase excitation of mode (2)

C = 0.5 in case of two phase excitation of mode (3)

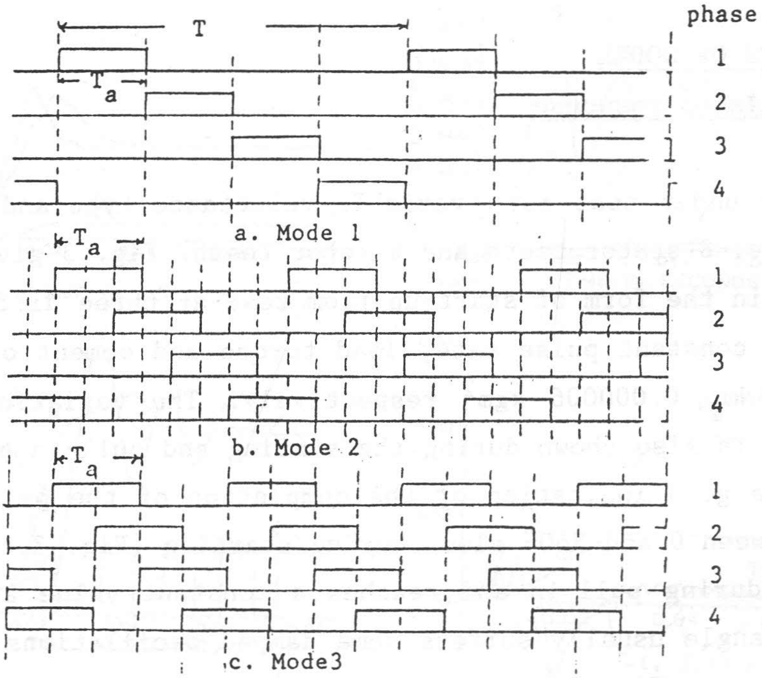


Fig. 2. Excitation modes

The angular frequency of the supply w_e is then

$$w_e = 2\Pi / C\, T_a\, Q \quad \text{or} \quad w_e = 2\Pi P_r / C\, Q \tag{15}$$

and the motor speed n in rev/min is given by

$$n = 60\, P_r / S_r\, C\, Q \tag{16}$$

Fig. 3 demonstrates the validity of equation 16, it shows the motor acceleration and pull-in at constant pulse rate of 200 P/S and load torque of 0.08 Nm. The motor reaches steady average speed of 500, 750, and 1000 rev/min at excitation modes 1, 2, and 3 respectively.

4. RESULTS OF MODEL

4.1 Multistep response

The motor under test is a variable reluctance type and rated 1 Nm, 15^{o} step angle, 8 stator teeth and 6 rotor teeth. Fig. 3 gives the multistep response in the form of start-up from rest at three different excitation modes and constant pulse rate, load torque and moment of inertia of 200 P/S, 0.08Nm, 0.000006 kgm^2 respectively. The variation of load angle with time is also shown during the staring and pull-in process. The load angle is a good indication of the completion of the pull-in process; it slips between 0 and 360^{o} elec. during starting (Fig. 3.c), tends to stop slipping during pull-in and reaches a constant value after pulling-in. The load angle usually suffers some damped oscillations before reaching steady state. Fig. 3.c shows the oscillations in load angle between -4 and 4^{o}. The motor experiences a pronounced oscillation in speed after synchronisation, it may reach $\pm$ 7% (Fig. 3.a) of the steady state speed especially at lower inertial loads. This oscillation can be greatly reduced by increasing the moment of inertia and phase resistance. Fifty percent increase in the phase resistance will reduce oscillation in speed to about 1%. The effect of phase resistance is shown in Fig. 3.a-c and Table (1) summarises this effect with 3 excitation modes. It should be noted that the higher values of phase resistance will reduce current, pull-out torque and steady state torque and increase the run-up time. Therefore a compromise has to be made in determining the value of the external resistance to be connected in series with the motor phase. The setting time is defined here as the time taken for the oscillation to decay so that the system is within 5%

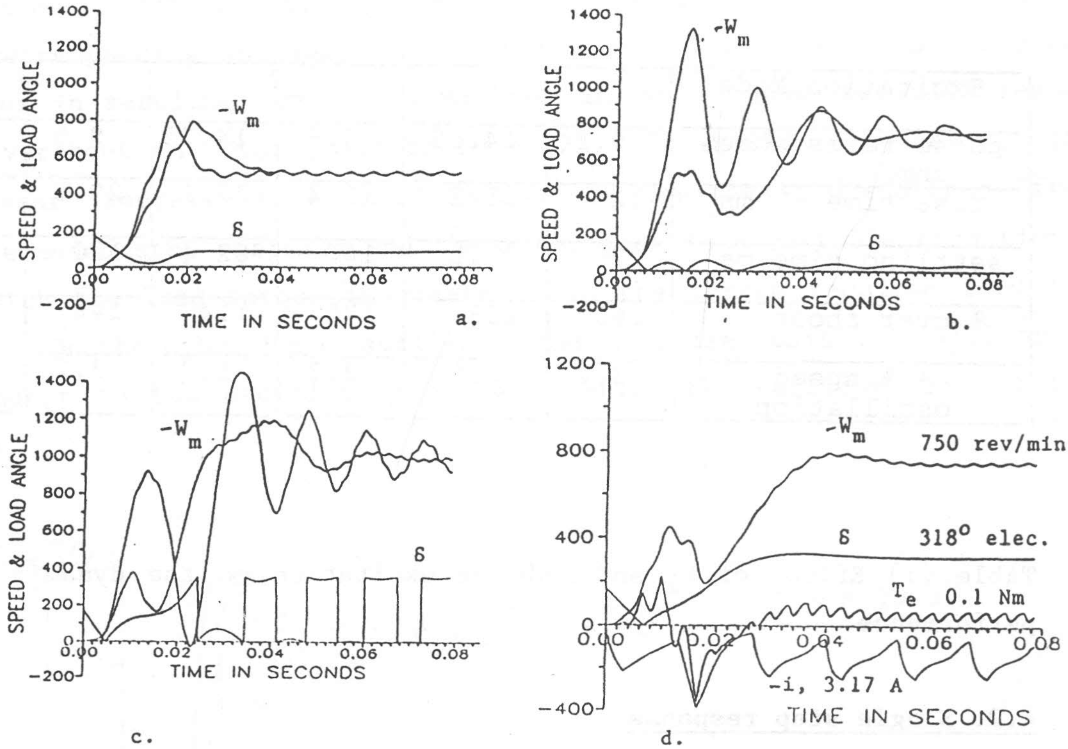


Fig. 3 Dynamic starting and pull-in at V=30 V, J=0.000006 kgm^2, T_L=0.08 Nm W_m in rev/min, S in elec. deg. - ______ R=9.5 Ohm- .... R=14.25 Ohm a. Mode (1), P_r=200 P/s b. Mode (1), P_r=200 P/s c. Mode (3), P_r = 200P/s d. Mode (1), P_r= 300 P/s

of the target. Fig. 3.d shows the speed, load angle torque and current of the motor during the dynamic starting and pull-in process at P_r = 300 P/s, T_L = 0.000006 kgm^2 and V_1 = 30 V with excitation mode (1). The effect of switching the phases can be clearly noticed in the ripples in torque and speed. The magnitude of these ripples may be reduced by increasing the phase resistance as shown in Fig. 3.a-c.

Excitation Mode	1		2		3	
phase resistance ohm	9.5	14.25	9.5	14.25	9.5	14.25
rise time ms	12	15.2	11.4	39.1	29.3	25
settling time ms	27	33.6	100	62	120	76
% over shoot	160	158	175	114	144	120
± % speed oscillation	3.2	2.8	3.3	1.7	3.2	1.3

Table (1) Effect of R_1 and mode of excitation on the dynamic starting performance.

4.2. Single step response

Fig. (4) presents the single step response of the motor with mode (1) and mode (3) excitation. The response of the mode (1) is simulated by switching on one phase from rest and tracing the subsequent changes in speed, rotor position, current and torque at T_L = 0.2 Nm, J = 0.000006 kgm^2 and V_1 = 30 V. The response of mode (3) is simulated by switching on two phases simultaneously from rest under the same operating conditions of mode (1). Table (2) gives a comparison between the two modes of excitation. It shows that mode (3) is superior to mode (1) since a reduction of 47.5% in setting time and 95% overshoot in rotor position has been achieved.

Fig. 5 gives the effect of phase resistance on the single step response characteristics. The increase of 20% in R_1 with mode (3) will lead to a reduction of 33% in settling time and of 19.4% in the speed overshoot. The corresponding increase in R_1 with mode (1) will not lead to great changes in settling time and overshoot in rotor speed but will reduce the overshoot in rotor position by 43.2%. It should be noted that the unnecssary increase in R_1 will lead to a significant disturbance from the demanded step position and the motor goes into unstable conditions, in which the load torque is greater than the torque required from the motor. On the other hand, smaller values of phase resistance will bring the motor to the oscillatory state in which the settling time is too large.

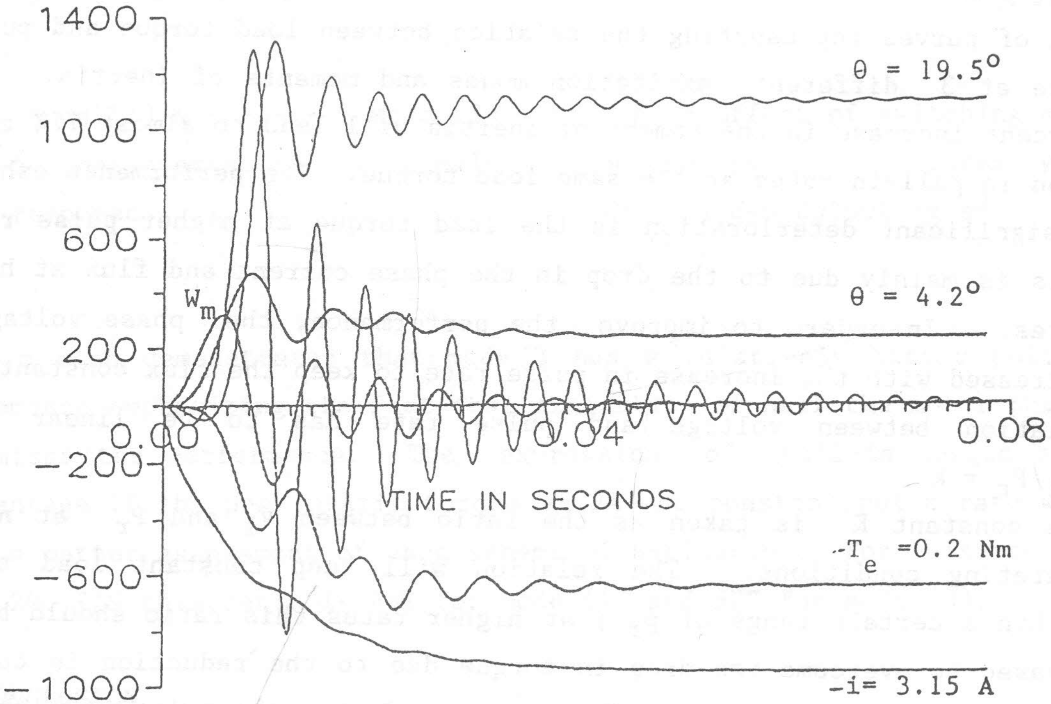


Fig. 4 Single step response at V=30, J=0.000002 kgm^2 & T_L= 0.2 Nm - W_m in rev/min. ___ Mode (1) _____ Mode (3)

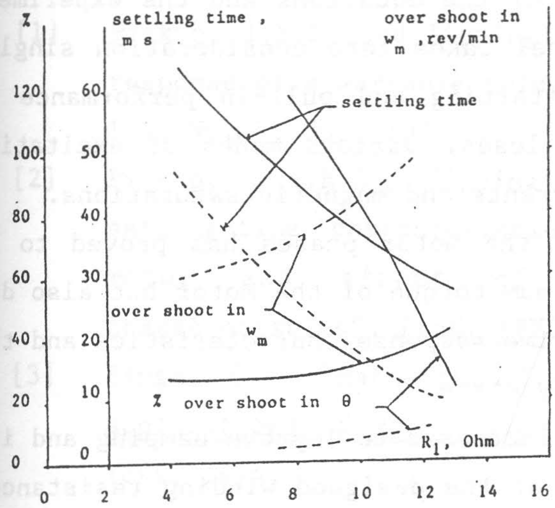
Mode	% overshoot in θ	overshoot in w_m rev/min	settling time, m_s
1	95	312	38.1
3	64	1200	20

Table (2) Effect of excitation mode on the single step response.

4.3 Pull-in performance

Fig. 6 shows the pull-in performance of the stepper motor in the form of set of curves representing the relation between load torque and pull-in rate at 3 different excitation modes and moments of inertia. Fifty percent increase in the moment of inertia will lead to almost 17% reduction in pull-in rates at the same load torque. The performance exhibits a significant deterioration in the load torque at higher pulse rates. This is mainly due to the drop in the phase current and flux at higher rates. In order to improve the performance, the phase voltage is increased with the increase in pulse rate to keep the flux constant; the relation between voltage and pulse rate has to be linear i.e., $V_1/P_r = K$.

The constant K is taken as the ratio between V_1 and P_r at normal operating conditions. The relation will keep constant load torque within a certain range of p_r ; at higher rates this ratio should be increased to overcome the drop in torque due to the reduction in current at higher switching rates. From the manufacturer's point of view, the increase of voltage at higher switching rates will be limited by the power requirements of the drive circuit.



____ Mode (1) & - - - - - Mode (3)

Fig. 5 Characteristics of single step response.

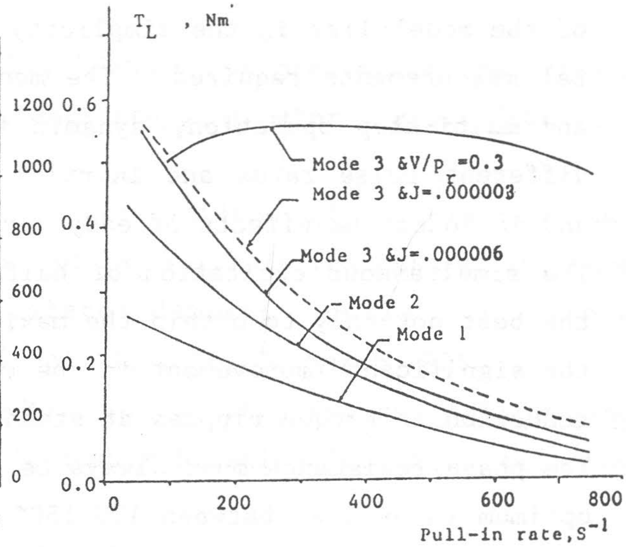


Fig. 6 Effect of switching mode on the pull-in performance at V=30V & J=0.000006 kg m^2.

Fig. 6 also demonstrates that mode 3 has a relatively better pull-in perfomance emphasizing the fact that excitation of half the motor phases optimises the performance. The expression of pull-in torque as a percentage if the peak static torque at 3A and constant pulse rate will give a better assessment of each scheme of excitation. For instance at P_r = 200 P/S this ratio is 45% for mode (1) and 57% for mode (3).

5. CONCLUSION

A model has been presented for predicting the dynamic starting and pull-in performance of variable reluctance step motors. The advantage of the model lies in the simplicity of the equations and the experimental measurements required. The model takes into consideration single- and multi-step operation, dynamic starting and pull-in performance at different pulse rates and inertial loads, various modes of excitation and includes the effects of eddy currents and magnetic saturations.

The simultaneous excitation of half the motor phases has proved to be the best not only to obtain the maximum torque of the motor but also due the significant improvement in the time response characteristics and the reduction in torque ripples at steady state.

The phase resistance must always be increased to improve damping and its optimum value lies between 120-150% of the designed winding resistance. A compromise has to be made between acceptable damping and rapid response.

The pull-in performance at relatively high pulse rates can be greatly improved if the voltage is increased linearly with pulse rate with a constant of proportionality equal to the ratio between V_1 and P_r at normal operating conditions. This constant has to be increased at higher pulse rates.

Further developments are required to include the effect of mutual inductance between phases and permanent magnets motors.

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