Alex. Eng. J., Alex. Univ., Volume 28, pp. 65 - 94 *(1989)*

STEERING CONTROL OF SHIPS-STATE SPACE AND ANALOG MODELS

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Abstract

The steering mechanism-ship system model was derived as a multi-variable directional control system. The analytical simulation includes the helm angle and hydrodynamic torque and disturbance as inputs, and the sway and yaw motions of the ship as outputs. Comprehensive state space model representations together with the solution were established. Further, an analog simulation model for the steering problem was developed.

Nomenclature

Steering control mechanism parameters

a	pinion to quadrant gear ratio
B_{μ}	coefficient of viscous friction in motor
E_c	feedback signal voltage
E_e	error signal voltage
E_f	voltage applied to motor's field
E_r	reference signal voltage
I_a	armature current (constant)
I_f	field current
J	mass polar moment of inertia of motor and rotating parts
K_c	feedback potentiometer's factor
K_r	input potentiometer's factor
K_m	motor's gain = $K_1' \, K_2'$
K_1	amplifier's gain
K_1'	proportionality constant of linearization
K_2'	proportionality constant of linearization
L_f	motor's field inductance
R_f	motor's field resistance
T	motor's driving torque
T_L	load torque
θ_c	angular position of armature or motor's shaft
θ_r	command signal = steering wheel angle
τ_f	motor's field time constant = $L_f \,/\, R_f$
ϕ	flux

Ship's parameters

I_{zz}	mass moment of inertia of ship about a vertical axis through the ship's centre of gravity
m	ship's mass
N	hydrodynamic moment acting on ship about a vertical axis through midship

N_r	$\frac{\partial N}{\partial r}$
N_v	$\frac{\partial N}{\partial v}$
N_δ	$\frac{\partial N}{\partial \delta}$

r	yaw rate
v	drift velocity
V	ship's resultant velocity
x_G	x-coordinate of ship's centre of gravity in xyz-system
Y	hydrodynamic force acting on ship perpendicular to her centre plane of symmetry
Y_r	$\frac{\partial Y}{\partial r}$
Y_v	$\frac{\partial Y}{\partial v}$
Y_δ	$\frac{\partial Y}{\partial \delta_r}$
δ_r	rudder angle

States of the system

$$X_1 = T$$

$$X_2 = \int_0^t (T - T_L)\, dt$$

$$X_3 = \theta_c$$

$$X_4 = v$$

$$X_5 = r$$

General

D	differeniating operator = d/dt
( )˙	differentiation w.r.t. time t
s	laplacian operator
t	time
xyz	coordinate system through midship; x pointing forward, y pointing to starboard, z pointing down

1. Introduction

The ship's response to directional control depends not only on the ship's dynamics but it is also affected by the characteristics of the steering mechanism.

It is evident that the yaw and sway of a ship, due to directional control or manoeuvre, are affected by the rudder angle which, in turn, depends on the helm angle. The rudder angle is affected also by the hyrodynamic and wave torque disturbance. Therefore, the steering system plays an important role in what concerns the directional stability and the dynamic behavior of the ship during manoeuvring. This justifies the importance of considering steering mechanism together with the ship dynamics for a thorough study of the directional control of a ship.

In [1] the rudder-ship system as a control plant was analysed by means of the state space technique. The application of this technique is further developed in the present work to include the whole control loop of the steering mechanism-rudder-ship system.

The adoption of the state space method renders an advantageous and powerful tool in treating dynamic and control systems with respect to digital and analog simulation and optimization problem.

2. Overview on Ship Steering Control Systems

Considerations to be taken into account while choosing the steering

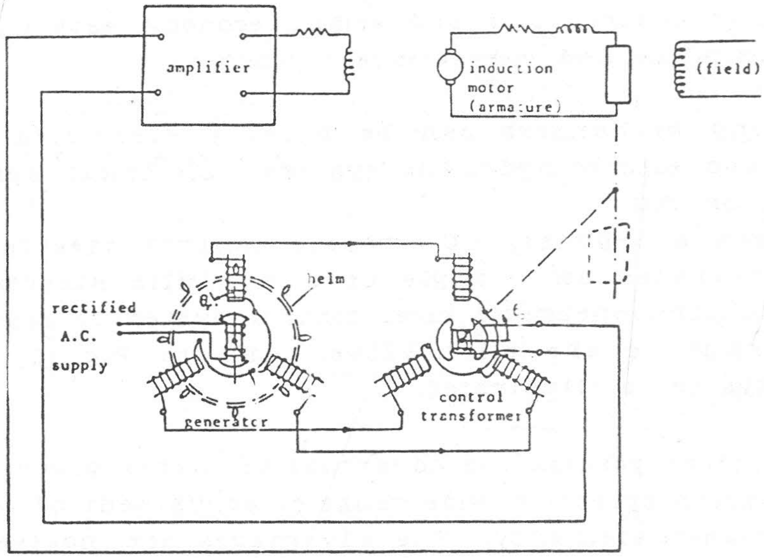

Fig. 1 A 3-phase, A.C. remote control ship steering servo-mechanism

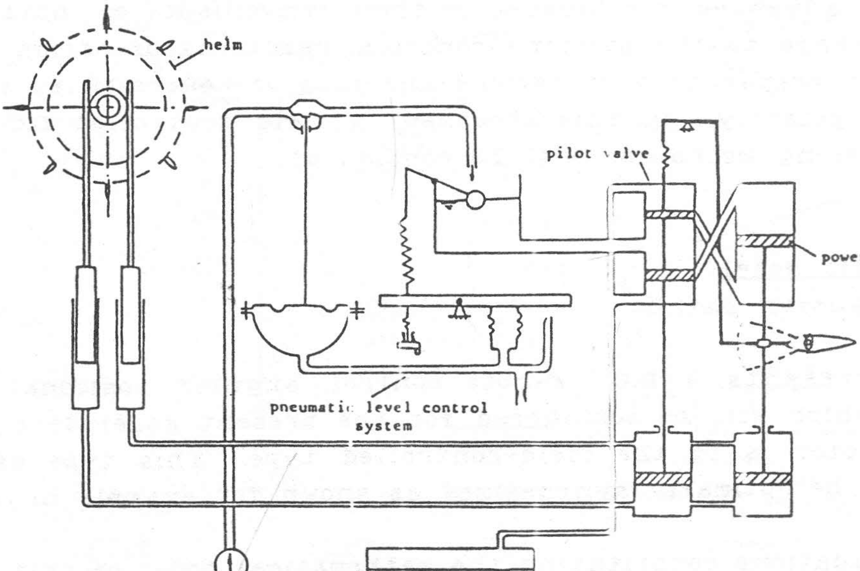

Fig. 2 A hydraulic ship steering servo-mechanism

control system of a ship are, briefly, reliability under any navigation conditions, the required power for rudder turning, convenience of control, size and weight, economy, ease of maintenance, quick reversibility and convenience of control.

Ship steering mechanisms can be broadly classified as electric, hydraulic and electro-hydraulic systems. Electrical systems may be either A.C. or D.C.
Fig. (1) shows a 3-phase, A.C. remote control steering mechanism. Fig. (2) illustrates an example of a hyraulic steering mechanism with an auxiliary pneumatic level control system to regulate the oil pressure input to the pilot valve. And in Fig. (3) an electro-hydraulic system is illustrsted.

Hydraulic systems possess the advantage of higher power transmission, silent and smooth operation, wide range of adjustment of rudder motion speed and higher efficiency. The advantages are, however, offset by the complicated manufacture, installation and adjustment as well as the required careful maintenance and servicing.

On the other hand, electrically operated steering mechanisms are, in many cases, advantageous because of their convenience of installation, reliable linkage to the steering controls, readiness of action at any time without preparatory operations and ease of reversibility through changes of polarity. In this treatment a field controlled D.C. remote control steering mechanism will be considered.

3. Mathematic Model

3.1 The transfer matrix:

Fig. (4) represents a D.C. remote control, angular postional servo-mechanism which will be considered for the present model development. The D.C. motor is of the field-controlled type. This type of motor control can be optimally synthesized as shown for example in [2].

The basic equations constituting the mathematical model of this system could be derived refering to e.g. [3]. The linearized forms of these equations could be summarized as follows:

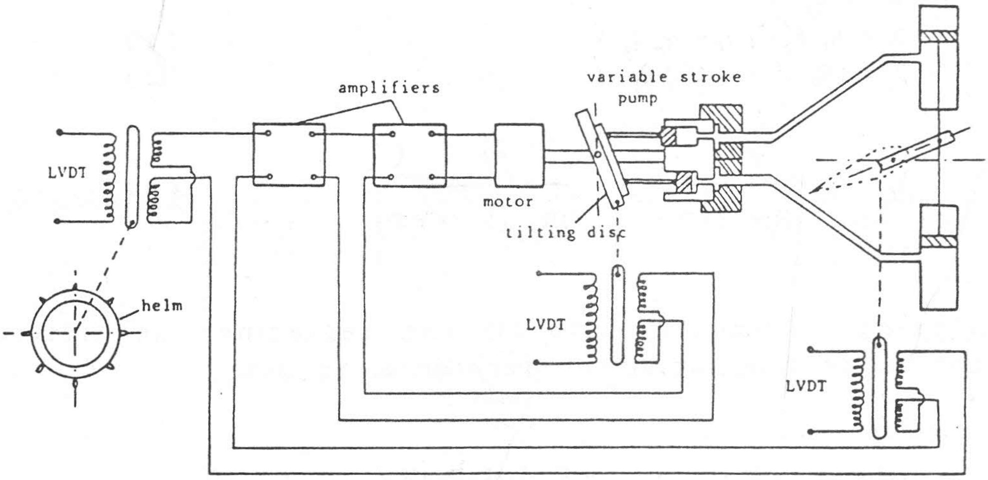

Fig. 3 An electro-hydraulic ship steering servo-mechanism

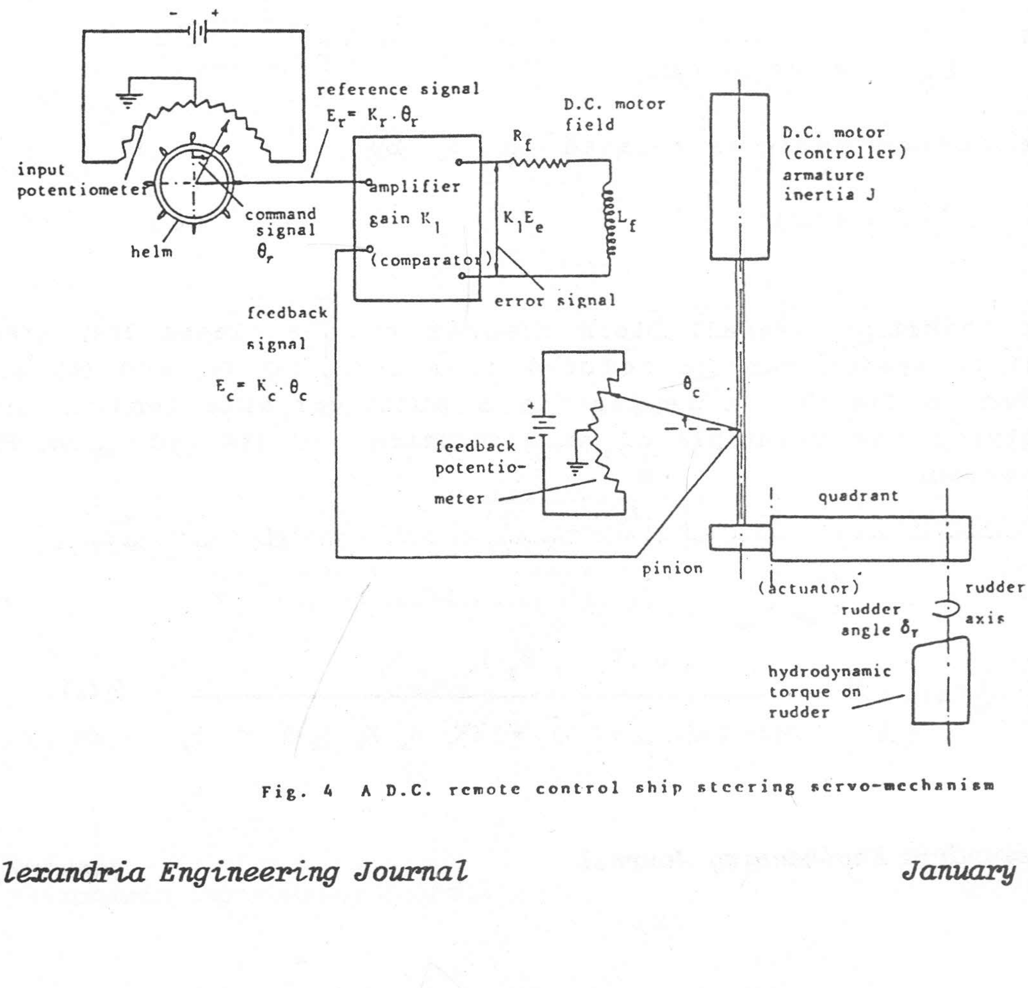

Fig. 4 A D.C. remote control ship steering servo-mechanism

$$T = K_1' \varphi I_a,$$
$$\varphi = K_2' I_f$$
$$T = K_1' K_2' I_a I_f = K_m I_a I_f \quad (1)$$
$$T - T_L = (B_\nu D + J D^2)\,\theta_c \quad (2)$$

$$I_f = \frac{E_f}{R_f + L_f D} = \frac{E_f}{R_f\,(1 + \tau_f D)} \quad (3)$$

Substituting (1) and (2) into (3) and replacing the differential operator D by the laplacian operator s, we get

$$\theta_c(s) = \frac{1}{s(B_\nu + J.s)} \left[\frac{K_m I_a E_f / R_f}{1 + \tau_f . s} - T_L \right] \quad (4)$$

and

$$E_f(s) = K_1 [K_r \theta_r(s) - K_c \theta_c(s)] \quad (5)$$

The rudder angle is related to θ_c by

$$\delta_r(s) = a\theta_c(s) \quad (6)$$

The resulting overall block diagram of the closed loop steering control system can be deduced from Eqns. (4), (5) and (6), and is shown in Fig.(5). It represents a multi-variable control system. Applying the principle of superposition for its reduction, Fig.(6), we obtain;

$$\delta_r(s)\Big|_{T_L=0} = \frac{a\;K_r\;K_1\;K_m\;I_a\;/\;R_f}{s(1+\tau_f s)(B_\nu + J.s) + (K_1\;K_m\;K_c\;I_a)\;/\;R_f} \cdot \theta_r(s)$$

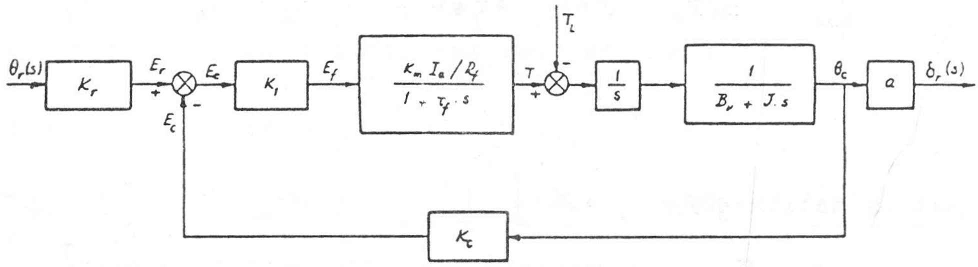

Fig. 5 Block diagram of the steering mechanism shown in Fig. 4

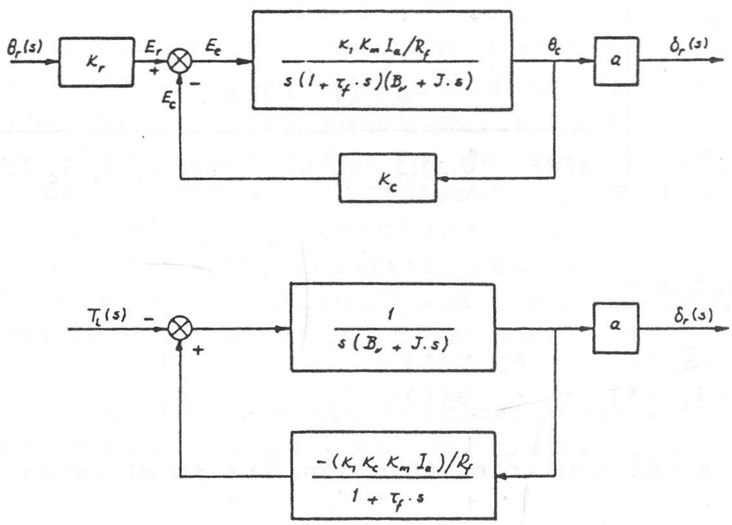

Fig. 6 Reduction of the block diagram shown in Fig. 5 by the superposition principle

and

$$\delta_r(s)\Big|_{\theta_r=0} = \frac{-a\ (1+\tau_f.s)}{s(B_\nu + J.s)(1+\tau_f s) + (K_1\ K_m\ K_c\ I_a)\ /\ R_f} .\ T_L\ (s)$$

or put in matrix form

$$\delta_r(s) = \begin{bmatrix} \dfrac{a.K_r\ K_1\ K_m\ I_a\ /\ R_f}{s(B_\nu + J.s)(1+\tau_f.s) + (K_1\ K_m\ K_c\ I_a)/R_f} \\ \\ \dfrac{-a\ (1+\tau_f.s)}{s(B_\nu + J.s)(1+\tau_f.s) + (K_1\ K_m\ K_c\ I_a)/R_f} \end{bmatrix}^T \begin{bmatrix} \theta_r(s) \\ \\ T_L(s) \end{bmatrix} . \qquad (7)$$

For step inputs

$$\theta_r\ (t) = A_1\ u(t)$$
$$T_L\ (t) = A_2\ u(t)$$

where u(t) is the unit step function, the final value theorem yields

$$\lim_{t\to\infty} \delta_r(t) = \lim_{s\to 0}\ s\ \delta_r(s) = a\left[A_1 + \frac{A_2}{(K_1\ K_r\ K_m\ I_a)/R_f}\right]$$

The second term in this expression tends to be negligible compared to the first one, hence

$$\lim_{t \to \infty} \delta_r(t) \approx a \ . \ A_1$$

In [1] the ship dynamic model was derived, namely

$$\begin{bmatrix} (m - Y_{\dot{v}}) & (mx_G - Y_{\dot{r}}) \\ (mx_G - N_{\dot{v}}) & (I_{zz} + mx_G^2 - N_{\dot{r}}) \end{bmatrix} . \begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} -Y_v & (mV - Y_r) \\ -N_v & (mx_G V - N_r) \end{bmatrix} . \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} Y_\delta \\ N_\delta \end{bmatrix} \delta_r \qquad (8)$$

for convenience of the following manupulation, Eqn. (8) is written as:

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} \\ \bar{\alpha}_{21} & \bar{\alpha}_{22} \end{bmatrix} . \begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} \bar{\beta}_{11} & \bar{\beta}_{12} \\ \bar{\beta}_{21} & \bar{\beta}_{22} \end{bmatrix} . \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \end{bmatrix} . \delta_r \qquad (9)$$

Eqn. (9) is in a dimensional form. The treatment of the problem of ship dynamics, based on the so called hydrodynamic derivatives [4,5,6] which appear in the governing equations, is usually carried out in a nondimentional form. This is due to the adoption of model testing technique for determining the hydrodynamic properties of full scale ships, where nondimensional quantities simplify the correlation of the results. In this work, however, the mathematical model is to simulate both the steering mechanism and the dynamics of a full scale ship. Therefore it is convenient to treat both systems in dimensional form.

In order to obtain the overall transfer matrix of the steering mechanism-ship system, Eqns. (7) and (9) are combined. This yields:

$$\begin{bmatrix} v(s) \\ r(s) \end{bmatrix} = \begin{bmatrix} \dfrac{(\bar{\alpha}_{22}\bar{\gamma}_1 - \bar{\alpha}_{12}\bar{\gamma}_2).s + (\bar{\beta}_{22}\bar{\gamma}_1 - \bar{\beta}_{12}\bar{\gamma}_2)}{\Delta} \\ \dfrac{(\bar{\alpha}_{11}\bar{\gamma}_2 - \bar{\alpha}_{21}\bar{\gamma}_1).s + (\bar{\beta}_{11}\bar{\gamma}_2 - \bar{\beta}_{21}\bar{\gamma}_1)}{\Delta} \end{bmatrix} \begin{bmatrix} \dfrac{(a K_r K_1 K_m I_a)/R_f}{\Delta_1} \\ \dfrac{-a(1 + \tau_f . s)}{\Delta_1} \end{bmatrix}^T . \begin{bmatrix} \theta_r(s) \\ T_L(s) \end{bmatrix}$$

$$\begin{bmatrix} v(s) \\ r(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\{(\bar{\alpha}_{22}\bar{\gamma}_1 - \bar{\alpha}_{12}\bar{\gamma}_2).s + (\bar{\beta}_{22}\bar{\gamma}_1 - \bar{\beta}_{12}\bar{\gamma}_2)\}\{(a K_r K_1 K_m I_a)/R_f\}}{\Delta . \Delta_1} & -\dfrac{\{(\bar{\alpha}_{22}\bar{\gamma}_1 - \bar{\alpha}_{12}\bar{\gamma}_2).s + (\bar{\beta}_{22}\bar{\gamma}_1 - \bar{\beta}_{12}\bar{\gamma}_2)\}\, a(1 + \tau_f . s)}{\Delta . \Delta_1} \\ \dfrac{\{(\bar{\alpha}_{11}\bar{\gamma}_2 - \bar{\alpha}_{21}\bar{\gamma}_1).s + (\bar{\beta}_{11}\bar{\gamma}_2 - \bar{\beta}_{21}\bar{\gamma}_1)\}\{(a K_r K_1 K_m I_a)/R_f\}}{\Delta . \Delta_1} & -\dfrac{\{(\bar{\alpha}_{11}\bar{\gamma}_2 - \bar{\alpha}_{21}\bar{\gamma}_1).s + (\bar{\beta}_{11}\bar{\gamma}_2 - \bar{\beta}_{21}\bar{\gamma}_1)\}\, a(1 + \tau_f . s)}{\Delta . \Delta_1} \end{bmatrix} . \begin{bmatrix} \theta_r(s) \\ T_L(s) \end{bmatrix}$$

$$\Delta = (\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21}).s^2 + (\bar{\alpha}_{22}\bar{\beta}_{11} + \bar{\alpha}_{11}\bar{\beta}_{22} - \bar{\alpha}_{12}\bar{\beta}_{21} - \bar{\alpha}_{21}\bar{\beta}_{12}).s + (\bar{\beta}_{11}\bar{\beta}_{22} - \bar{\beta}_{12}\bar{\beta}_{21})$$

$$\Delta_1 = s(B_v + J.s)(1 + \tau_f . s) + (K_1 K_c K_m I_a)/R_f$$

or

$$\begin{bmatrix} v(s) \\ r(s) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} . \begin{bmatrix} \theta_r(s) \\ T_L(s) \end{bmatrix} \quad (10)$$

All the roots of the overall characteristic equation $\Delta . \Delta_1 = 0$ should have negative real parts as a condition for absolute stability. Eqn.(10) is graphically displayed in a block diagram from in Fig.(7).

3.2 State space modelling

State variables describing the state of a system need not be physically measurable or observable quantities. They may be any mathematical variables. However, it is convenient to choose the state variables as measurable quantities in order to facilitate the optimal directional control for course keeping, whereby feedback gyros are to be used and all the states of the system are fed back.

It is worth mentioning to state that the number of state variables is equal to the degree of the characteristic equation. The states represent the minimum number of variables which describe completely the system, provided that their initial values are known together with the inputs of the system at initial time.

Refering to the block diagram shown in Fig.(8), the states are selected to be :

$X_1 = T$	driving torque of the motor.
$X_2 = \int_0^t (T - T_L) dt$	integral of the net torque
$X_3 = \theta_c$	angular position of the motor shaft
$X_4 = v$	drift velocity
$X_5 = r$	yaw rate

Refering to the same figure, the following equations could be deduced for the steering mechanism part :

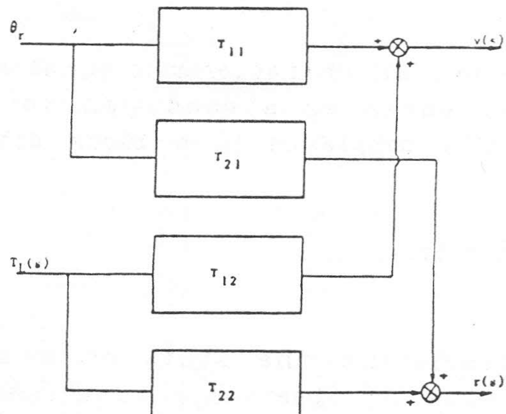

Fig. 7 Block diagram of the steering mechanism-ship system (Eqn. (10))

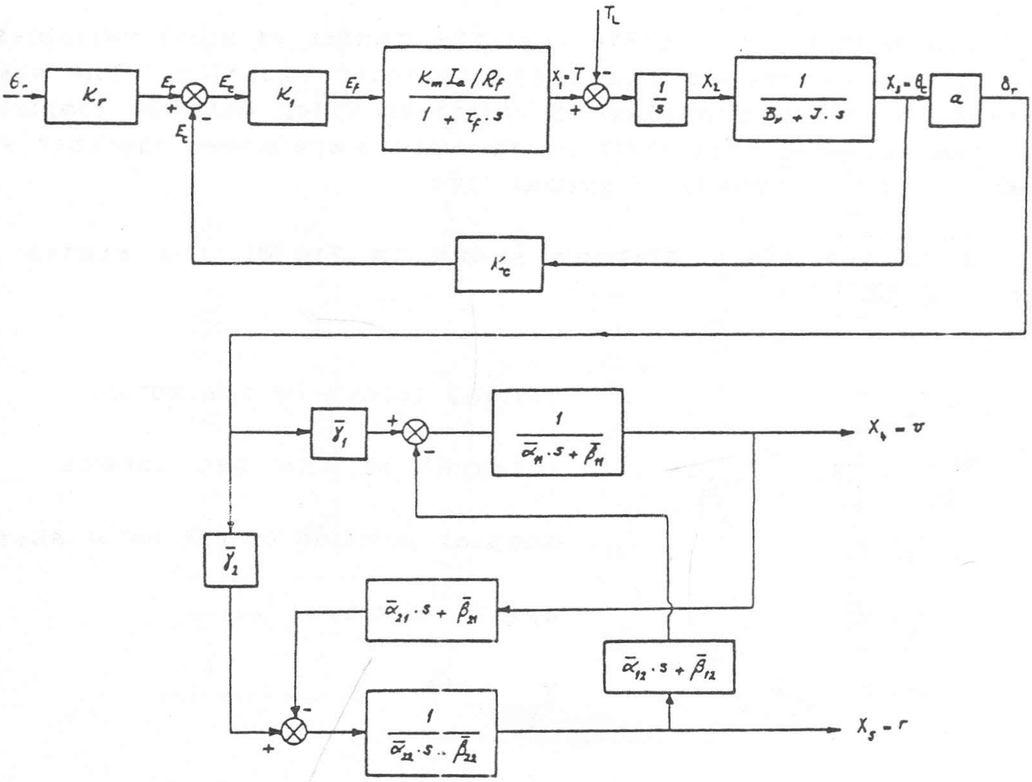

Fig. 8 Overall block diagram of the steering mechanism-ship system from Eqns. (4), (5), (6) and (9)

$$E_f = (K_r \theta_r - K_c X_3) K_1$$

$$\frac{X_1}{E_f} = \frac{K_m I_a / R_f}{1 + \tau_f . s}$$

$$\frac{X_2}{X_1 - T_L} = \frac{1}{s}$$

$$\frac{X_3}{X_2} = \frac{1}{B_\nu + J.s}$$

From these relations the following state variable equations are obtained :

$$\dot{X}_1 = (K_r \theta_r - K_c X_3) \frac{K_1 K_m I_a}{\tau_f \cdot R_f} - \frac{1}{\tau_f} X_1 \qquad (11)$$

$$\dot{X}_2 = X_1 - T_L \qquad (12)$$

$$\dot{X}_3 = \frac{1}{J} (X_2 - B_\nu X_3) \qquad (13)$$

The rudder angle δ_r is given by

$$\delta_r = a X_3$$

The states for the ship dynamics were derived in [1]. The equations to be used here are hence completely analogous to those in [1] except that the coefficients here are dimensional, and denoted by "-".

$$\dot{X}_4 = \frac{1}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} \Big\{ a(\bar{\alpha}_{22}\bar{\gamma}_1 - \bar{\alpha}_{12}\bar{\gamma}_2) X_3 + (\bar{\alpha}_{12}\bar{\beta}_{21} - \bar{\alpha}_{22}\bar{\beta}_{11}) X_4 + $$
$$ + (\bar{\alpha}_{12}\bar{\beta}_{22} - \bar{\alpha}_{22}\bar{\beta}_{12}) X_5 \Big\} \qquad (14)$$

$$\dot{X}_5 = \frac{1}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} \Big\{ a(\bar{\alpha}_{11}\bar{\gamma}_2 - \bar{\alpha}_{21}\bar{\gamma}_1) X_3 + (\bar{\alpha}_{21}\bar{\beta}_{11} - \bar{\alpha}_{11}\bar{\beta}_{21}) X_4 + $$

$$+ (\bar{\alpha}_{21}\bar{\beta}_{12} - \bar{\alpha}_{11}\bar{\beta}_{22}) X_5 \Big\} \qquad (15)$$

The outputs of the overall control system are :

$$\left.\begin{aligned} v &= X_4 \\ r &= X_5 \end{aligned}\right\} \qquad (16)$$

and the vector $[X_4 \ X_5]^T$ is identical to the vector $[y_1 \ y_2]^T$ in [1].

From Eqns. (11), (12), (13), (14) and (15) the state matrix equations can be written as follows:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_f} & 0 & \frac{-K_c K_1 k_m I_a}{R_f \cdot \tau_f} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{J} & -\frac{B_v}{J} & 0 & 0 \\ 0 & 0 & \frac{a(\bar{\alpha}_{22}\bar{\gamma}_1 - \bar{\alpha}_{12}\bar{\gamma}_2)}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} & \frac{(\bar{\alpha}_{12}\bar{\beta}_{21} - \bar{\alpha}_{22}\bar{\beta}_{11})}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} & \frac{(\bar{\alpha}_{12}\bar{\beta}_{22} - \bar{\alpha}_{22}\bar{\beta}_{12})}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} \\ 0 & 0 & \frac{a(\bar{\alpha}_{11}\bar{\gamma}_2 - \bar{\alpha}_{21}\bar{\gamma}_1)}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} & \frac{(\bar{\alpha}_{21}\bar{\beta}_{11} - \bar{\alpha}_{11}\bar{\beta}_{21})}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} & \frac{(\bar{\alpha}_{21}\bar{\beta}_{12} - \bar{\alpha}_{11}\bar{\beta}_{22})}{(\bar{\alpha}_{11}\bar{\alpha}_{22} - \bar{\alpha}_{12}\bar{\alpha}_{21})} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} +$$

$$+\begin{bmatrix} \frac{K_r K_1 K_m I_a'}{R_f \cdot \tau_f} & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta_r \\ T_L \end{bmatrix} \qquad (17)$$

or $$[\dot{X}] - [P][X] + [B][U] \qquad (18)$$

From Eqn. (16) the output matrix equation can be written as follows:

$$\begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta_r \\ T_L \end{bmatrix} \qquad (19)$$

or

$$[C] = [L]\,[X] + [E]\,[U] \qquad (20)$$

where [E], in our case, is the null matrix since there exists nc feedforward signals in the system. Eqns. (18) and (20) are represented in the form of generalized state space block diagram and signal flow graph in Figs. (9) and (10), respectively.

The transfer matrix of the system given in Eqn. (10) could be synthesized as [7,8]

$$[L]\,[s[I] - [P]]^{-1}\,[B] + [E],$$

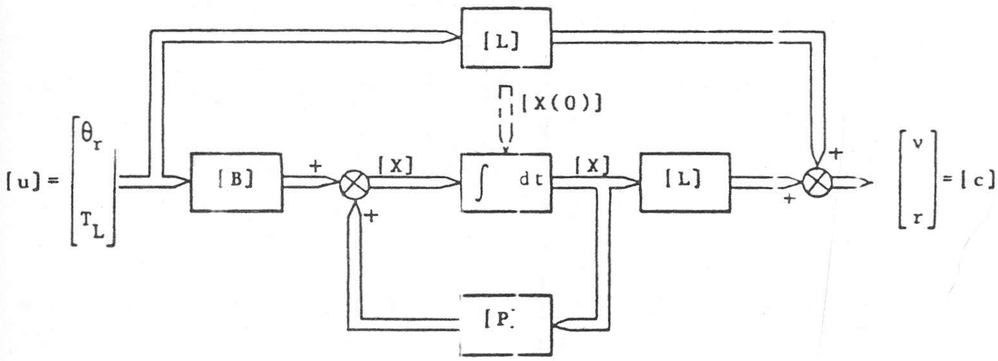

Fig. 9 Generalized state space block diagram

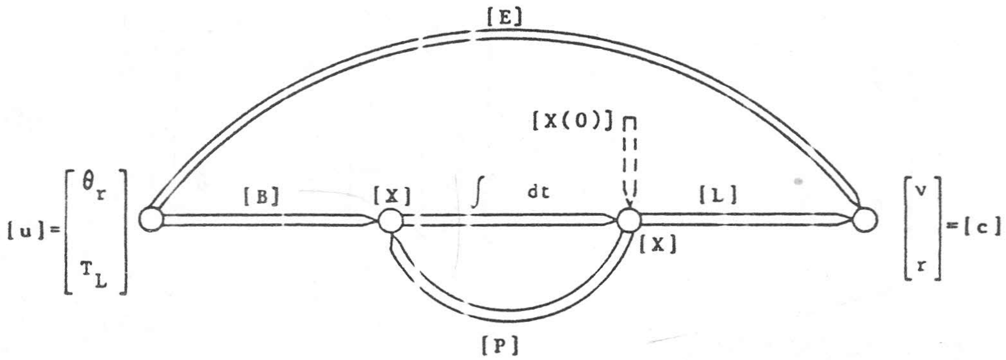

Fig. 10 Generalized state space signal flow graph

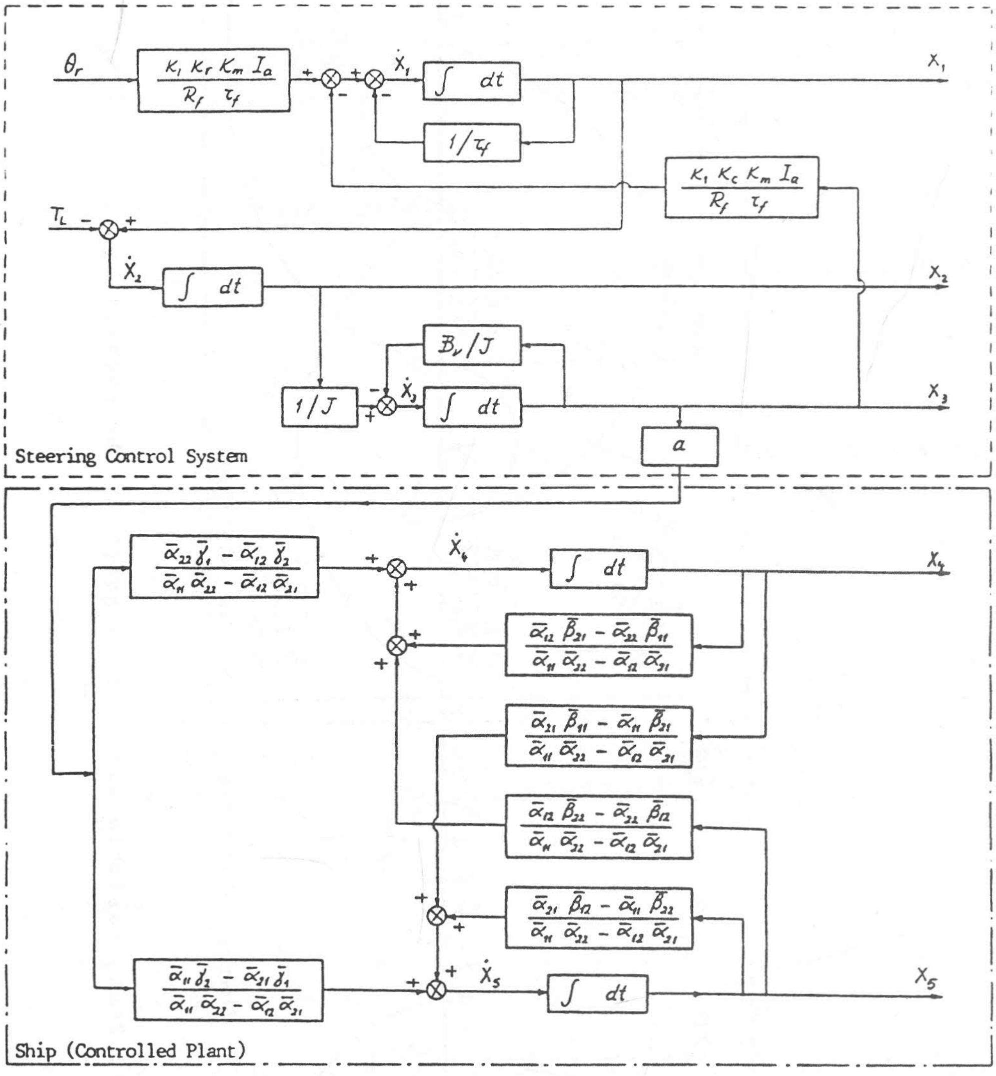

Fig. 11 State variable block diagram of the overall system

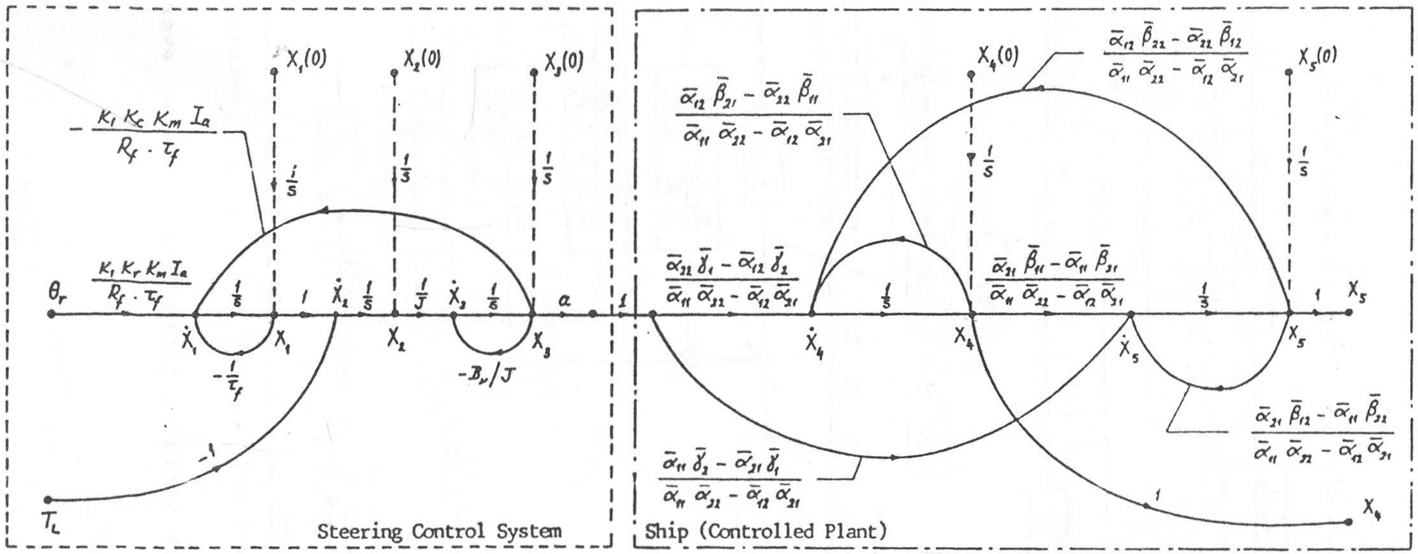

Fig. 12 State variable signal flow graph of the overall system

where [I] is the unity matrix of the same order of [P].

Figs. (11) and (12) represent, the state variable block diagram and state variable signal flow graph of the overall system, respectively.

3.3 Solution of the state space model

The solution in the time domain requires the determination of the transition matrix [7,8]

$$[\phi(t)] = \mathcal{L}^{-1} [s[I] - [P]]^{-1} .$$

For numerical computation, however, the transition matrix is preferably expressed in the form :

$$[\phi(t)] = e^{[P]t} = \sum_{k=0}^{\infty} [P]^{k} t^{k} / k! .$$

According to Leverrier's algorithm [9] the roots of the characteristic equation $\Delta . \Delta_1 = 0$, i.e. the system poles, are identical with the eigenvalues of the matrix [P],i.e. the roots of the equation det [s[I] - [P]] = 0,and with the poles of det $[\phi(s)] = 0$.

The general solution of Eqn. (18) for the states of the system could be proved to be expressable as [7,8,9]

$$[X(t)] = [\phi(t)][\phi(t_0)]^{-1}[X(t_0)] + [\phi(t)]\int_{\tau=t_0}^{\tau=t}[\phi(\tau)]^{-1}[B][U(\tau)]d\tau ,$$

where t_0 is the initial time $t \geqslant 0$ and $t \geqslant t_0$. For the manipulation of the above equation the following properties of the transition matrix [9] are useful:

The general output vector [c] is given by :

$$[c] = [L][X] + [E][U],$$

where in our case, as [E] is zero,

$$[c] = [L][X].$$

Hence,

$$[c] = [L][\phi(t)][\phi(t_0)]^{-1}[X(t_0)] + $$
$$+ [L][\phi(t)]\int_{\tau=t_0}^{\tau=t}[\phi(\tau)]^{-1}[B][U(\tau)]d\tau .$$

For a ship commencing a manoevre from a straight line course at constant velocity, all the states of the system at $t_0 = 0$ are zero, hence

$$[c] = \begin{bmatrix} v(t) \\ r(t) \end{bmatrix} = \int_{\tau=0}^{\tau=t}[L][\phi(t - \tau)][B][U(\tau)]\; d\tau. \qquad (21)$$

For a general element $\phi_{i,q}(s)$ of $[\phi(s)]$ and using the Heaviside's expansion theorem, the following can be written, whereby the subscripts i and q are dropped for convenience :

$$\phi(s) = \frac{\sum_{i=0}^{m} b_i \,.\, s^i}{\prod_{i=1}^{r}(s + p_i)^{n_i}} = \varepsilon_0 + \sum_{i=1}^{r}\sum_{k=1}^{n_i}\frac{C_{ik}}{(s + p_i)^k},$$

where

$$C_{ik} = \frac{1}{(n_i - k)!}\,\frac{d^{n_i - k}}{ds^{n - k}}\left[(s + p_i)^{n_i} \,.\, \phi(s)\right]\Big|_{s = -p_i}$$

where

- m = degree of numerator of $\phi(s)$,
- r = number of poles of $\phi(s)$,
- p_i = the i-th pole of $\phi(s)$, real or complex,
- n_i = number of repetitions of p_i,
- k = the k-th multiplicity of p_i,
- C_{ik} = the Heaviside's coefficient corredponding to pole p_i at its k-th repetition,
- a_0 = constant resulting from dividing the numerator by denominator when both are of the same degree, otherwise $a_0 = 0$.

Through inverse Laplace transform, the element 1,q of the matrix $\phi(t-\tau)$ is given by

$$\phi(t-\tau) = a_0\delta(t-\tau) + \sum_{i=1}^{r}\sum_{k=1}^{n_i}\frac{C_{ik}}{(k-1)!}(t-\tau)^{(k-1)}e^{-p_i(t-\tau)}$$

where $\delta(t)$ is the Dirac function.

The element p of the output vector [c(t)] is given by:

$$c_p(t) = \int_{\tau=0}^{\tau=t}\left(\sum_{i=1}^{5} L_{pi}\sum_{k=1}^{5}\phi_{ik}(t-\tau)\sum_{j=1}^{2}B_{kj}U_j\right)d\tau \,, \qquad (22)$$

where p = 1,2.

The diagonalization technique may be used for expressing the output vector [c] in case of distinct eigenvalues of the system. The state vector [X] of the system is expressed in terms of another vector [Y] through the transformation:

$$[X] = [A]\;[Y]$$

The transformation matrix [A] (modal matrix) is chosen such that the product

$$[A]^{-1}\,[p][A]$$

yields a diagonal matrix [11].

The output vector [c(t)] is given by:

$$[c(t)] = [L][A]\, e^{[A]^{-1}[P][A]t}\, [A]^{-1}[X(0)] + [L][A] \int_{\tau=0}^{\tau=t} e^{[A]^{-1}[P][A](t-\tau)}\, [A]^{-1}\, [B][U(\tau)]\, d\tau \qquad (23)$$

For determining the states of the system through digital computation the finite difference method may be applied. This can be given in the form:

$$[X((m+1)\tau)] = (\tau[p] + [I])[X(m\tau)] + \tau[B][U(m\tau)], \qquad (24)$$

where

$m = 0,1,2,\ldots\ldots$

τ = discretizing time interval, i.e. $t = m\tau$.

The output vector is then given by

$$[c((m+1)\tau)] = [L][X((m+1)\tau)]. \qquad (25)$$

4. Analog Simulation Model

Another mathematical simulation of the dynamics of the steering mechanism-ship is developed using an analog model.

Linear analog circuits are used to simulate the elements of the transfer matrix represented by the block diagram in Fig. (8). The problem of needing to build differentiators in the circuit was encountered. It is well known that a conventional differentiator of D_0 -property, Fig. (13a), tends to amplify noise, drift and hence should be avoided in analog simulation. There exists, however, several alternative approximations of the conventional differentioator

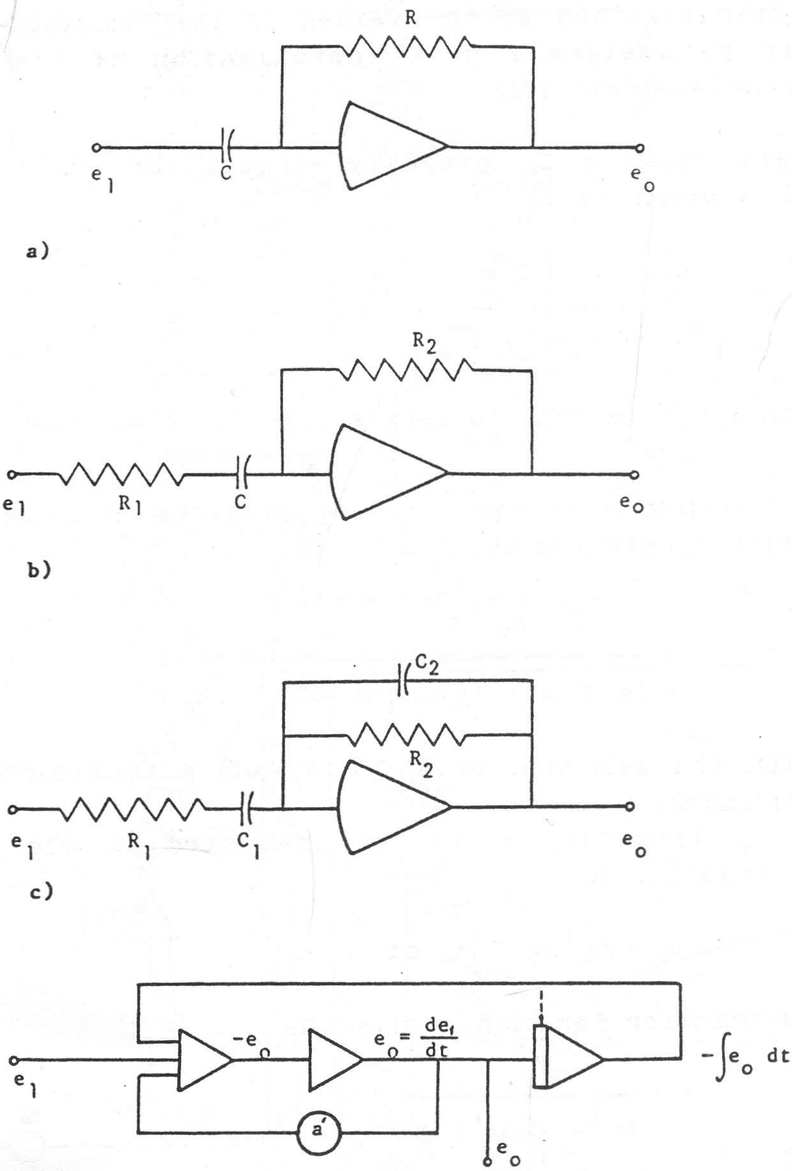

Fig. 13 Differentiating element (a) and its improved alternatives (b), (c) and (d)

which are more stable, and do not amplify signal noise.
With proper selection of the values of the resistances, capacitances or other parameters a good approximation of the differentiating process is achieved [10].

Fig. (13b) shows a D_1 -property circuit for which the transfer function is given by

$$-\frac{R_2 C s}{R_1 C s + 1}$$

By choosing $R_1 C$ tending to zero a true differentiator is approached.

Another alternative circuit, with D_2 -property, is shown in Fig.(13c). Its transfer function is given by

$$-\frac{R_2 C_1 s}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

Again, with the selection of $R_1 C_1 = R_2 C_2 \approx 0$, a true differentiator can be accomplished.
Another D_1 -property circuit is presented in Fig. (13d), whose integral equation is;

$$e_0 = a' e_0 - \int_0^t e_0 \, dt + e_1$$

and whose transfer function is given by

$$-\frac{s}{(1 - a')s + 1}$$

With a chosen value of a' in the proximity of unity a true differentiator is realized.
The circuits shown in Fig. (13c,d) were adopted for the analog simulation model of the steering mechanism-ship system shown in Fig.(14). In accordance with the coefficients appearing in the block

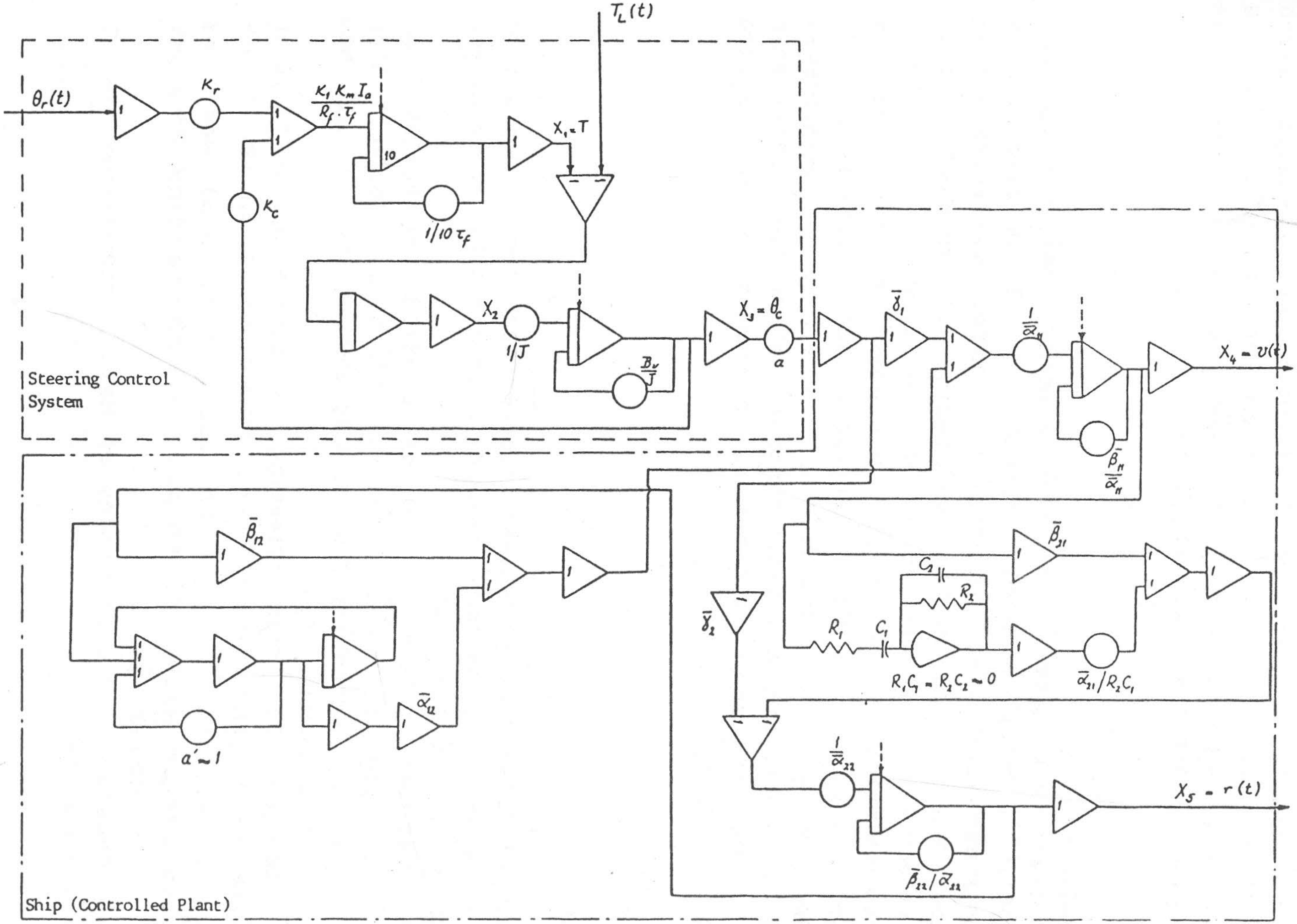

Fig. 14 Analog simulation model of the overall system

diagram, Fig.(8), positive signs of all variables were considered in the analog circuit. At the time of execution, any negative valued variables should be taken into account in the circuit by changing their corresponding polarity. Amplitude and, if necessary, time scaling should be performed as well.

5. Conclusion

A thorough state space model of the steering mechanism-ship system was mathematically developed. Several forms of the block diagram and signal flow graph were given. The state and output matrix equations were derived. The solution in the time domain was presented yielding the drift velocity and yaw rate of the ship in response to helm angle and hydrodynamic loading torque on the rudder. While the steering control loop is a closed one, the rudder-ship control loop is an open loop. Further improvement to course keeping qualities may be achieved by adding a subsidiary automatic control loop to the rudder-ship system [12]. Another dynamic model through analog circuit was also simulated.

References

[1] A.M. El Iraki and M.H.Soliman, "State Space Representation of Ship Steering Dynamics", Alexandria Engineering Journal, Vol. 28, No.1, January 1989.

[2] M. Pachter, "Speed Control of a Field Controlled D.C. Traction Motor", Automatica, Vol. 17, No.4, pp 627-630, 1981.

[3] F.H. Raven, Automatic Control Engineering, McGraw-Hill, New York, 2nd Edition, 1968.

[4] H. Eda and C.L. Crane, "Steering Characteristics of Ships in Calm Water and Waves", Trans. SNAME, Vol.73, pp 135-177,1965.

[5] A. Ogawa and H.Kasai, "On the Mathematical Model of Manoeuvring Motion of Ships", Int. Shipbuilding Progress, Vol. 25, No.292, pp 306-319, 1978.

[6] J.P. Comstock (Ed.), Principles of Naval Architecture, SNAME, New York, 1974.

[7] K. Ogata, State Space Analysis of Control Systems, Prentice Hall Inc., New Jersy, 1967.

[8] F. Csáki, State Space Methods for Control Systems, Akadémiai Kiadó, Budapest, 1977.

[9] D.M. Wiberg, State Space and Linear Systems, McGraw-Hill, New York, 1971.

[10] A.S. Jackson, Analog Computation, McGraw-Hill, New York, 1960.

[11] R. Zurmühl, Matrizen, Springer, Berlin, 1964.

[12] T. Koyama, "Improvement of Course Stability by the Subsidiary Automatic Control", Int. Shipbuilding Progress, Vol. 19, No. 212, April 1972.