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ON ORIENTATION PROBLEMS IN PHOTOGRAMMETRY

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Abstract

This paper introduces new suitable coordinate systems of reference to solve orientation problems in photogrammetry. Solution of such problems yield image coordinates as well as certain orientation parameters. The so called equations of transformation from the terrain-as an inclined plane-to the image plane are derived. A chosen orientation problem is solved accordingly both analytically and graphically. The graphical solution depends on the principals of descriptive geometry.

Nmenclature

A : Principal point of image plane.

B : Point of intersection (ρ , Z-axis).

c : Focal length

O : Centre of projection

OA : Camera axis.

P : Point on the terrain

P_1 : Image of P

X_P, Y_P, Z_P : Space coordinates of P.

X_{P1}, Y_{P1}, Z_{P1} : Space coordinates of P_1.

x,y : Ordinary image coordinate system

x_1, y_1 : Image coordinate system.

Z_o : Hight of fly.

π : Terrain plane.

π_1: Horizontal plane of reference.

ρ : Image plane

ν : Angle between OA and Z-axis. (Tilt angle).

φ : Angle between CA and Y-axis.

θ : Swing angle (angle between (x,y) and (x_1, y_1) - axes).

1. Introduction

The so called orientation problems play a very important role in photogrammetry. The main profits of solving orientation problems are deriving formulae for the determination of the image coordinates of the image point P_1 of any space point P on the terrain as well as some orientation elements such as tilt angle ν and the swing angle θ, when sufficient data are known. Several coordinate systems are used depending on the type of photos [1], [2], [3].

New suitable coordinate systems of reference are established in this paper. The surface of the terrain is approximated to an inclined plane since relatively small domains of the earth are concerned. Equations of transformation from the plane to the image plane were then derived on the assumption that the photographic process is a central projection from lens as centre of projection O, On the image plane ρ as plane of projection. Analytical solution of an orientation problem is derived to determine some orientation elements when sufficient data are given. Numerical values for the given data are assumed then graphical solution using principals of descriptive geometry is presented. The results of the graphical solution are then compared with those obtained by numerical substitutions in the analytical solution.

2. Coordinate Systems

Since the terrain is represented on the image by central projection, the properties of this representation depend on the shape of the terrian, the position of the centre of projection and the direction of the camera axis at the moment of exposure, but it is independent of any chosen coordinate system. In this work the terrain will be considered as an inclined plane [4].

2.1 Space coordinate system of reference

The following systems of reference are chosen and have been proved to be very suitable to solve the problems which are dealt with hereafter.

In Figure (1) let :

O : the centre of projection,

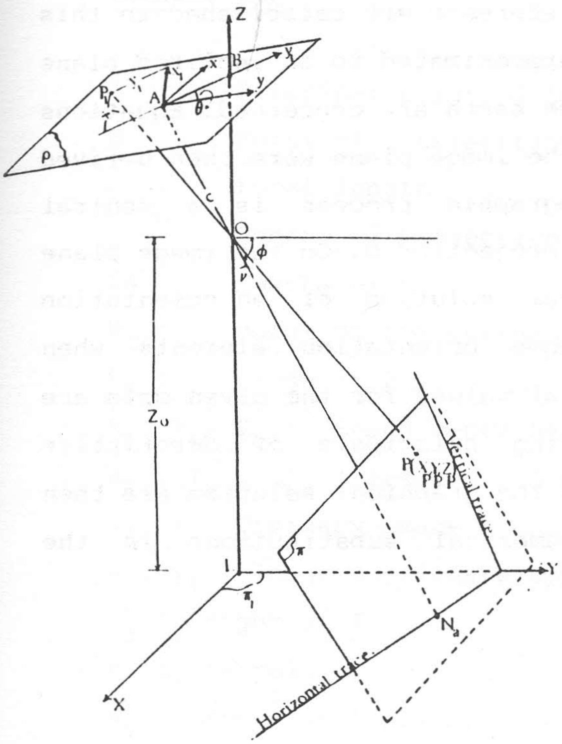


Fig. 1

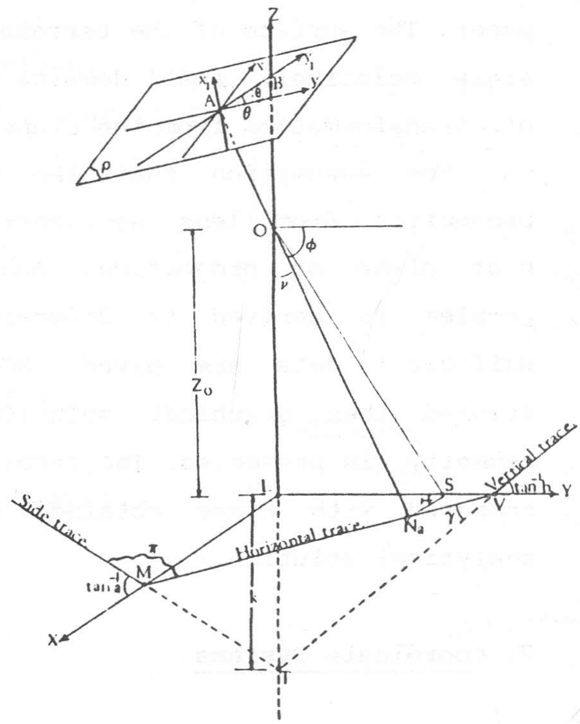


Fig. 2

OA : the camera axis

π : the terrain plane,

ρ : the image plane,

ν : the angle between OA and the Z-axis,

B : the point of intersection (ρ , Z),

The vertical line through O is the Z-axis of reference system. The camera axis OA meets π in N_a through which passes the horizontal plane of reference π_1 (XY-plane). π_1 meets the Z-axis in L. The Y-axis passes through L making an arbitrary angle φ with the camera axis OA where $\varphi + \nu \geq 90^{\circ}$. The X-axis will be determined.

2.2 Image coordinate system

The image plane ρ passes through A and is perpendicular to the camera axis OA. In ρ, the following two coordinate systems are considered.

i) The ordinary image coordinate system with A as origin and the lines joining the fudicial marks as x-and y-axes .

ii) The image coordinate system is consisted of the y_1-axis which is the line AB and x_1-axis which is perpendicular to it through A.

2.3 Representation of the surface of the terrain

The surface of the terrain is generally topographic which can not be represented analytically. In areal survey by single image measurements, it is sufficient to consider this surface as a plane, since relatively small domains of earth are considered.

In this work, this plane is denoted by π and its equation is,

$$Z = aX + bY - k \tag{1}$$

The choice of an inclined plane instead of the usually taken horizontal plane as an approximating surface is a better approximation to the real surface of the terrain and also leads to more accurate results.

In Figure (2), MVT is the trace-traingle of π and the angle TVM is denoted by γ, then after equation (1)

$$VM = (k\sqrt{a^2 + b^2})/ab\ , \quad VT = (k\sqrt{1 + b^2})/b,$$

$$MT = (k\sqrt{1 + a^2})/a \tag{2}$$

$$\overline{MT}^2 = \overline{VT}^2 + \overline{VM}^2 - 2\overline{VT}.\ \overline{VM}\cos\gamma \tag{3}$$

$$\text{Then } \gamma = \cos^{-1}[a/\sqrt{(1+b^2).\ (1 + a^2)}\ ] \tag{4}$$

$$\overline{TN}_a^2 = Z_o^2 \tan^2\nu + k^2 \tag{5}$$

$$\overline{VN}_a^2 = (k^2/b^2) - (2kZ_o \cos\varphi/b\cos\gamma) + Z_o^2\tan^2\nu$$

$$\text{But } \overline{TN}_a^2 = \overline{VN}_a^2 + \overline{VT}^2 - 2\overline{VN}_a\ .\ \overline{VT}\cos\gamma \tag{7}$$

Substitution from equations (2), (4), (5) and (6) into equation (7) yields :

$$k = Z_o\ (b\cos\varphi \pm a\sqrt{\sin^2\nu - \cos^2\varphi})\ /\ \cos\nu \tag{8}$$

Equation (1) of π becomes

$$Z = aX + bY - Z_o\ (b\cos\varphi \pm a\ \sqrt{\sin^2\nu - \cos^2\varphi})/\cos\nu \tag{9}$$

2.4 Equations of transformation

By equations of transformation it is meant the equations which express the following coordinates in terms of the space coordinates (X_p, Y_p, Z_p) of any point P in π :

i) The space coordinates (X_{P1}, Y_{P1}, Z_{P1}) of the image P_1 of P

ii) The image coordinates (x_1, y_1) of P_1 in ρ .

iii) The ordinary image coordinates (x,y) of P_1 in ρ

These equations are derived on the assumption that photographic process is a central projection from the lens as point O on ρ as the plane of projection.

2.4.i Determination of the coordinates (X_{P1}, Y_{P1}, Z_{P1}):

In figure (1) let OA = c (Camera constant or the focal length), the coordinates of O are $(0,0,Z_o)$ and the coordinates of A are

$$(-c\sqrt{\sin^2\nu - \cos^2\varphi}, -c\cos\varphi, Z_o + c\cos\nu) \tag{10}$$

Equation of the plane ρ is

$$\alpha_1 (X - X_A) + \beta_1 (Y - Y_A) + \gamma_1 (Z - Z_A) = 0 \tag{11}$$

where the direction ratios of the camera axis OA are

$$(\alpha_1, \beta_1, \gamma_1) = (\sqrt{\sin^2\nu - \cos^2\varphi}, \cos\varphi, -\cos\nu) \tag{12}$$

Then equation (11) will be

$$X\sqrt{\sin^2\nu - \cos^2\varphi} + Y\cos\varphi - Z\cos\nu + Z_o\cos\nu + c = 0 \tag{13}$$

Parametric equations of OP are:

$$\begin{aligned} X &= tX_P \quad , \quad 0 \le t \le \infty \ , \\ Y &= tY_P \\ Z &= t(Z_P - Z_o) + Z_o \end{aligned} \tag{14}$$

Solution of equations (12) and (13) yields

$$
\begin{aligned}
X_{P1} &= -\,cX_P/H \\
Y_{P1} &= -\,cY_{P/H} \\
Z_{P1} &= Z_o + c(Z_o - Z_P)/H,
\end{aligned} \tag{15}
$$

$$H = X_P \sqrt{\sin^2\nu - \cos^2\varphi} + Y_P \cos\varphi + (Z_o - Z_P)\cos\nu$$

2.4.ii **Determination of the coordinates (x_1, y_1) of the image P**

The y_1-axis is the line AB where the coordinates of B are $(0,0,Z_o + c\sec\nu)$ and its direction ratios are

$$(\alpha_2, \beta_2, \gamma_2) = (\sqrt{\sin^2\nu - \cos\varphi^2}, \cos\varphi, \sin\nu\tan\nu) \tag{16}$$

In the image plane ρ Fig. (3):

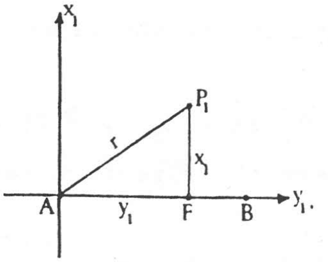


Fig. (3)

$$\overline{P_1A}^2 = (X_A - X_{P1})^2 + (Y_A - Y_{P1})^2 + (Z_A - Z_{P1})^2, \tag{17}$$

$$y_1^2 = [\alpha_2(X_A - X_{P1}) + \beta_2(Y_A - Y_{P1}) + \gamma_2(Z_A - Z_{P1})]^2/(\alpha_2^2 + \beta_2^2 + \gamma_2^2) \tag{18}$$

Solution of equations (15), (16), (19) yields:

$$y_1 = \pm\, c\{\cot\nu - [(Z_o - Z_P)\operatorname{cosec}\nu/H]\}, \tag{19}$$

Since $x_1 = \pm\sqrt{r^2 - y_1^2}$ (20)

then $x_1 = \pm\, c\,(X_P \operatorname{cosec}\nu\cos\varphi - Y_P \operatorname{cosec}\nu\sqrt{\sin^2\nu - \cos^2\varphi})/H$ (21)

2.4.iii Determination of the coordinates (x,y) of the image P_1:

Since $x = x_1 \cos\theta + y_1 \sin\theta$,

$y = y_1 \cos\theta - x_1 \sin\theta$,

Then using (19), (21) yields

$$x = \pm (c \cos\theta / H) . (X_P \operatorname{cosec}\nu \cos\theta - Y_P \operatorname{cosec}\nu \sqrt{\sin^2\nu - \cos^2\varphi})$$

$$\pm c \sin\theta [\cot\nu - ((Z_o - Z_P)/H) . \operatorname{cosec}\nu]$$

$$y = \pm c \cos\theta [\cot\nu - ((Z_o - Z_P)/H) \operatorname{cosec}\nu] \qquad (22)$$

$$\mp (c \sin\theta / H) [X_P \operatorname{cosec}\nu \cos\varphi - Y_P \operatorname{cosec}\nu \sqrt{\sin^2\nu - \cos^2\varphi}]$$

3. Analytical and graphical solution of an orientation problem

Here we introduce the analytical solution of an orientation problem applying directly the above derived equations. This solution is followed by a graphical solution depend mainly on the principals of descriptive geometry using numerical values. The results are then compared with those obtained by numerical substitutions in the formulae derived in the analytical solution.

Given are: ν , φ , Z_o, (X_P, Y_P, Z_P) of P and r of P_1,

where $r = \sqrt{x^2 + y^2}$

Required are: (X_{P1}, Y_{P1}, Z_{P1}), (x_1, y_1) of P_1, c and θ .

3.1 The analytical solution Figure (4)

1. The determination of the coordinates (X_{P1}, Y_{P1}, Z_{P1}):

Point P_1 has two locii in space. The first is the line OP whose parametric equations are given in (14). The second is the right circular cylinder with OA as axis and its equation is:

$$X^2 + Y^2+(Z-Z_o)^2-[\alpha_1 X+ \beta_1 Y+\gamma_1 (Z-Z_o)]^2/(\alpha_1^2+\beta_1^2+\gamma_1^2) = r^2 \qquad (23)$$

where $r = \sqrt{x^2 + y^2}$ is the radius of the cylinder and α_1, β_1 and γ_1 are the direction cosines of the line OA given by equation (12).

The intersection of these two locii gives two positions of P_1 as:

$$X_{P1} = \pm rX_P/E$$

$$Y_{P1} = \pm rY_P/E$$

$$Z_{P1} = Z_o \pm r(Z_P - Z_o)/E, \qquad (24)$$

$$E^2=X_P^2+Y_P^2+(Z_o-Z_P)^2-[Y_P \cos\varphi+X_P \sqrt{\sin^2 \nu - \cos^2 \varphi}$$
$$+ (Z_o- Z_P) \cos \varphi]^2$$

The required point P_1 satisfies the condition $Z_P > Z_o$.

2. The determination of the focal length c.

In Figure (4) : $c^2 = \overline{OP}_1^2 - r^2$

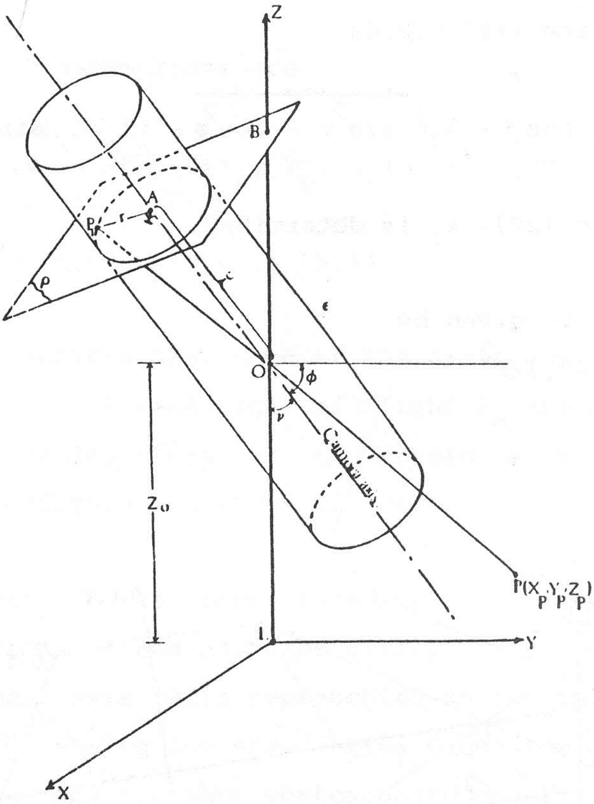


Fig. 4

where $\overline{OP}_1^2 = X_{P1}^2 + Y_{P1}^2 + (Z_{P1} - Z_o)^2$

Hence :

$$c = r\,[Y_P \cos\varphi + X_P\sqrt{\sin^2\nu - \cos^2\varphi} + (Z_o - Z_P)\cos\nu]/E \qquad (25)$$

3.The determination of the coordinates (x_1, y_1) of the image P_1:

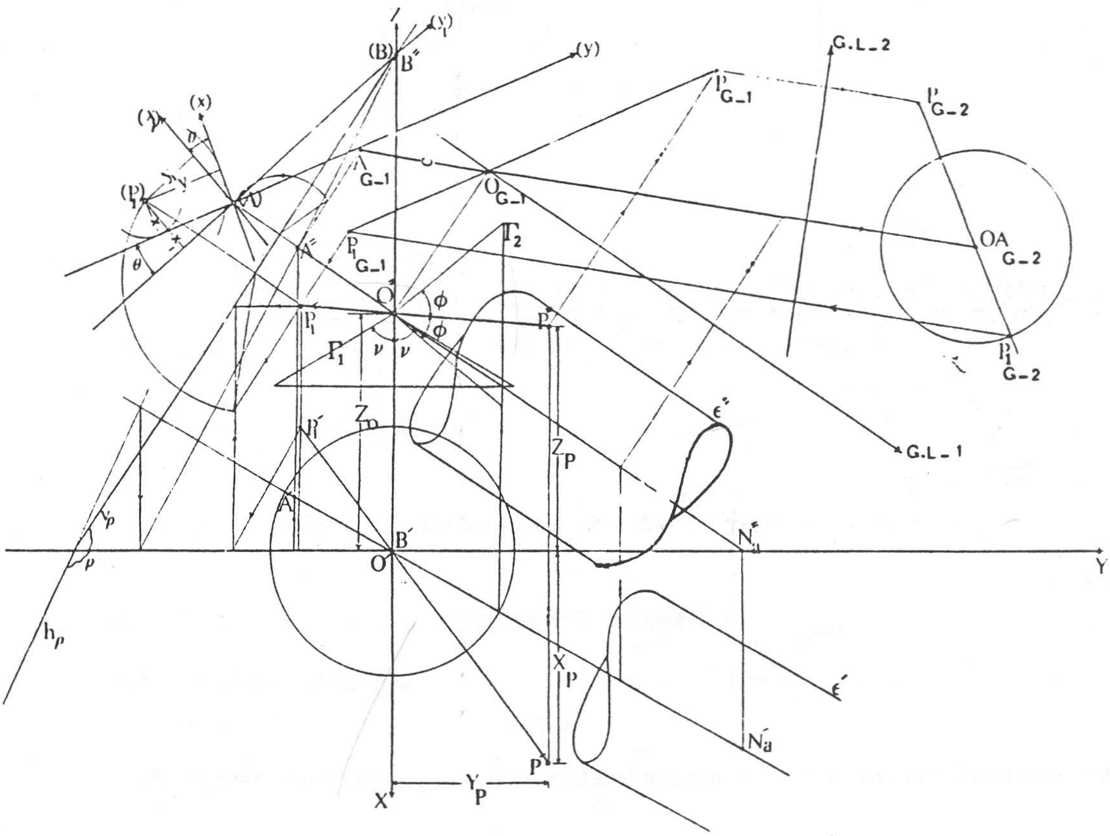
Substitution from equations (16) , (24) and using the coordinates of A (from (10)) in equation (18) yields:

$$y_1 = \pm\, r \cot \nu \,[Y_P \cos \varphi + X_P \sqrt{\sin^2 \nu - \cos^2 \varphi} - (Z_o - Z_P) \sin \nu \tan \varphi]/E \qquad (26)$$

Then by using equation (20), x_1 is determined.

4. The swinge angle φ is given by

$$\varphi = \sin^{-1} [(xy_1 - yx_1)/r^2] \qquad (27)$$



Scale:

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Fig. (5)

3.2 The graphical solution, Fig. (5)

The numerical assumptions are:

$\nu = 60^{o}$, $\varphi = 40^{o}$, $P(X_P, Y_P, Z_P) = (40, 30, 42.5)$,

$Z_o = 45$, $P_1(x,y) = (8.1, -16.4)$

It may be noticed that some of the above numerical values e.g. space coordinates of P and hight of flight Z_o are somewhat inexpressive and inapplicable, that is to obtain a suitable and acceptable graphical configuration steps of solution:

1. The point o is represented by its horizontal and vertical projections O' and O" respectively.
2. The camera axis OA is represented as the intersection of two cones Γ_1 and Γ_2 having the same vertex O and the axis of Γ_1 is the Z- axis and its semi vertex angle equals ν, and the axis of Γ_2 is the line passing through O and parallel to the Y-axis and its semi vertex angle equals φ. These two cones intersect in real generators if $\nu + \varphi \geq 90^{o}$.
3. The point P is represented by its horizontal and vertical projections P' and P" respectively.
4. A right circular cylinder ε is represented with the camera-axis as its axis and its radius is r.
5. The intersection of ε with the line OP gives point A as shown in Fig. (4),this intersection is obtained using the auxiliary planes.
6. The image plane ρ is represented by its horizontal and vertical traces, h_ρ and v_ρ respectively where ρ passes through A and is perpendicular to OA.

7. The image P_1 is found as the intersection of ρ with the line OP. Hence (X_{P1}, Y_{P1}, Z_{P1}) of P_1 are obtained.
8. The image plane ρ is revolved about v_ρ untill it coincide with the (Y,Z)-plane. Point A takes the position (A) and the image coordinate axes x_1 and y_1 appear as shown in Figure (5). The image P_1 takes the position (P_1) and its image coordinates appear in their true lengths.
9. The ordinary coordinate axes x and y are drawn using the coordinate x or y and then, the swing angle θ is determined

3.3 The Comparison

Graphical results are obtained from Figure (5). The numerical results based on the analytical solution are obtained by substituting the assumed numerical values used in the graphical solution.

The results are summed up in the following table:

Subject	Graphical	Analytical
c	24.9	24.975
$P_1(X_{P1}, Y_{P1}, Z_{P1})$	(-24.72, -18.52,46.56)	(-24.734,-18.55,46.549)
$P_1(x_1, y_1)$	(13.25,-12.62)	(13.226, - 12.634)
θ	19.8° (=190 48')	19° 59' 53"

As shown in the table, there appears a slight difference in results due to the unavoidable natural inaccuracy in graphical procedure . The above techniques can be applied to many practical problems [5].

References

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