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MONTHLY AND DAILY OPTIMUM TILT ANGLES FOR SOUTH FACING SOLAR COLLECTORS; THEORETICAL MODEL, EXPERIMENTAL AND EMPIRICAL CORRELATIONS

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Abstract

An analysical model for optimum tilt angle (β_{op}) versus the latitude angle (L), for every day number (N), is derived. A computer mode is used to obtain β_{opt} for each month. Using least squares method, four empirical formulas were obtained for calculating β_{opt} for each month. In an attempt to verify the theoretical model, an experimental set up was built, which consists essentially of a photovoltaic module (2.3 w fixed on a hinged flat plate facing south direction. An electric circuit, having a resistance of 39 Ω is connected to the terminals of the module. The reading of output power is recorded at different times during the day, for different tilt angles. The optimum tilt angle is predicted as the angles at which the maximum average output were obtained. The experimental results were found to tie up reasonably well with the prediction.

Nomenclature

I_{day}	Daily total extraterrestrial radial radiation, kw hr/m^2.
h_{ss}	Sunrise-hour angle, rad.
M	Month number starting from January.
N	Day number starting from first of January.
L	Latitude angle, deg.

Greek Symbols

β_{opt}	Optimum tilt angle
δ	Solar declination angle.

1. Introduction

One of the methods for increasing the thermal efficiency of any type of collectors is to use the optimum tilt angle [1]. This is due to the fact that, the orientation and tilt angle cf solar collector are the two most important factors usually considered in solar energy system design.

Most researchers express β_{opt} empirically .For example, Hottel [2], stated that β_{opt} = L+20, where L is the latitude angle, while Heywood [3], suggested β_{opt} =L-10. Yellott [4], has reported a wide range of β_{opt} =L$\pm$20, where the plus and minus signs are used in winter and summer, respectively. Moreover, theoretical models for β_{opt} are suggested by some other researches like, for instance, Lewis [5] who suggested two different models for

β_{opt}, as $\beta_{opt} = L \pm 8^{\circ}$. This was concerned for a narrow of L between 30° -35° (Alabama USA). The defect of Lewis' experimental work is that he measured the total radiation at the inclination calculated from his model. This does not mean, in one way or another, that it is the optimum inclination.

El-Kassaby [1], reported a theoretical model as well as two empirical formulas for β_{opt}. He concluded that a change of tilting angle twice a year provides 50% increase in efficiency of solar collector as compared with a horizontal one. The work presented here is an extension of that reported in Ref. [1]. It describes an experimental apparatus designed to provide the daily optimum tilt angle at Mu'tah university, Jordan (L=31 N). A computer model is developed to calculate β_{opt} versus L for every day and month. Finally the paper presents empirical correlations to fit the results obtained from computer model.

2. Theoretical Analysis

The daily total extraterrestrial radiation intercepted on south facing surface, tilted by an angle β with horizontal, can be expressed as [1]

$$I_{day} = \frac{24}{\pi} I_{o} [1+0.034 Cos(\frac{2\pi N}{365})]*[Cos(L-\beta)Cos(\delta)Sin(h_{ss}) + h_{ss} Sin(L-\beta) Sin(\delta)] \quad (1)$$

where:

$$\delta = -23.45 Cos[(N+10.5)\frac{360}{365}] \quad (2)$$

= The solar declination angle.

and

$$h_{ss} = Cos^{-1}[-tan(L) Tan(\delta)] \quad (3)$$

Therefore, the total monthly radiation can be obtained as:

$$I_M = \sum_{N=N_1}^{N=N_2} I_{day} \quad (4)$$

where:

M is the month number, N_1 and N_2 are the first and the last days of the M th month as counted from January first, respectively.

2.1 The Optimum Daily Tilt Angle

In some practical applications, for example a medical services car uses a solar-panel installed on its roof to generate the necessary electrical power for medical equipments, it is important to know the daily optimum angle. Referring to equation (1) at a certain location in a particular day N, all the parameters are considered constant except β. For optimum tilt angle (β_{opt}), at $dI_{day}/d\beta = 0$ from which we find

$$\beta_{opt} = L - Tan^{-1}\left[\frac{h_{ss}}{sin(h_{ss})} Tan(\delta) \right] \quad 5)$$

where, δ and h_{ss} are defined in Eqs.(2) and (3),respectively. The results obtained by Eq. (5) are tabulated in Table 1 and plotted in Fig. 1.

2.2 Monthly Optimum Tilt Angle

It is not practical to design a solar collector for which the tilt

TABLE 1 The Optimum Daily Tilt Angle, for Different Latitude Angles. (Eq. 5).

		L						
N	Date	0	10	20	30	40	50	60
1	1/1	33.68	42.47	51.30	60.07	68.72	77.10	84.99
10	10	32.40	41.28	50.71	59.02	67.75	76.22	84.22
20	20	30.14	39.15	48.17	57.16	66.02	74.65	82.85
30	30	27.00	36.19	45.38	54.54	63.58	72.43	80.90
32	2/1	26.27	35.50	44.73	53.92	63.01	71.91	80.44
42	10	22.11	31.55	40.98	50.39	59.71	68.87	77.74
52	20	17.17	26.82	36.47	46.09	55.65	65.11	74.35
60	3/1	12.74	22.54	32.34	42.12	51.87	61.54	71.09
69	10	7.36	17.29	27.22	37.14	47.05	56.93	66.76
79	20	1.09	11.09	21.08	31.08	41.08	51.08	61.07
89	30	- 5.23	4.74	14.70	24.66	34.60	44.53	54.41
91	4/1	- 6.47	3.47	13.42	23.35	33.26	43.15	52.96
100	10	-11.91	- 2.09	7.72	17.48	27.18	36.75	46.00
110	20	-17.47	- 7.85	1.72	11.21	20.52	29.49	37.63
120	30	-22.37	-12.98	- 3.68	5.46	14.29	22.48	29.00
121	5/1	-22.82	-13.46	- 4.18	4.92	13.70	21.81	28.15
130	10	-26.48	-17.33	- 8.30	.48	8.80	16.13	20.72
140	20	-29.75	-20.80	-12.02	- 3.56	4.29	10.80	13.41
150	30	-32.14	-23.35	-14.76	- 6.56	.91	6.74	7.62
152	6/1	-32.51	-23.74	-15.19	- 7.03	.38	6.1	6.69
161	10	-33.74	-25.06	-16.61	- 8.59	- 1.39	3.95	3.50
171	20	-34.26	-25.62	-17.22	- 9.25	- 2.15	3.02	2.10
181	30	-33.9	-25.24	-16.80	- 8.79	- 1.63	3.66	3.07
182	7/1	-33.82	-25.15	-16.70	- 8.69	- 1.50	3.81	3.29
191	10	-32.65	-23.9	-15.36	- 7.21	.17	5.84	6.31
201	20	-30.52	-21.62	-12.90	- 4.52	3.20	9.50	11.59
211	30	-27.5	-18.41	- 9.46	- .78	7.40	14.49	18.51
213	8/1	-26.80	-17.66	- 8.66	.10	8.37	15.63	20.05
222	10	-23.19	-13.85	- 4.60	4.47	13.21	21.24	27.42
232	20	-18.44	- 8.86	.67	10.09	19.32	28.16	36.03
242	30	-12.98	- 3.19	6.57	16.29	25.93	35.4	44.49
244	9/1	-11.82	- 2.00	7.81	17.58	27.28	36.85	46.12
253	10	- 6.-8	3.57	13.51	23.44	33.36	43.25	53.06
263	20	- .09	9.91	19.91	29.91	39.91	49.91	59.91
273	30	6.21	16.16	26.11	36.06	45.99	55.91	65.78
274	10/1	6.83	16.77	26.71	36.65	46.57	56.46	66.31
283	10	12.24	22.06	31.88	41.67	51.44	61.13	70.71
293	20	17.77	27.40	37.03	46.62	56.16	65.58	74.78
303	30	22.63	32.04	41.45	50.83	60.12	69.26	78.09
305	11/1	23.51	32.88	42.25	51.58	60.83	69.90	78.66
314	10	27.05	36.24	45.43	54.58	63.63	72.47	80.93
324	20	30.18	39.19	48.21	57.19	66.05	74.68	82.88
334	30	32.43	41.30	50.20	59.05	67.77	76.24	84.24
335	12/1	32.61	41.47	50.35	59.19	67.90	76.36	84.35
344	10	33.79	42.58	51.39	60.16	68.80	77.17	85.06
354	20	34.27	43.02	51.81	60.55	69.16	77.50	85.34
364	30	33.86	42.64	51.45	60.21	68.85	77.22	85.09

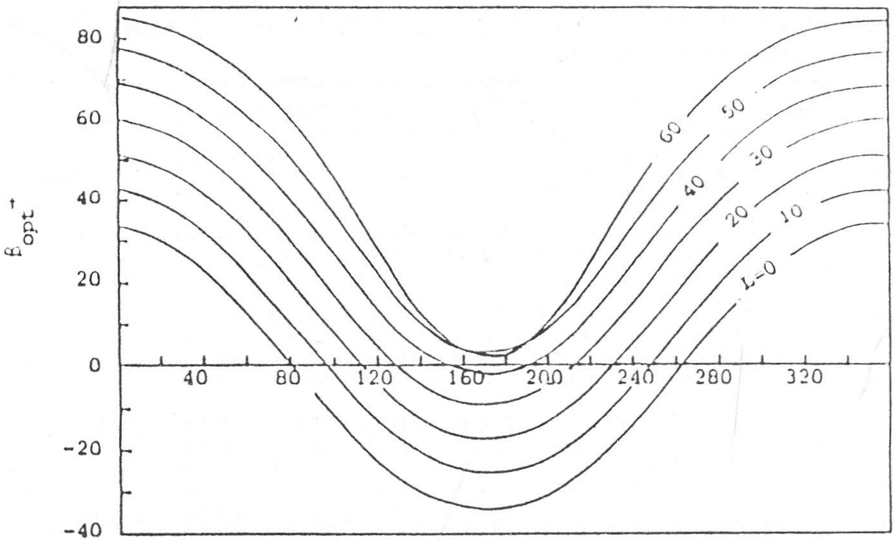

Fig. 1. Shows the variation of β_{opt} with the day number N , for different latitude angles L.

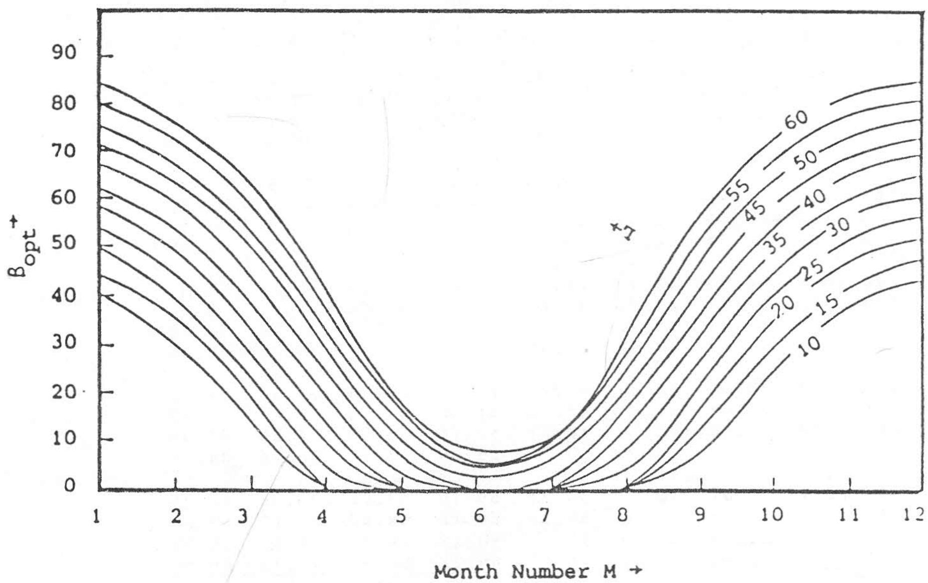

Fig. 2. Shows the monthly β_{opt} versus the latitude angle L.

angle changes every day. However, it may be possible to change it once a month. The question that may arise is that, is it worth to change the tilt angle once a month?. To answer this question, a computer program was performed, using Eq. (4) to obtain the optimum tilt angle for each month as well as the total extraterresterial insolation. Fig. 2. and Table 2 show the results obtained from this program, where it is to be noted that the negative values of β_{opt} are considered as zero. This is due to the fact that it is impractical to design a solar collector with negative tilt angles.

In order to generalise the solution presented here, a least squares method is implemented to the above results and empirical formulas for β_{opt} are obtained as:

a) From January up to March ($1 \leq M \leq 3$)

$$\beta_{opt}=60.00012+1.49986\,M+3.49996M^2+(L-30)(0.7901+0.01749M+0.0165M^2) \qquad (6)$$

b) From April up to June ($3 < M \leq 6$)

$$\beta_{opt}=216.0786-72.03219M+6.00321M^2+(L-40)(1.07515+0.11244M-0.03749M^2) \qquad (7)$$

providing that if $\beta_{opt} < 0$ then $\beta_{opt} = 0$

c) From July up to September ($6 < M \leq 9$)

$$\beta_{opt}=29.11831-20.52981M+2.50186M^2+(L-50)(-11.17256+2.70569M-0.15035M^2) \qquad (8)$$

providing that if $\beta_{opt} < 0$ then $\beta_{opt} = 0$

d) From October up to December ($9 < M \leq 12$)

$$\beta_{opt}=-441.2385+84.533M-3.50196M^2+(L-40)(4.2137-0.54834M+0.0223M^2) \qquad (9)$$

The computer model results and the corresponding results obtained from the empirical equations (6,7,8 and 9) are compared as shown in Table 3, from which it can be shown that the empirical correlations fit the

L	10		15		20		25		30		35		40		45		50		55		60	
Mon.	β	I*	β	I	β	I	β	I	β	I	β	I	β	I	β	I	β	I	β	I	β	I
1	40	355.22	44	352.38	49	349.07	53	345.14	58	340.39	62	334.53	67	327.07	71	317.26	75	303.71	79	283.8	83	251.57
2	30	305.81	35	304.97	39	303.96	44	302.73	49	301.2	53	299.27	58	296.81	63	293.54	67	289.03	72	282.54	76	272.49
3	14	322.95	19	322.91	23	322.84	28	322.76	33	322.64	38	322.49	43	322.29	48	322.01	53	321.61	58	321.03	63	320.14
4	0	312.16	0	313.47	4	313.43	9	313.31	14	313.07	19	312.66	24	312.03	28	311.1	33	309.67	37	307.42	41	303.73
5	0	319.41	0	328.03	0	334.46	0	338.67	0	340.09	2	340.76	6	340.27	10	339.11	13	336.91	16	333.01	17	326.2
6	0	304.56	0	316.07	0	325.6	0	333.11	0	338.62	0	342.83	0	343.83	2	343.9	4	343.12	5	341.25	8	338.29
7	0	315.71	0	325.98	0	334.13	0	340.13	0	344.	0	345.77	2	345.8	6	345.08	8	343.39	10	340.19	10	334.61
8	0	319.11	0	323.23	0	325.04	3	325.09	8	324.87	13	324.39	17	323.6	21	322.29	25	320.2	29	316.83	33	311.12
9	7	306.93	12	306.91	17	306.87	22	306.82	27	306.75	32	306.64	37	306.5	42	306.3	47	306.01	52	305.75	57	304.9
10	25	328.98	30	328.5	35	327.9	40	327.16	44	326.22	49	325.05	54	323.52	59	321.48	64	318.65	68	314.57	73	308.29
11	38	337.75	42	335.54	47	332.93	51	329.83	56	326.07	60	321.41	65	315.48	69	307.67	73	296.9	78	281.2	82	256.17
12	43	360.1	47	356.41	51	352.13	56	347.11	60	341.05	65	333.58	69	324.12	73	311.63	77	294.32	81	268.58	85	225.73
Total		3888.69		3918.4		3928.36		3931.86		3924.97		3912.35		3884.32		3841.37		3783.52		3696.17		3553.24

* The total extraterrestrial beam radiation in kw hr/m².

TABLE 3 Comparison Between the Coputer Results and Empirical Formulas Results for Monthly Optimum Tilt Angle.

M	β_{opt}																					Eq.
	L=0			L=10			L=20			L=30			L=40			L=50			L=60			
	C*	E**	D†	C	E	D	C	E	D	C	E	D	C	E	D	C	E	D	C	E	D	
1	31	38.2	2.2	40	41.5	1.5	49	49.7	0.7	58	58.0	0.0	67	66.2	0.8	75	74.5	-0.5	83	82.7	-0.3	
2	20	22.2	2.2	30	31.2	1.2	39	40.0	1.0	49	49	0.0	58	57.9	-0.1	67	66.8	-0.2	76	75.7	-0.3	6
3	4	3.2	0.8	14	13.2	-0.8	23	23.1	0.1	33	33	0.0	43	42.9	-0.1	53	52.8	-0.2	63	62.7	-0.3	
4	0	0	0	0	0	0	4	5.5	1.5	14	14.7	0.7	24	24	0	33	33.2	0.2	41	42.5	1.5	
5	0	0	0	0	0	0	0	0	0	0	0	0	6	6	0	13	13	0	17	20.0	3.0	7
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	0	8	8	0	
7	0	0	0	0	0	0	0	0	0	0	0	0	2	4	2	8	8	0	10	12	2.0	
8	0	0	0	0	0	0	0	0	0	8	8	0	17	16.5	-0.5	25	25	0	33	33.5	0.5	8
9	1	0	1.0	7	7	0	17	17	0	27	27	0	37	37	0	47	47.0	0	57	57	0	
10	16	15.6	-0.4	25	25.1	0.1	35	34.8	-0.2	44	44.4	0.4	54	54	0	64	63.6	-0.4	73	73.2	0.2	
11	29	29.8	0.8	38	38.6	0.6	47	47.4	0.4	56	56.2	0.2	65	65	0	73	73.8	0.8	82	82.6	0.6	9
12	34	35.2	1.2	43	43.6	0.6	51	52.1	1.1	60	60.5	0.5	69	69	0	77	77.4	0.4	85	85.9	0.9	

* Coputer results.
** Empirical formulas
† Difference in degree.

computer theoretical model results within an error $\pm 1.5^{o}$ which can be accepted as a tolerable discrepancy.

The total yearly extraterrestrial insolation is calculated at $\beta=0$, β_{opt} on a daily as well as on a monthly basis. The ratio between insolation on a tilted surface to insolation on a horizontal one for the same period of time, (tilt factor) is calculated for each case. The results are presented in Table 4, which shows that the total insolation all over the year received by collector using β_{opt} on daily basis is almost the same as that received by a collector using β_{opt} on a monthly basis. This means that, it is worth to design a collector for which the tilt angle changes once a month.

Fig. 3. shows a recommended design for a solar collector with optimum efficiency, which can be used at latitude angle 30 N (Cairo, Egypt). For this design it is important to notice that we suggested to change the tilt angle in five positions. The first position for November, December and January ($\beta=58$), the second for February and October ($\beta=46$), the third for March and September ($\beta=30$), the fourth for April and August ($\beta=10$). The last position for May, June and July ($\beta=0$). We recommended this design to provide a less expensive solar collector with optimum possible efficiency.

3. Experimental Work

In order to provide a further check on the proposed daily optimum angle model, an experimental test apparatus is built, as shown in Fig. 4. The mainpart of the apparatus is the photovoltaic solar-panel which has a power of 2.3 watts, 18 solar cells connected in series, the open circuit voltage is 10.2 V, the short circuit current is 300 m.A,

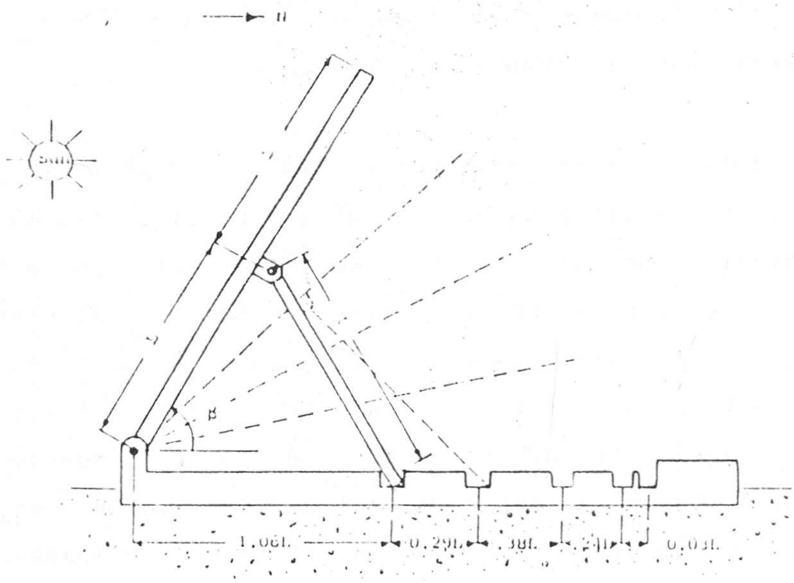

Fig. 3. Shows the recommended design for a solar collector at L=30° N.

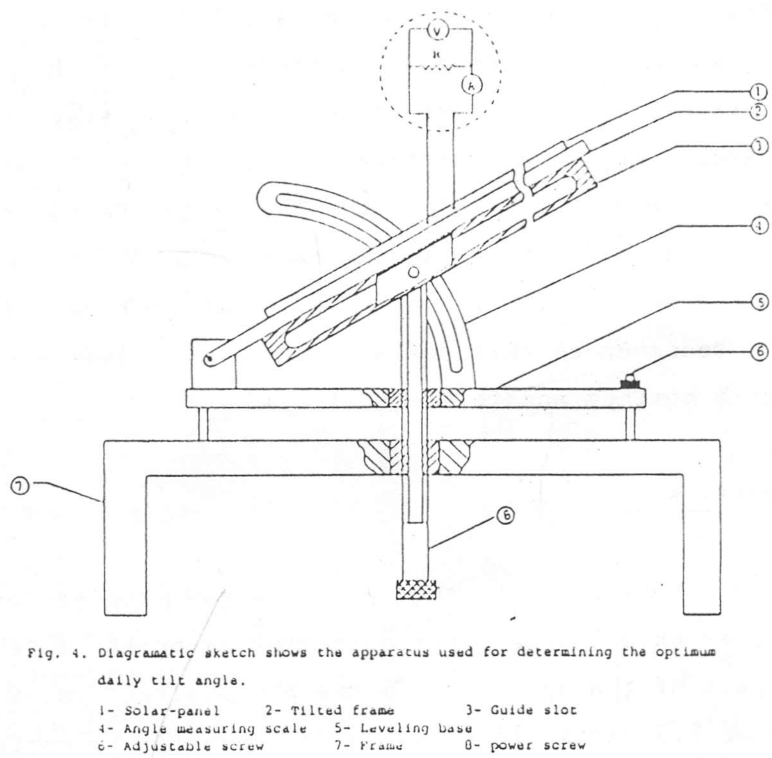

Fig. 4. Diagramatic sketch shows the apparatus used for determining the optimum daily tilt angle.

1- Solar-panel 2- Tilted frame 3- Guide slot
4- Angle measuring scale 5- Leveling base
6- Adjustable screw 7- Frame 8- power screw

TABLE 4 The Tilt Factor for both Daily and Monthly Optimum Tilt Angle.

L	β=0	β=β_opt (daily)		β=β_opt (Monthly)	
	I*	I	F**	I	F
10	3559.49	3894.8	1.094	3888.69	1.092
15	3494.06	3922.0	1.122	3918.40	1.121
20	3404.82	3935.6	1.155	3928.36	1.1537
25	3292.66	3940.4	1.196	3931.86	1.194
30	3158.51	3934.6	1.245	3924.97	1.242
35	3004.82	3918.6	1.304	3912.35	1.302
40	2832.63	3891.6	1.373	3884.32	1.371
45	2532.63	3852.2	1.521	3841.37	1.516
50	2444.21	3795.2	1.552	3783.52	1.548
55	2235.68	3708.5	1.658	3696.17	1.653
60	2026.04	3568.6	1.760	3553.24	1.753

F is the tilt factor

I is the total yearly extraterresterial beam radiation in kw hr/m².

TABLE 5 Shows a Comparision Between the Experimental Results and the Theoretical Ones Given by Eq. (5).

Date	N	Exp.	Eq.(5)	% error
25/11/87	329	57.66	59.1	2.49
26	330	60.00	59.28	-1.2
27	331	61.33	59.45	-3.06
28	332	61.50	59.61	-3.07
29	333	61.33	59.92	-2.3
30	334	61.33	59.92	-2.3
1/12/87	335	59.4	60.07	1.12
3	337	62.00	60.33	-2.69
7	341	59.5	60.78	2.15
8	342	61.0	60.87	-0.21
11	345	62.33	61.11	-1.95
15	349	62.6	61.32	-2.04
16	350	63.25	61.35	-3.00
18	352	61.00	61.40	0.65
21	355	61.00	61.42	0.66
28	362	60.75	61.21	0.75
31	365	59.00	61.01	3.40
1/1/88	1	61.00	60.94	0.09
2	2	59.00	60.86	3.1

nominal voltage 8.1 V, and nominal current is 282 m.A. The solar-panel is manufactured by Solar Trend Company (W. Germany). To measure the output power of the solar-panel, an electric circuit (shown in dotted circle of Fig. 4), was connected to output terminals. The resistance R was so chosen that its value gives maximum power transfer.

To get the optimum daily angle the following steps are followed:

1. Start with a tilt angle (β) of a value less than the one obtained from Eq. 4, say by 10 degrees.
2. Read the values of both voltage V and current I .
3. Increase β by one degree, then record V and I.
4. Keep increasing β until you get maximum power output. At this point β will be considered the optimum tilt angle at this time. Let us denote it by β_{opt}.
5. Repeat the previous steps several times per day and record the corresponding values of β_{opt}.

The optimum tilt angle during a certain day is thus taken as the average value of all the recorded values of β_{opt}.

The experimental results obtained at Mu'tah university, Jordan (L=31 N) are given in Table 5, as well as the values obtained from equation 5. The Table shows that the experimental results tie up reasonably well with the theoretical ones.

4. Discussion

Provided that the negative values of β in Table 1, are considered zero, it is easy to notice that the optimum monthly tilt angle given in Table 2, is the average daily optimum tilt angle during that month.

Therefore, for any period of time, started with day number N, and ending with N_4, the optimum tilt angle can be expressed as:

$$\beta_{opt} = \frac{1}{N_3 - N_4 + 1} \sum_{N=N_3}^{N=N_4} (\beta_{opt})_{daily} \qquad (10)$$

To check its validity, Eq. (10) was checked against the already available data [1]. For just one sample result, letting $N_3=1$, $N_4=365$, and with L=30 , Eq. (10) gives β_{opt} = 28.5 while the corresponding value found in Ref. [1] is 28.

In designing a solar collector for heating purposes, it is impractical to let the collector have a negative angle, therefore in Table 2 and Fig. 2, the negative values are considered zero. It must be pointed out that for higher values of negative angles ($L=0^{o}$ up to $L=\pm 25^{o}$) it is important to think about a new design which can provide such negative tilt angles as well as the positive ones.

6. Conclusion

The following conclusive remarks can be mentioned:

1. Designing a solar collector in which the tilt angle can change monthly, provides better efficiency (see Table 4).
2. The daily optimum tilt angle at any location can be given by a simple expression (Eq. 5). Equally simple expression (Eq. 10) can be used for the optimum tilt angle over any required period.
3. The recommended design shown in Fig. 3 gives higher efficiency.
4. It is worth to design a collector having negative tilt angles between latitude zero and $\pm 25^{o}$.

References

[1] El-Kassaby, M.M., The Optimum Seasonaly and Yearly Tilt Angle for South-facing Solar Collectors, ISES Solar World Congress, Sept. 1987, Hamburg, W. Germany, Paper No. 2.2.20.

[2] Hotel, H. C., Performance of Flat-Plate Solar Energy Collectors. In space Heating with Solar Energy, Proc. Course-Symposium. MIT Press, Cambridge, (1954).

[3] Heywood, H., Operating Experience with Solar Water Heating. J. Instn Heat Vent. Eng. 39, 63-69 (1971).

[4] Yellott, H.J. Utilization of Sun and Sky Radiation for Heating and Cooling of Buildings. ASHRAE Jl 15,31-42, (1973).

[5] Lewis, G., Optimum Tilt of Solar Collector, Solar and Wind Technology, Vol. 4, No. 3. PP. 407-410, (1987).