

HEAT TRANSFER DURING THE SOLIDIFICATION OF LATENT HEAT THERMAL ENERGY STORAGE MATERIAL

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Abstract

A mathematical model is developed to predict the temperature distribution in both solid and liquid phases as well as the motion of the liquid-solid interface boundary during the solidification of latent heat thermal energy storage material. The effect of molten initial temperature (superheat), heat transfer surface temperature, and natural convection, on the heat transfer rate, accumulative energy stored and interface growth with time is presented. The developed model is used to predict the experimental results of recent published investigation. Both experimental results and model predictions are in good agreement.

Nomenclature

A	Area ratio at interface	
C	Specific heat	$\text{kJ/kg} \cdot ^\circ\text{C}$
E	Accumulative energy stored	kJ
Gr	Grashoff number $\text{Gr} = \beta g \Delta T (R_o - R_{inf}(t))^3 / \nu^2$	
K	Thermal Conductivity	$\text{W/m} \cdot ^\circ\text{C}$
h_{sl}	Latent Heat of fusion	kJ/kg
i	Integer representing number of nodes	
IMAX	Maximum number of nodes	
INTL	Interface liquid node number	
INTS	Interface Solid node number	
L	Length of Cooling surface	m
m	mass	kg
n	Superscript representing time	
Pr	Prandtl number, $\text{Pr} = \nu / \alpha$	
Q	Heat transfer rate	W
r	Radial coordinate	
R	Radius	
Ra	Rayleigh number, $\text{Ra} = \text{Gr} \cdot \text{Pr}$	
t	time	s
T	Temperature	$^\circ\text{C}$

Greek Letters

α	Thermal diffusivity	m^2/s
β	Coefficient of thermal expansion	$1/^\circ\text{K}$
γ	Value defining problem coordinate ($\gamma = 0$ cartesian coordinate, $\gamma = 1$ cylindrical coordinate)	
ρ	Denisty	kg/m^3

Subscripts

e	equivalent
i	inner
int	interface
L	liquid
m	melting
o	outer
s	solid
w	wall

1. Introduction

Solidification or melting heat transfer processes are important problems and of many engineering applications, for example, the charging and discharging processes of latent heat thermal energy storage. Heat transfer analysis in PCM is rather difficult as it involves the analysis of a moving solid-liquid interface boundary whose behaviour is non-linear with respect to time. Previous analytical approach followed to tackle this problem is based on the fact that the movement of this interface boundary is very slow as compared to the temperature response of the PCM. This is because the latent heat effect is more predominant than that of the sensible heat. So, a stationary average solid-Liquid interface boundary was assumed instead of a moving boundary by Prakash et. al. [1]. Another traditional analytical simplification is the assumption that conduction is the sole mean of heat transport across the liquid layer. Investigations have shown, however, that natural convection in the melt region plays a major role in the heat transfer processes, and significantly influences the

solidification (or melting) rate and heat transfer coefficient [2-5]. Moreover, some investigators considered the PCM to be initially near the melting temperature, however, if PCM is initially at different temperature, conduction in the solid during melting or conduction-convection in the liquid during solidification would also play an important role. Solomon [6] has made his analysis based on a semi-infinite PCM slab, while Gupta [7] used the steady state temperature profile ($T = A \ln r + B$) in the frozen zone or melt.

The foregoing discussion serves to motivate the study that was performed here. A mathematical model will be developed to predict the transient temperature distribution in both Solid and Liquid phases as well as the interface motion during solidification heat transfer process. The heat transfer rate and accumulative energy stored as a function of time will also be presented.

2. Theoretical Consideration

The heat transfer problem with a phase change from liquid to solid phases and transient moving solid-liquid interface, may be described by the unsteady heat conduction equations (convection effect will be introduced later) applied to both phases as follows, see Figure (1);

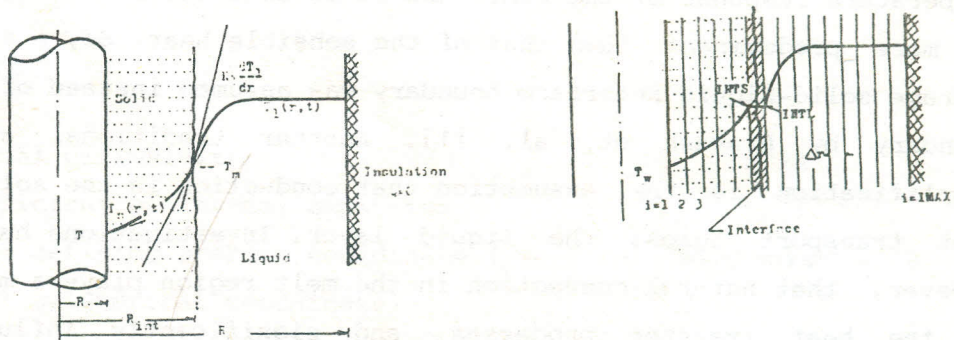


Figure (1) Solidification Process of PCM

$$\frac{\partial T_s(r,t)}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s(r,t)}{\partial r^2} + \frac{\gamma}{r} \frac{\partial T_s(r,t)}{\partial r} \right) \quad (1)$$

Solid Phase

for $t > 0$ and $R_i < r < R_{int}$

with the following initial and boundary conditions

$$T_s(r,0) = f_1(r), \quad T_s(0,t) = T_w, \quad \text{and} \quad T_s(R_{int},t) = T_m$$

Liquid Phase

for $t > 0$ and $R_{int} < r < R_o$

with the following initial and boundary conditions

$$\partial T_L(r,0) = f_2(r), \quad T_L(R_{int},t) = T_m, \quad \text{and} \quad \frac{\partial T_L(R_o,t)}{\partial r} = 0$$

The interface boundary conditions

$$T_s(R_{int},t) = T_L(R_{int},t)$$

$$- \left(K_L \frac{\partial T_L}{\partial r} + K_s \frac{\partial T_s}{\partial r} \right)_{int} = h_{sL} \frac{dR_{int}(t)}{dt} \quad (2)$$

Numerical Solution

In the present study, a simplified explicit finite difference scheme is

employed for the numerical solution of the above phase change problem. The computation region is divided into a number of (IMAX) nodes of incremental distance Δr . The differential equation (1) is defined in a finite difference form as follow:

Solid Phase

$$TS_i^{n+1} = TS_i^n + \frac{\alpha_s \Delta t}{(\Delta r)^2} (TS_{i+1}^n - 2TS_i^n + TS_{i-1}^n) + \frac{\gamma (\Delta r) (TS_{i+1}^n - TS_i^n)}{(R_{i+i} * \Delta r)} \quad (3)$$

with the following initial and boundary conditions

$$TS_1^1 = T_m, \quad TS_1^{n+1} = T_w \quad \text{and} \quad TS_{INTS}^{n+1} = T_m$$

The moving Solid-liquid interface is defined between the two nodes INTS and INTL. The transient interface radius is defined from equation (2) as:

$$R_{int}^{n+1} = R_{int}^n + \frac{\Delta t}{h_{sL}} (K_s \frac{TS_{INTS}^n - TS_{INTS-1}^n}{\Delta r} - k_e \frac{TL_{INTL+1}^n - TL_{INTL}^n}{\Delta r}) \quad (4)$$

Calculation of the interface radius will define the interface solid and liquid nodes (INTS and INTL) so that when:

$$(R_{i+(i-1)} * \Delta r) \leq R_{int}^{n+1} \leq (R_i + i * \Delta r) \quad (5)$$

Then, INTL = i and INTS = i - 1

The heat transfer rate to the cold wall is calculated using the Fourier conduction equation in the solid near the wall as:

$$Q = -K_s A \left(\frac{\partial T}{\partial r} \right)_w = -K_s \cdot A \cdot \frac{(T_2^{n+1} - T_1^{n+1})}{\Delta r} \quad (6)$$

The accumulated energy discharged during solidification E is calculated by integrating equation (6) with respect to time.

Natural Convection Effect

The effect of natural convection have been introduced by replacing the liquid thermal conductivity K_L by an effective thermal conductivity K_e which decreases as the melt layer thickness decreases. A correlation which was developed for pentacosane-hexacosan mixture by Farid [4] will be used. This correlation can be written as:

$$K_e/K_L = 0.0159 Ra^{0.34}$$

In this equation, Rayleigh number is based on the melt thickness ($R_o - R_{int}(t)$) which varies with time. The equation indicates also that the equivalent thermal conductivity K_e is almost a linear function of ($R_o - R_{int}(t)$). Experimental results of Webb and Viskonta [3] show that the equivalent thermal conductivity ratio (K_e/K_L) could be between 1.5 to 2.

A computer program is developed to carry-out the computation using the above equations and the computation steps are summarized in Figure (2).

3. Results and Discussion

The results that will be presented here are for the case of

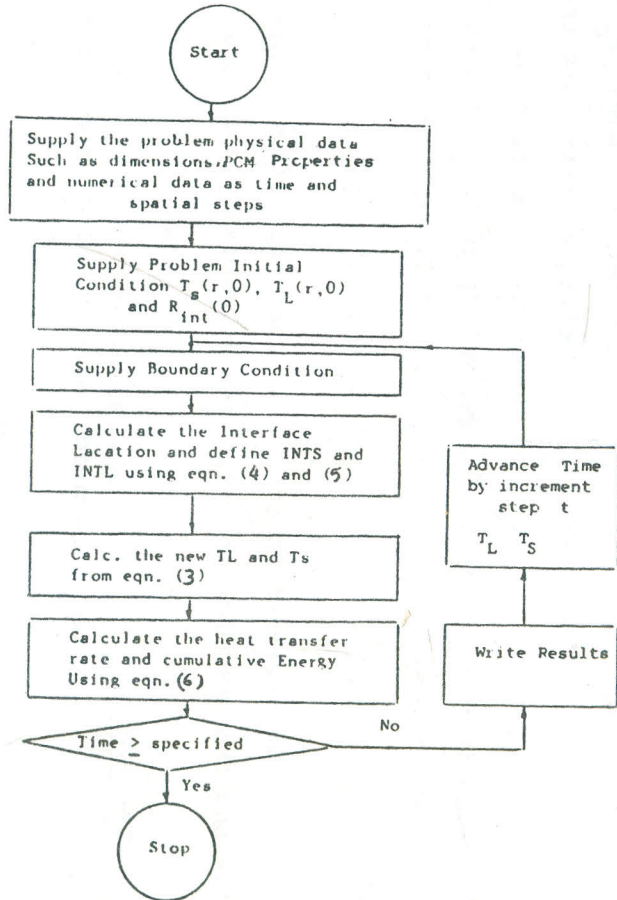


Figure (2) Computation steps

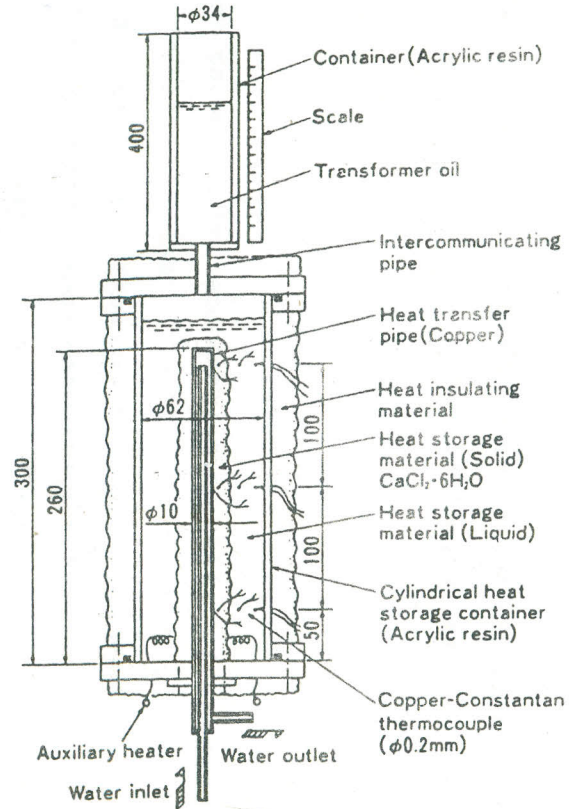


Figure (3) Experimental Set-up of [9], Simulated By The Present Model

solidification of calcium-chloride PCM around a cylindrical vertical tube experimentally carried-out by Yanadori and Masuda [9]. Table (1) gives the physical properties of calcium-chloride while the heat transfer apparatus is shown in Figure (3).

Table 1 Proparety of Calsium Chloride

Property	PCM	
	CaCl ₂ . 6H ₂ O	
Melting point T_m	(°C)	28
Latent Heat h_{sL}	(kJ/kg)	200
Density of solid	(kg/m ³)	1680
Density of liquid	(kg/m ³)	1500
Thermal Conductivity K_s	(W/m.°C)	0.626
Thermal Conductivity K_L	(W/m.°C)	0.455
Specific heat C	(kJ/kg.C)	1.25
Specific heat C_1^s	(kJ/kg.C)	2.13
Coefficient of viscosity	(Pa.s)	2.25×10^{-3}
Coefficient of thermal expansion (1/K)		5.24×10^{-4}

Interface Movement

The interface radius growth is shown in Figure (4) as a function of time. The figure indicates that although the interface growth velocity (slopes of $R(t)$ - t relation-ship) is high at the beginning of the solidification process, it tends to drop rapidly within the first half hour after which the interfacial velocity tends to be almost constant. These results are in agreement with the results of Farid [2] and Yanadari and Mosuda [9].

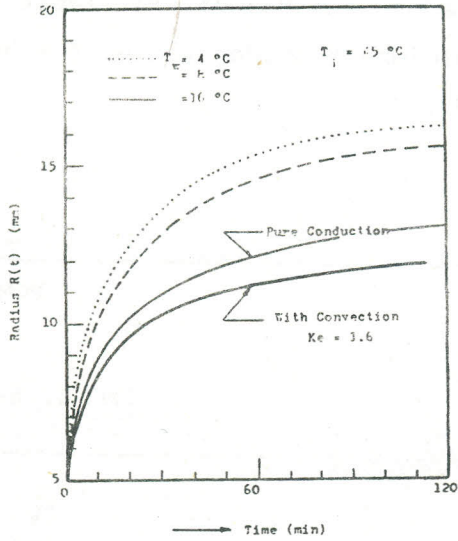


Figure (4.a) Effect of T_w and Convection on Interface Radius $R(t)$

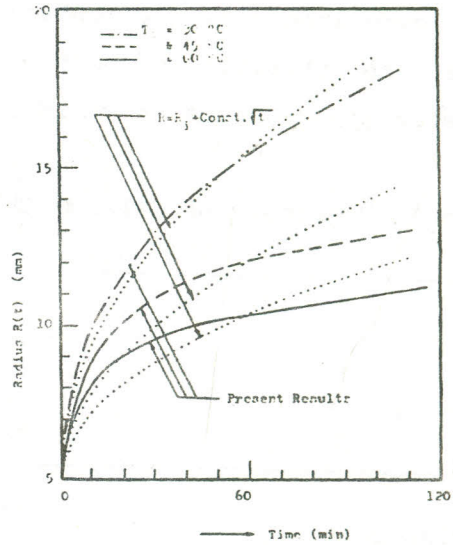


Figure (4.b) Interface Radius Growth Relationship

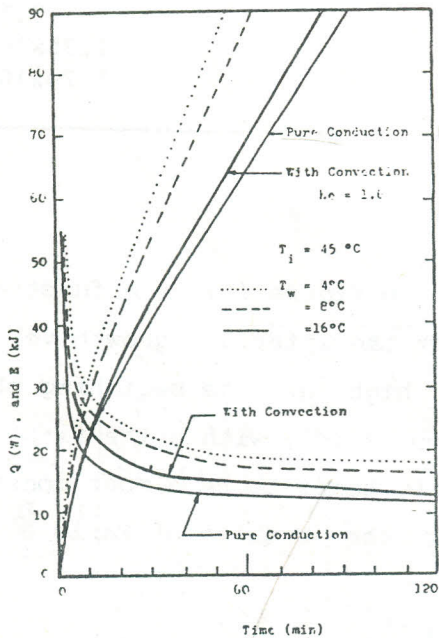


Figure (5.a) Heat Transfer Ratio & Accumulated Energy (Effect of T_w)

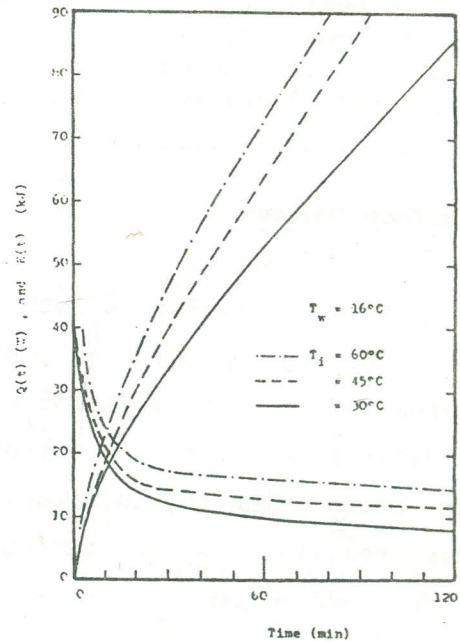


Figure (5.b) Heat Transfer Ratio & Accumulated Energy (Effect of T_i)

The effect of wall temperature T_w is shown in figure (4.a) where decreasing the wall temperature results in an increase in the growth rate mainly due to the higher heat rate leaving the interface boundary to solid zone. On the other hand, Figure (4.b) shows that decreasing the initial molten temperature increases the growth rate and interface radius, mainly due to lower heat transfer rate from the molten zone to the interface boundary. The relationship, $R(t)-t$, does not however follow the square root function proposed by Neumann [8]. The deviation between the present results and this function is shown in Figure (4.b). The Neumann's [8] assumption of semi-infinite solid and liquid is the main drawback of his theoretical prediction. The effect of natural convection in the molten zone, caused by increasing the effective thermal conductivity of the liquid, leads to a higher heat transfer from the molten zone to the interface. This leads to a slower growth of the interface boundary as shown in figure (4.a).

Heat Transfer and Accumulative Energy

The computed values of heat transfer rate near the cold wall as well as the accumulative energy discharged during the solidification process are shown in Figure (5) as a function of time. These two figures show that the heat transfer rate decreases rapidly within the first half hour after which it decreases slowly with almost constant rate. The later constant heat transfer rate is caused by the "almost steady state" temperature profile near wall so that "conduction", the dominated heat transfer mode, is "almost" constant. The rapid reduction in the heat transfer rate is due to the influence of the rapid increase in the thermal resistance caused by the rapid increase in the solid thickness near the heat transfer wall. The trend of these results are very much in agreement with most of the published theoretical and

experimental results, for example Yanadori and Masuda [19]. Decreasing the heat transfer wall temperature or increasing the initial molten temperature, as well as increasing the molten zone effective thermal conductivity (caused by natural convection effect) tends to increase the heat transfer rate and consequently the accumulated discharged energy. Figure (6) shows a comparison between the experimental results of Yanadori and Masuda [9] and the present predictions. The experimental results and the predictions are in good agreement.

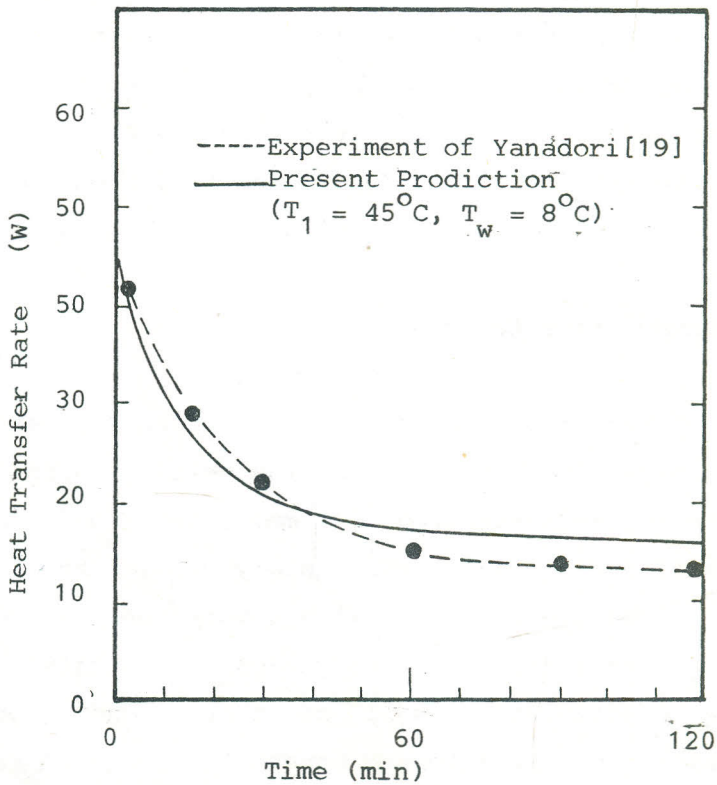


Figure (6) Comparison Between Exp. & Theoretical Results.

4. Conclusion

A transient heat transfer model for the process of solidification, with solid-liquid moving interface boundary is developed. The model is used to predict the interface growth rate, heat transfer rate and accumulated energy as a function of time for the solidification of calcium-chloride around vertical tube. The model is also used successfully to predict the rate of heat transfer experimentally obtained by recent published investigation of Yanadori and Masuda [9]. The developed model is simple, flexible and is now under extension for analysing the two dimensional conduction-convection solidification (or melting) heat transfer process.

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