

## A SIMPLIFIED TECHNIQUE OF GORMAN'S EIGENVALUES FOR CALCULATING CRITICAL SPEEDS OF SPINDLES

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### Abstract

The spindles are the main element of the twisting mechanism of the; roving machines, ring spinning frames, open-end spinning machines and twisting frames whether ring or two-for-one. They are responsible for twist insertion in single or plyed yarns. The productivity of these machines depend upon the working speed of the spindles. The choice of working speed depends upon technological and mechanical factors. One of these mechanical factors are the critical speed of the spindle. Knowing the critical speed, it will be easy to choose the working speed according to a well-known mathematical relation.

In the present work a simplified technique for calculating critical speed of spindles was used. This technique depends upon the eigenvalues of Gorman's determinates.

In the present work, a survey for dimensions of some of spindles of different machine makers has been carried out.

Depending on the dimensions of the spindle, the corresponding Gorman's eigenvalue were chosen.

In the present work, the critical speeds of spindles were calculated according to their eigenvalues. These calculated values were compared by another calculated or measured values (reference values) to know the performance of the simplified technique.

### 1. Introduction

The spindle of the machines for yarn production has different types according to the type of the machine on which they are mounted for example, on the roving machines, there are spindles with flyers. In rotor type open-end spinning machines there are spindles of rotors and opening rollers.

The productivity of the machine is greatly dependant on the working speed of the spindle with it's attached masses or bodies. The working speed depends upon technological and mechanical factors. Concerning the mechanical factor, the working speed must be due to the following equation (1),

$$0.70^{-n} n_{cr} \leq n_w \leq 1.40 n_{cr} \quad (1)$$

Where,

$n_{cr}$  - Critical speed of the spindle with it's connected masses or bodies in r.p.m.

$n_w$  - Working speed of the spindle with it's masses or rigid bodies.

According to equation (1), the spindle with masses and bodies can be worked safely far away from their resonance region.

To calculate, the critical speeds of the spindles, there are several methods such as; Karitiski [4], Makarov [1], Efros [2], Popov [7], Malshev [6], Kyshel [8], Moscow Textile Institute [5]... etc.

In the present work an attempt is made to apply the eigenvalues of Gorman [3] to calculate critical speeds of some spindles of machines of yarn production.

Gorman [3] in his work "free vibrations analysis of beams and shafts" has studied carefully, many different cases of beams and shafts with classical and nonclassical supports. For each case, the determinate of free vibration was written. For each determinate, he tabulated and graphed different numerical values-Eigenvalues - which lead to Zero values of the determinate.

The eigenvalue " $\beta$ " depend upon many factors such as: elasticity of the supports, number of supports, distance between supports...etc.

In the present work, technical data of spindles of flyer frames, and open-end spinning machines-rotor type has been tabulated in tables (1) & (3). For each type of the spindles, the values of  $EI$ ,  $A$ ,  $U_0$ ,  $U_1$  and  $U_2$  are, also calculated in table (2) & (4). Where  $EI$ -bending stiffness of the spindle,  $A$ -mass per unit length of the spindle,  $U_0$ -ratio between length of the spindle and it's total length,  $U_1$ ,  $U_2$ -ratio between both of overhanging length and the supported length and the supported length of the spindle and its total length.

These values are essential for calculating the eigenvalue " $\beta$ " and the critical speeds. Where the critical speed can be calculated by formula

given by Gorman (3).

$$w_{cr} = \frac{\beta^2}{L} \sqrt{\frac{EI}{\rho A}} \quad (2)$$

Where,

- $w_{cr}$  - Can be the first or second critical speed in  $\text{Sec}^{-1}$ , depending upon the value of  $B$  (first mode or second mode).
- $\beta$  - numerical value which leads to Zero value for Gorman's determinate.
- $L$  - Total length of the spindle.
- $EI$  - Bending stiffness of the spindle blade.
- $E$  - Young's modulus of elasticity. For steel  $E=2 \times 10^6 \text{ Kg/cm}^2$

Table 1. Dimensions of Spindles of Different Rotors.

Type of Rotor	Dimensions of the spindle in mm.			
- Sussen	9	- 56	- 10	, $\varphi = 5$
- BD 200	17.5	- 32.5	- 17.5,	$\varphi = 10$
- BD 200 M69	10	- 40	- 10,	$\varphi = 10$
- BD 200 R(RC)	28	- 38	- 28 -	$\varphi = 10$
- Spx spin Box Rull (Rotor)	30.7	- 35.1	- 30.7,	$\varphi = 11.5$
- Spx spin Box Rull (opening Roller)	29	- 37.5	- 27.5,	$\varphi = 14.15$
- SPE spin Box	Rotor and opening Rotor spindle's as in spx spin Box Rull.			
- Rieter	15	- 57	- 12.5,	$\varphi = 10$

\* In this table, the middle dimension is the length between supports of spindle while the side dimensions are the length of over-hanging extensions of the spindle. All lengths in millimeters.

\*  $\varphi$  - the diameter of the spindle in millimeters.

Table 2. Critical speeds of Shaft's of Rotors

Typ of Toror	Mass per unit length & Bending stiffness of the shaft		Eigen value				critical speed	
	(Kg. Cm )	(Kg.)	U	U	W	W		
- Sussen	6.133x10	1.562x10	0.12	0.75	4.122	7.7805	18927	67436
- BD 200	0.09813x10	6.2480x10	0.26	0.48	4.609	5.859	58421	94406
- BD 200 M69	0.09813x10	6.2480x10	0.17	0.67	4.447	7.210	68843	180964
- BD 200 R(RC)	0.9813x10	6.2480x10	0.304	0.391	4.426	5.317	29005	41858
- Spx spin Box Rull (rotor)	0.1716x10	8.2629x10	0.318	0.364	0.329	5.204	29001	41410
(Opening Roller)	0.3934x10	12.5098x10	0.308	0.399	4.317	5.317	39315	56737
- SPE spin Box as in spx spin	0.3934x10	12.5098x10	0.308	0.399	4.317	5.317	39315	56737
Box Rull.	0.3934x10	12.5098x10	0.308	0.399	4.317	5.317	39315	56737
-- Rieter spin Trainer.	0.09813x10	6.2480x10	0.178	0.675	4.447	7.560	34709	100312

\* E - Young's modulus of elasticity, for steel  $E = 2 \times 10^6 \text{ Kg/cm}^2$ .

I - moment of inertia of cross-section, for circular spindl  $I = \pi \phi^4 / 64$ .

$\rho$  - density of shaft's material, for steel =  $7800 \text{ Kg/m}^3$ .

A - cross-sectional area of spindle for circular shaft it equals to  $\pi \phi^2 / 4$ .

The calculated values of eigenvalues of " $\beta$ " for different spindles are tabulated in table (2) and table (3). According to these values, the critical speeds (first and second) have accordingly been calculated using formulas (2).

The values of calculated critical speeds has been tabled in table (2). From these tables it can be shown that:

1. The first critical speeds of the spindles of rotors and opening rollers ranges from 18927 rad/sec to 68843 rad/Sec. While the second critical speeds of the same spindles lies between 41858 rad./Sec. and 100312 rad./Sec. The spindles of rotors of sussen type given the lowest value in the first critical speed while the spindles of rotors of BD 200 R(RC) give the lowest value in the second critical speed.

This is because of the square ratio between  $\beta_2$  and  $\beta_1$  are different for both type of construction (sussen) & (BD 200 R (RC)).

In Moscow Textile Institute at Department of Design of Textile Machines (5) the value of the first critical speed for the spindle of rotor of BD 200 M 69, has been calculated. This value was found to be 70800 rad./sec. In the present work, the first critical speed was found to be 68843 rad./sec. which represent about 97.24% of that obtained by the Moscow Textile Institute value.

If it is known that, practically the rotor of open-end spinning machine has a running speed ranges between 30,000 r.p.m and 70.000 r.p.m. m and some designers claim that it may reach 100.000 r.p.m., then the critical speed of rotor's spindle whether the

Table 3. Dimensions and Eigenvalues of some spindles.

type of	Dimensions of $u_o=b/L$ of the spindle		Eigenvalues $\beta$ (dimensionless)		
	lower	total	1	2	-
<b>Roving Machines</b>					
- Spindle A	68	770	0.09	2	5
- Spindle B	68	910	0.08	2	5
- Spindle C	68	910	0.08	2	5
- Spindle D	70	1010	0.07	2	5

I. Moment of inertia of cross-section of the spindle, for circular cross-section  $I = \frac{\pi \phi^4}{64}$  ( $\phi$  - diameter of the spindle).

$\rho A$  - Mass per unit length of the spindle.

$\rho$  - Density of spindle's material, for steel = 7800 Kg/m<sup>3</sup>.

A - Cross-sectional area of the spindle, for circular cross-section  $A = \pi \phi^2/4$  ( $\phi$  - diameter of the spindle).

**Results & Discussions**

The eigenvalues " $\beta$ " for spindles of flyer frames have been calculated by the aid of graph demonstrated by Gorman (3) Fig. (3-2) P. 74 -

The eigenvalues " $\beta$ " for spindles of rotors and opening rollers of open-end spinning machines has been calculated by the aid of (Table 4.3 in P. 124) the same text (3). It must be mentioned here, that eigenvalues " $\beta$ " for first mode and second mode of vibrations, have only been chosen.

first or the second critical speeds are; too high with respect to the working speed. This can be shown clearly in formula (1).

2. The spindles of flyers of roving machines has first critical speed lies between 93 rad./sec. and 135 rad./sec., while the second critical speed lies between 584 rad./sec. and 846 rad/sec. Usually the flyers of flyer-frame runs under the first critical speed due to technological limitations.

According to calculation of Efros (2) the lower values of critical speeds of the spindle with flyer ranges between 100 rad./sec. and 150 rad./sec. while the higher values ranges between 300 rad./sec. and 800 rad./sec., depending on the position of bolster rail (bobbin rail).

Table 4. Comparison between measured and calculated values of critical speeds.

Type of spindle	Calculated critical speeds $w_1$ (Efros)	Ratio between Gorman's calculated values and Efros calculated and values $[w_1/w_2]$
- Spindle A	- lower values of critical	
- Spindle B	speeds lies between	
- Spindle C	100 rad./sec and 150	
- Spindle D	rad./sec.	1.08-1.30
	- Higher values of critical	
	speeds lie between	
	300 rad./sec. and 800	
	rad./sec. This depends	
	upon position of bobbin	
	rail.	



Comparing the values obtained by Efros (2) to the present work values as shown in table 4., it can be shown that the difference increases by 8 to 30 % in favour of effors values depending on the construction of the flyer and this difference may be reduced if Efros (2) has calculated the critical speeds of the same type flyers as these of our present study.

### Conclusions

1. The first critical speeds of the spindles of rotors and opening rollers of rotor-type open-end spinning machines ranges from 18927 rad./sec. to 6843 rad./sec. depending on the dimensions of the spindle. The working speed of these spindles with rotors or opening rollers is relatively low.
2. The spindles of the flyers or roving machines has first critical speed lies between 93 rad./sec. and 135 rad./sec. These values is slightly higher than that of the working speeds.
3. The Eigenvalues of Gorman for calculating critical speeds of beams and shafts can be used, safely for calculating critical speeds of spindles or rotor, and opening rollers of the rotor spinning machines and the spindles of flyers of roving machines.
4. The presented simplified method for estimating the critical speeds of shafts has proved to be very useful tool to facilitate the calculation of the working speed of those spindles.

### References

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