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TRANSIENT HEATING OF A HOMOGENUOUS STRONGLY IONIZED PLASMA BY A MICROWAVE FIELD

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ABSTRACT

We analytically solved the average electron energy balance equation of a homogenuous strongly ionized plasma heated by a microwave field. The exact variation of the average normalized electron energy θ with time is calculated for different field frequency ω and different field amplitude E_o. For each case, the heating time τ_h necessary for θ to reach 98 % of its steady state value θ_{ss} is found exactly. It was found that there is an increase in the value of both θ_{ss} and τ_h with the increase of E_o and the decrease of ω.

NOMENCLATURE

E_o	Electric field amplitude
N	Number of steady state values of average electron energy
T	Average electron temperature at time t
T_o	Average electron temperature at time t = 0
m	Electron mass
m_i	Ion mass
E_p	Ginzburg's plasma field
C	Normalized electric field frequency $= \frac{\omega}{\nu_{co}}$
e	Electron charge
x	
A	$1 + \frac{\Gamma^2}{C^2}$
B	$\frac{1}{C^2}$
x_i	Zero of on algebraic 4^{th} degree function
C_i	Constants whose values are given by $\frac{Ax_i^3 + B}{\prod_J (x_i - x_J)}$, $J \neq i$, J = 1,2,3,4
C_t	constant equal to $\sum_{i=1}^{4} C_i$
t	time

Greak symbols

θ	Average normalized electron energy = T/T_o
ω	Electric field frequency
τ_h	Heating time
θ_{ss}	steady average normalized electron energy
ν_{co}	Equilibrium collision frequency of electrons with ions
η	normalized electric field amplitude = E_o/E_p
δ	Fraction of electron energy lost in a collision

with the ions.

τ Normalized time = $_{co}t$

η Normalized field amplitude = E_o/E_p

Γ Variable = $(1 + C^2)^{\frac{1}{2}}$

θ_1 One value of the three steady state average normalized electron energies, i = 1,2,3

τ_{hi} One value of the three heating times, i= 1,2,3

INTRODUCTION

The problem of heating a strongly ionized homogenuous plasma by a microwave field is important because of its applications in the fields of high-power air-borne communication systems and fusion research. The particular case in which the field frequency ω is much greater than the equilibrium collision frequency of electrons with ions ν_{co} has been considered by Ginzburg and Gurevich [1] and El Khamy [2]. When $\omega << \nu_{co}$, the relation between the steady state value of the average electron energy θ_{ss} and the field amplitude E_o was found to be no longer one to one [3]. In a recent work [4], we considered the general case in which ω is not to be compared with ν_{co}. We investigated the effect of ω and E_o on the number N of steady state values of the average electron energy. In this work, for the case in which $\omega \leq \nu_{co}$ we found a necessary and sufficient condition for N to be equal to one and a necessary but not sufficient condition for N to be equal to 3. In a subsequent paper [5] we calculated the steady state values of the average electron energy θ_{ss} for different values of ω and E_o. We found that θ_{ss} increases with the increase of E_o and decreases with the increase of ω. Also we found that the cases in which N = 3 occurs for low values of ω and E_o. Comparison of the values of θ_{ss} with those of the particular cases considered by other investigators [1], [2] showed good agreement. In this paper we present the analytical exact solution of the electron energy balance equation. This solution gives the exact variation of the average electron energy θ with time. We identified the heating time τ_h of the plasma by the microwave to be the time necessary for θ to reach 98 % of its steady state value θ_{ss}. The transient response of θ θ , θ_{ss} and τ_h are calculated for different values of ω and E_o. The discussion of the stability of the plasma during the heating process as well as the stability of the steady state values are left for future investigation.

Basic Equations

The non linear autonomous total differential equation representing the electron energy balance equation of a homogenuous strongly ionized plasma heated by a microwave field was found to be [2], [4] given by

$$\frac{d\theta}{d\tau} = - \frac{[C^2 \theta^4 - (\Gamma^2 + C^2)\theta^3 + \theta - 1]}{\theta^{3/2}(1+C^2\theta^3)} \qquad (1)$$

where

$$\theta = \frac{T}{T_o}, \quad \tau = \delta \nu_{co} t, \quad C = \frac{\omega}{\nu_{co}}$$

$$\Gamma = \eta\ (1+C^2)^{\frac{1}{2}}, \quad \eta = \frac{E_o}{E_p}$$

T, T_o are the average electron temperatures at time t and at time $t = 0$ respectively, δ is the mean fraction of electron energy lost in a collision with the ions (for an elastic collision $\delta = \frac{2m}{m_i}$ where m, m_i are the electron and ion masses respectively), and E_p is the Ginzburg's plasma field given by

$$E_p = [3m\delta \nu_{co}^2 T_o (1+C^2)/e^2]^{\frac{1}{2}}$$

where e is the electron charge

Equation (1) can be written in the form

$$\frac{dx}{d\tau} = - \frac{[\ x^4 - Ax^3 + Bx - B\ ]}{x^{3/2}(x^3+B)} \qquad (2)$$

where $x = \theta$, $A = 1 + \frac{\Gamma^2}{C^2}$, $B = \frac{1}{C^2}$

The solution of equation (2) that satisfies the initial condition $\theta = 1$ when $\tau = 0$ is found to have the form

$$\tau = \frac{2}{3}(1-x^{3/2}) + 2C(1-x^{\frac{1}{2}}) + \sum_{i=1}^{4} C_i \sqrt{x_i}\ \ln \frac{(\sqrt{x_i} + \sqrt{x})\ (\sqrt{x_i} - 1)}{(\sqrt{x_i} - \sqrt{x})\ (\sqrt{x_i} + 1)} \qquad (3)$$

where x_i, i = 1,2,3,4 are the four zeros of the algebraic 4^{th} degree function represented by the numerator of equation (2) which has the form

$$f_o(x) = x^4 - Ax^3 + Bx - B \tag{4}$$

and C_i , i=1,2,3,4 are constants whose values are given by

$$C_i = \frac{A\,x_i^3 + B}{\prod_J (x_i - x_J)} \tag{5}$$

where $J \neq i$, J =1,2,3,4 and $C = \sum_{i=1}^{4} C_i$

The nature of the zeros of function (4) was discussed by El-Khoga and Negm [4]. These zeros satisfy one of the following two cases; x_1, x_2 are complex conjugate zeros x_3 +ve real zero, x_4 negative real zero or or x_1 , x_2 x_3 are all positivereal zeros and x_4 is a negative real zero. For the first case N=1, $\theta_{ss} = x_3$ and τ_h can be calculated from equation (3) by substituting 0.98 x_3 for x. For the second case we have 3 steady state values of θ namely θ_1, θ_2 and θ_3 which are equal to x_1, x_2 and x_3 respectively. Hence there are three corresponding heating times τ_{h1}, τ_{h2} and τ_{h3}. These heating times can be obtained from equation (3) by substituting 0.98 x_1, 0.98 x_2 and 0.98 x_3 for x respectively.

RESULTS

When C = 0.01, it was found by El-Khoga and Negm [5] that for $\eta^* = 0.1$, x_1, x_2, x_3 and x_4 are 1.015d, 9.90112, 100 and -9.9517 respectively. It has been found by Gurevich [3] that x_2 is unstable and x_3 is unacessible. Also since $x_1 \simeq 1$ hence no appreciable heating has happened.

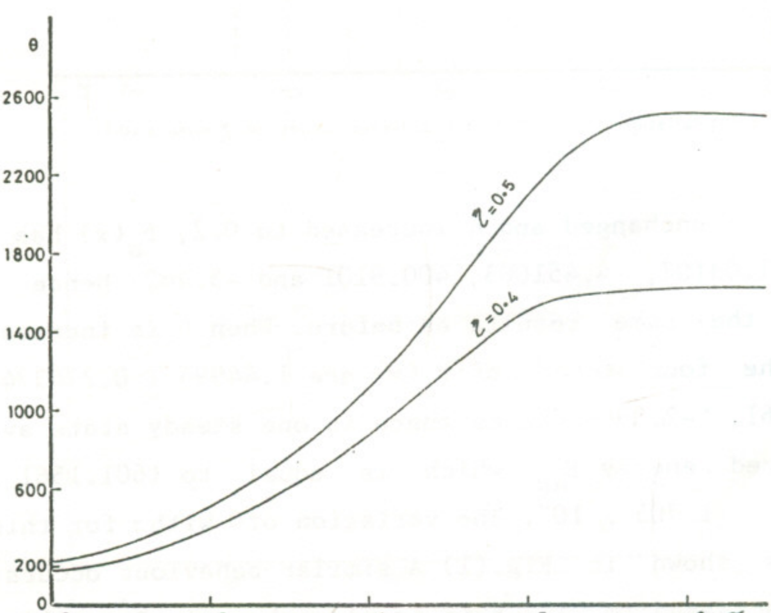

Fig.(1) Variation of θ with τ for tow values of ζ when c=0.01

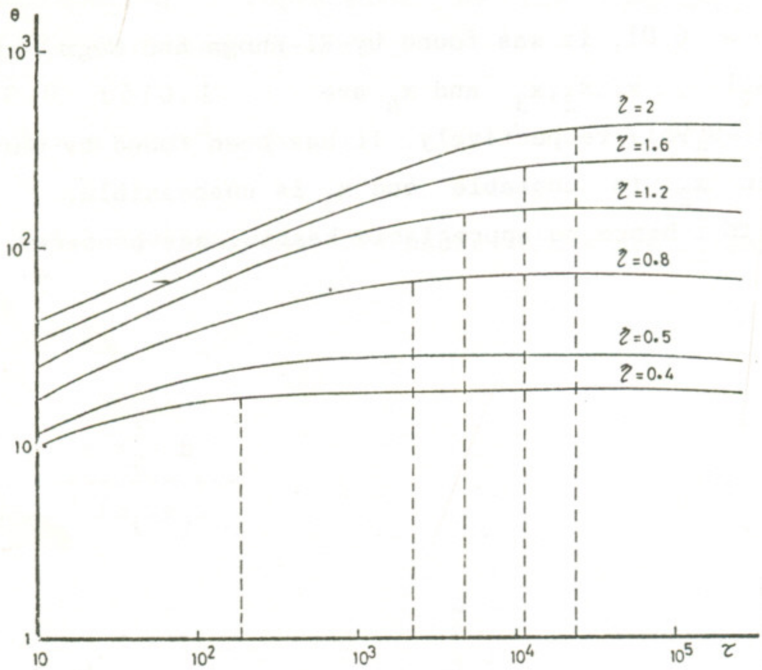

Fig. (2) Variation of θ with τ for different values of η when c = 0.1

When is unchanged and η increased to 0.2, $f_o(x)$ has the zeros 1.03102, 4.451003, 400.9101 and -5.442, hence we deduce the same results as before. When η is increased to 0.4, the four zeros of $f_o(x)$ are $1.44995 \pm 0.274374\ i$, 1601.1561, -2.8960 hence there is one steady state average normalized energy θ_{ss} which is equal to 1601.1561 for which $\tau_h = 1.705 \times 10^5$. The variation of θ with τ for this case is shown in Fig.(1) A similar behaviour occurs when η is increased to 0.5, this is also shown in Fig. (1). Both θ_{ss} and τ_h increases with the increase of η i.e. with the increase of the field amplitude E_o. When C is increased to 0.1, the variation of θ with τ for different values of η are shown in Fig. (2). It is obvious that both θ_{ss} and τ_h decreases with the increase of C i.e with the increase of the field frequency ω. These results are affirmed by the results presented in Fig. (3)&Fig. (4). In tables (1) to (7) we present accurate values of the results presented in Figs (1) to (4). Thus, it is theoretically possible to raise the electron energy to high levels by increasing the field amplitude and decreasing the field frequency; however the time needed for such a heating process will be comparatively long. By increasing C to values 1, θ_{ss} and τ_h are decreased and the results obtained agree with those previously obtained by El-Khamy [2].

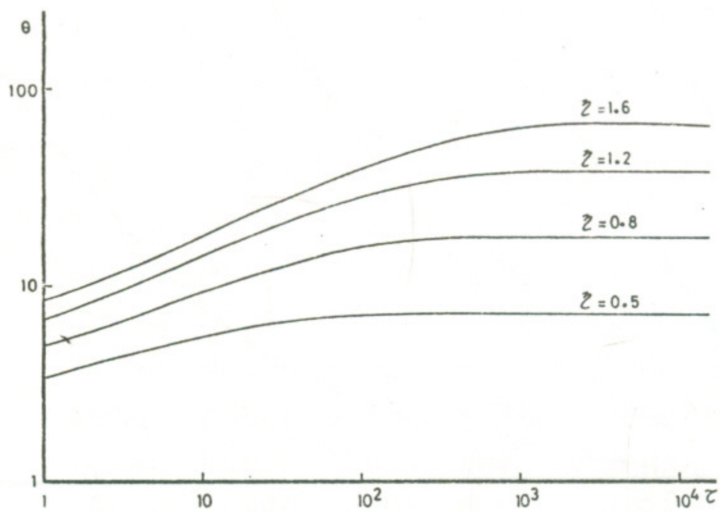

Fig. (3) Variation of θ with τ for different values of ζ when c=0.2

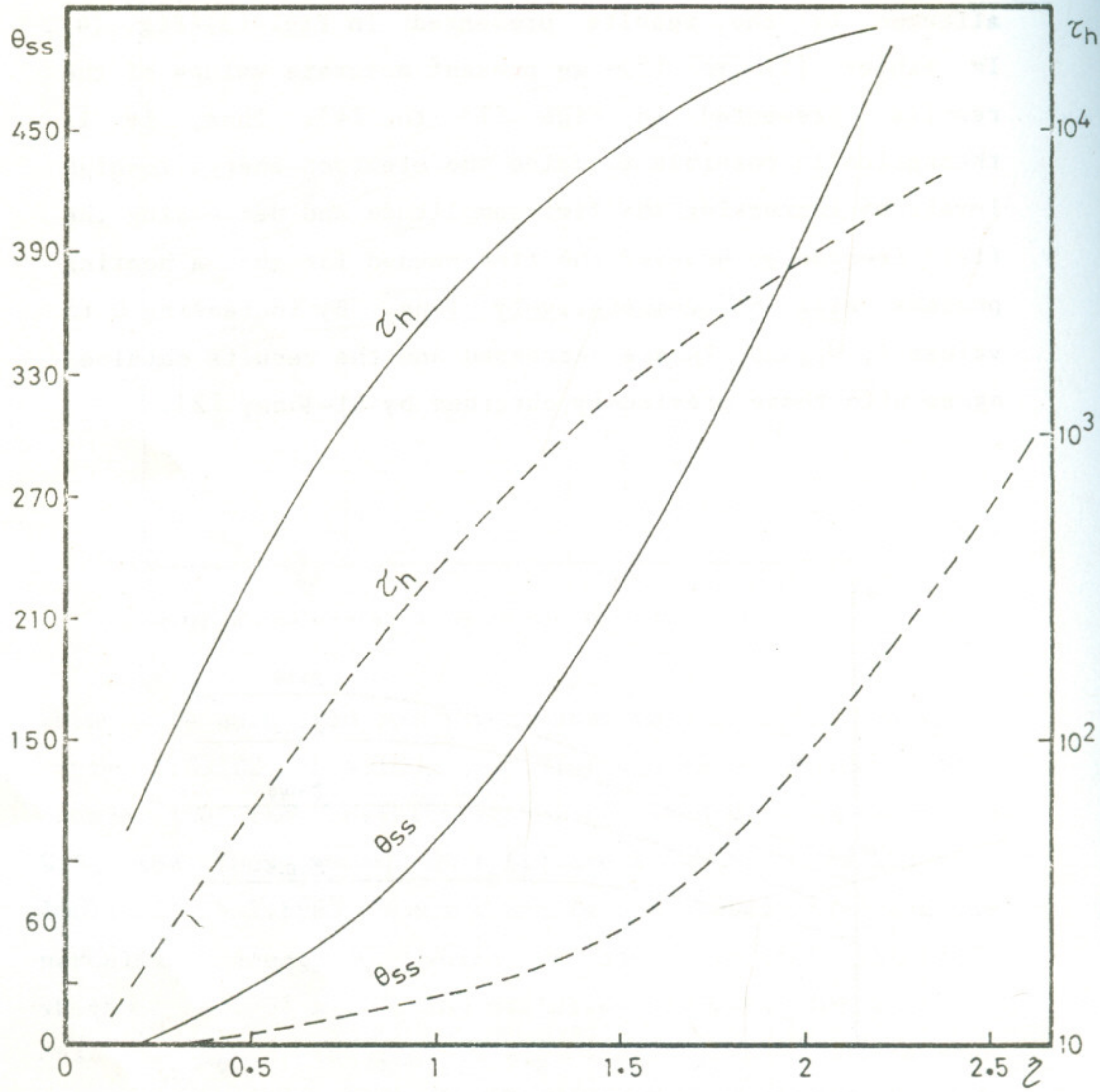

Fig. (4) Variation of θ_{ss} and τ_h with ℓ

——C = 0.1 , ----C = 0.2

Table 1: Variation of normalized average electron energy with normalized time τ when C = 0.01 and η = 0.4

θ	τ	θ	τ	θ	τ
69.0694	0.029309	144.104	111.969	240.173	220.788
288.208	373.866	336.243	578.432	400.289	944.758
448.324	1300.7	496.358	1736.94	560.405	2461.9
608.439	3128.99	656.97	3917.42	704.51	4843.47
752.543	5926	800.578	7186.83	848.613	8652.02
896.647	10352.4	944.682	12325.5	992.717	14616.9
104075	17283.8	1088.79	20398.4	1136.82	24054.3
184.86	28375.1	1232.89	33531.1	1280.92	39762.9
1328.96	47429.5	1376.99	57100.1	1425.03	69759.5
1473.06	87348.7	1521.1	114626	1569.13	170474

Table 2: As table 1, but for C = 0.01 and η = 0.5

θ	τ	θ	τ	θ	τ
75.0374	0.594412	150.075	35.6294	225.112	117.16
300.15	258.37	375.187	472.368	450.225	772.61
5252.262	1173.2	600.3	1689.37	700.349	2585.89
775.387	3438.6	850.424	4468.5	925.462	5699.14
1000.5	7157.19	1075.54	8872.62	1150.57	10880.6
1225.61	13221.7	1300.65	15944	1300.65	15944
1375.63	19104.1	1450.72	22770.5	1525.76	27026.8
1600.8	31972.2	1675.84	37748.6	1750.87	44511.2
1825.91	5283.8	1900.95	61963.6	1975.99	73367.3
2051.02	87304.6	2126.06	104721	2201.1	127200
2276.14	157707	2276.14	157707	2351.17	202906
2426.21	284048	2451.22	332906		

Table 3: As table 1, but for C = 0.1 and η = 0.4

θ	τ	θ	τ	θ	τ
1.0098	0.001812	1.5147	0.084485	2.0196	0.158145
2.5245	0.24614	3.0294	0.373254	3.5343	0.557891
4.2075	0.916078	4.7124	1.28296	5.2175	1.74637
5.7222	2.31804	6.2271	3.01141	6.732	3.84154
7.2369	4.82563	7.9101	6.4125	8.415	7.83977
8.9199	9.50242	9.4248	11.4352	9.9297	13.6802
10.4346	16.2894	10.7395	19.327	11.4444	22.875
11.9493	27.0389	12.4542	31.9591	12.9591	37.8279
13.464	44.9182	13.9689	536361	14.4738	64.6267
14.9787	79.0052	15.4837	98.9698	15.9885	129.91
16.9934	193.215				

Table 4: As table 1, but for C=0.1 and and η= 0.8

θ	τ	θ	τ	θ	τ
7.87406	0.06444	9.84257	0.845262	11.8111	2.01159
13.7796	3.61767	15.7481	5.72486	17.7166	8.40101
19.6851	11.7218	21.6537	15.7712	23.6222	20.64448
25.5907	26.4495	27.5592	33.3077	29.5277	41.3618
31.4962	50.7747	33.4647	61.7287	35.4333	74.4827
37.4018	94.7225	40.0265	112.784	41.995	133.795
43.9635	158.309	45.932	187.036	47.9005	220.91
49.869	261.198	51.8376	309.669	53.8061	368.919
55.7746	442.968	57.7431	538.551	59.7116	668.286
61.680	860.508	63.6486	1205.62	64.3048	1413.42

Table 5: As table 1, but for $C = 0.2$ and $\eta = 0.8$

θ	τ	θ	τ	θ	τ
3.33706	0.0261128	3.86397	0.227175	4.39088	0.511687
4.91778	0.886871	5.4469	1.362	5.97159	1.94808
6.4985	2.65817	7.0254	3.50748	7.5523	4.51334
8.07921	5.69633	8.60611	7.08038	9.13302	8.69379
9.65593	10.5702	10.1868	12.7502	10.7137	15.2835
11.2406	18.2315	11.7675	21.6716	12.2945	25.7033
12.8214	30.4576	13.3483	36.1116	13.8752	42.9137
14.4021	51.2273	14.925	61.616	15.4559	75.0237
15.9828	93.2188	16.5097	120.173	17.0366	168.556
17.2122	197.686				

Table 6: As table 1, but for $C = 0.2$ and $\eta = 1.6$

θ	τ	θ	τ	θ	τ
4.05328	0.008492	6.07992	0.306207	8.10656	0.880746
10.1332	1.78517	12.1598	3.07719	14.1865	4.81825
16.2131	7.07490	18.2398	9.91978	20.2664	13.4327
2.293	17.7025	24.3197	22.8291	26.3463	28.9244
28.373	36.1173	30.3996	44.5558	32.4246	54.4105
34.4529	65.8824	36.4795	79.21	38.5062	94.6778
40.5328	112.633	42.5594	133.501	44.5861	157.816
46.6127	186.256	48.6394	219.712	50.666	259.372
52.6926	306.881	54.7193	364.61	56.7459	436.159
58.7725	527.384	60.7992	648.802	62.8258	822.398
64.8525	1108.5	66.2036			

Table 7: Variation of $_{ss}$ and $_{h}$ with C and

C		$_{ss}$	$_{h}$
0.01	0.4	1601.156	170474
0.01	0.5	2501.2485	332906
0.1	0.2	1.05	1.14058
0.1	0.4	16.83	193.215
0.1	0.5	26.1106	357.933
0.1	0.8	65.61715	1413.42
0.1	1.2	146.4353	4714.25
0.1	1.6	259.5585	1127.5
0.1	2	404.99939	21690.8
0.2	0.5	7.0705	59.0279
0.2	0.8	17.5635	197.686
0.2	1.2	38.4235	633.719
0.2	1.6	67.55465	1476.86

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