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INFIUENCE OF PRANDTL NUMBER AND BOUNDARY CONDITIONS ON NATURAL CONVECTION HEAT TRANSFER IN VERTICAL AND INCLINED FLUID LAYERS

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ABSTRACT

A numerical study was carried-out to investigate the influence of Prandtl number and boundary conditions on natural convection heat transfer in vertical and tilted fluid layers. The fluid layer is bounded by two flat isothermal plates at different temperatures and either perfectly conducting or adiabatic boundaries at top and bottom ends of the layer. Calculations are reported for five values of Prandtl number ($Pr = 10, 0.72, 0.1, 0.01$ and 0.001) and four values of aspect ratio ($A = 1,5,10,$ and 20). Rayleigh number was varied between 10 and 10 , and the angle of tilt was varied between 90° (vertical) to 180° (horizontal) with heating upward. The flow pattern, velocity and temperature profiles, and Nusselt number–Rayleigh number relationship are presented. Wall heat conduction and decrease of Prandtl number was found to reduce the average values of Nusselt number. The present results are in good aggrement with other published experimental and numrical investigations.

NOMENCLATURE

A	aspect ratio of cavity, $A = H/L$	
C_p	specific heat at constant pressure	$kJ/kg.^{o}C$
g	gravitational acceleration	m/s^2
h	coefficient of heat transfer	$W/m^2.^{o}C$
H	height of cavity	m
k	thermal conductivity	$W/m.^{o}C$
L	width of cavity	m
Nu	average Nusselt number, $Nu = hL/k$	
Pr	Prandtl number, $Pr = \nu / \alpha$	
q	heat flux	W/m^2
Ra	Rayleigh number, $Ra = L^3 \beta g(T_h - T_c)/\nu\alpha$	
t	non-dimensional time	
T	temperature	^{o}C
u,v	non-dimensional veloctiy components in x,y directions	
x,y	non-dimensional coordinates	

Greek

α	thermal diffusivity , $\alpha = k/ \rho C_p$	m^2/s
β	coefficient of thermal expansion	$1/^{o}C$
φ	angle of tilt	deg.
ρ	density	kg/m^3
ν	kinematic viscosity	m /s
θ	non-dimensional temperature $\theta = T-T_c/T_h-T_c$	

Subscript

c	cold surface
h	hot surface
i,j	integer counters, at the center of cells in x,y-

directions

Abreviations

LTP linear temperature profiles (perfectly conducting B.C)

ZHF zero heat flux (adiabatic B.C.)

B.C. boundary conditions.

INTRODUCTION

Heat transfer by natural convection across vertical and inclined fluid layers of different values of Prandtl number and boundary conditions is of a great interest in many engineering fields. Much research have been carried-out for moderate values of Prandtl number and for adiabatic top and bottom enclosure boundary conditions. Equipment components with enclosed fluid layers, double glazed windows, solar collectors, and cavities in building materials are few examples. Batchelor (1954) was probably the first to study the flow regimes in such layers and gave approximate relations for heat transfer. Analytical studies were given by Raithby et al.,(1977) and Emery and Chu (1965) where conduction, laminar and turbulent flow regimes were defined. Several experimental studies were performed on these layers and a complete set of data for a wide range of Ra number and aspect ratio A were given by Elsherbiny et. al (1980). Less attension was, however, given to the study of low Prandtl numbers and for perfectly conducting boundary conditions which is the subject of the present study.

The fluid layer, as shown in Figure (1), consists of two flat

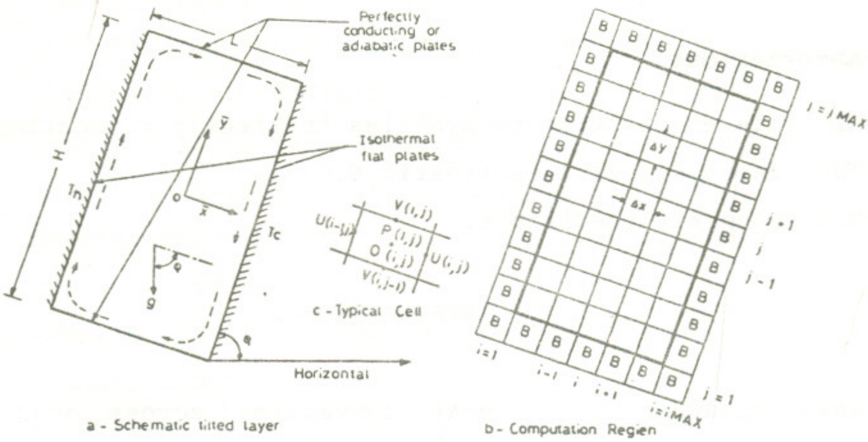


Fig.(1) Fluid Layer Geometry & Numerical Representation

and parallel isothermal plates of height H and spacing L. One plate is heated at T_h and the other is cooled at T_c. Two limiting cases of the boundary conditions can be used at both top and bottom ends of the fluid layer. The first is perfectly conducting with a linear temperature profile (LTP) between the two plates, and the second is adiabatic with zero heat flux (ZHF) between fluid and end walls.

In the present study, calculations will be given for aspect raties, A, of 1,5, 10 and 20 over a range of Rayleigh numbers from 10^3 to 10^7. The two limiting cases of top and bottom walls boundary conditions, LTP and ZHF, will be investigated over the range of tilting angle, from 90^o(vertical) to 180^o (horizontal with upwards heating). Fluid layers Prandtl numbers Pr = 10, 0.72, 0.1, 0.01, and 0.001 will be studied.

MATHEMATICAL ANALYSIS

For two-dimensional laminar natural convection inside cavities, with the Boussinesq approximation, the governing equations of mass, momentum, and energy equations can be written in a non-dimensional form as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x^2} + \frac{\partial uv}{\partial y} = - \frac{\partial P_d}{\partial x} + Pr.\ \nabla^2 u - Ra.Pr.\theta\ .\ \cos \qquad (2)$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial y} + \frac{\partial uv}{\partial x} = - \frac{\partial P_d}{\partial y} + Pr.\ \nabla^2 u + Ra.Pr.\theta\ .\ \sin \varphi \qquad (3)$$

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial\partial t} + u\ \frac{\partial\theta}{\partial\partial x} + v\ \frac{\partial\theta}{\partial y} = \nabla^2\theta \qquad (4)$$

The mathematical analysis and the numerical technique used in this study is similar to that given in reference [8].

Mesh Size, Accuracy, and Computation Time

In the present study, the fluid layer was divided into (10x10) cells and surrounded by a single layer of boundary cells making the computational matrix (12x12). The steady-state solution was obtained as the limit of the transient calculations using a non dimensional time, t as low as $8x10^{-4}$. This gave an average computational time per cell per time step of about 0.015 s of CPU on the PDP 70/11 computer available at Alexandria University. This time increased to about 0.02 s of CPU at A = 10 and Ra = 10^5. When the mesh size was reduced to give (20x20) cells, the

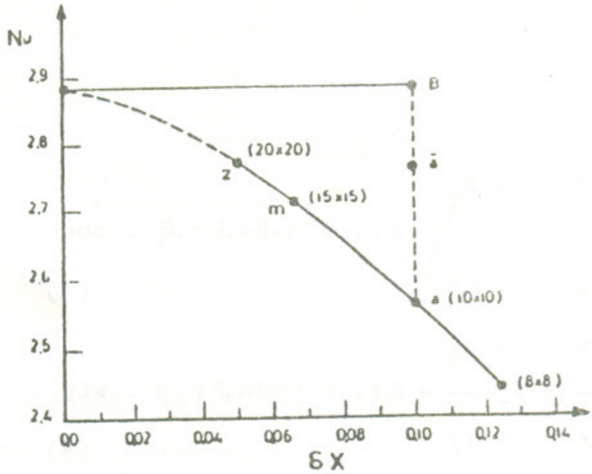


Fig. (2) Effect of Mesh Size on Average Nusselt Number

time per cell per time step was increased to 0.06 s of CPU. Due to the large CPU time required if (20x20) grid was used and since we need to study the effect of several parameters, it was decided to use (10x10) grid. In order not to loose the higher accuracy of (20x20) grid, an interplation procedure developed by Ozoe (1976) was applied. In this way, the computation was completed with (10x10) mesh to reach the steady state solution. Linear interpolation was used to obtain additional points for a smaller cell size of (20x20) grid. New transient computation was performed to reach a new steady state. It was found that, (see Fig. 2), the average Nusselt numbers computed from the (20x20) interpolated grid before, (point a), and after running the new transient solution, (point a'), were almost the same within less than 2%. In this way the computation time was reduced with the same higher accuracy attained for (20x20). The reported results are those of (20x20) interpoleted grid developed from the (10x10) steady state results.

NUMERICAL RESULTS

The effect of Prandtl number on both velocity and temperature profiles is shown in Figure (3). Decreasing Prandtl number retards the velocity of circulation which shifts towards conduction regime profiles. This trend is more apparent in the temperature profile where linear profile occurs almost at Prandtl number equals 0.001. A sample of the flow pattern inside square cavity is shown in Figure (4). Although the flow is characterised by a single vortex fills the whole cavity, the circulation velocity was found to diminishes as Prandtl number decreases.

Heat Transfer Results

The results for vertical fluid layers (of Pr = 10.0 and Pr = 0.72) are shown in Figure (5) for different aspect ratios and boundary conditions. Increasing aspect ratio decreases the average values of Nusselt number. In addition, the computed Nusselt number for ZHF are usually higher than that for LTP.

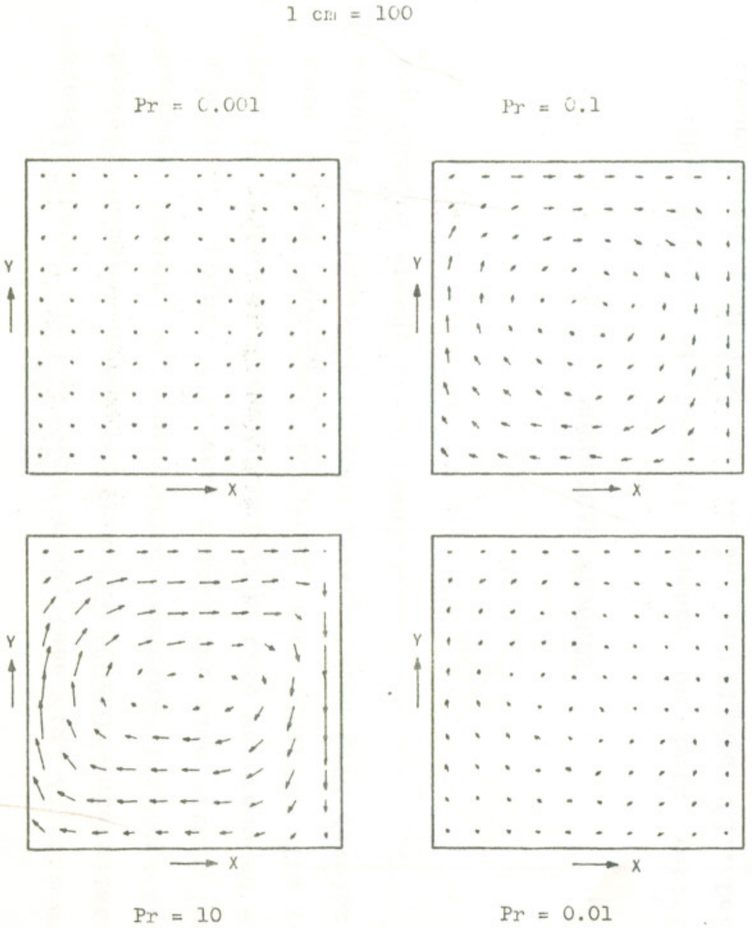


Fig. (3) Effect of Prandtl Number on Flow Pattern
(A=1 , ϕ=90° , and Ra=10^5, ZHF)

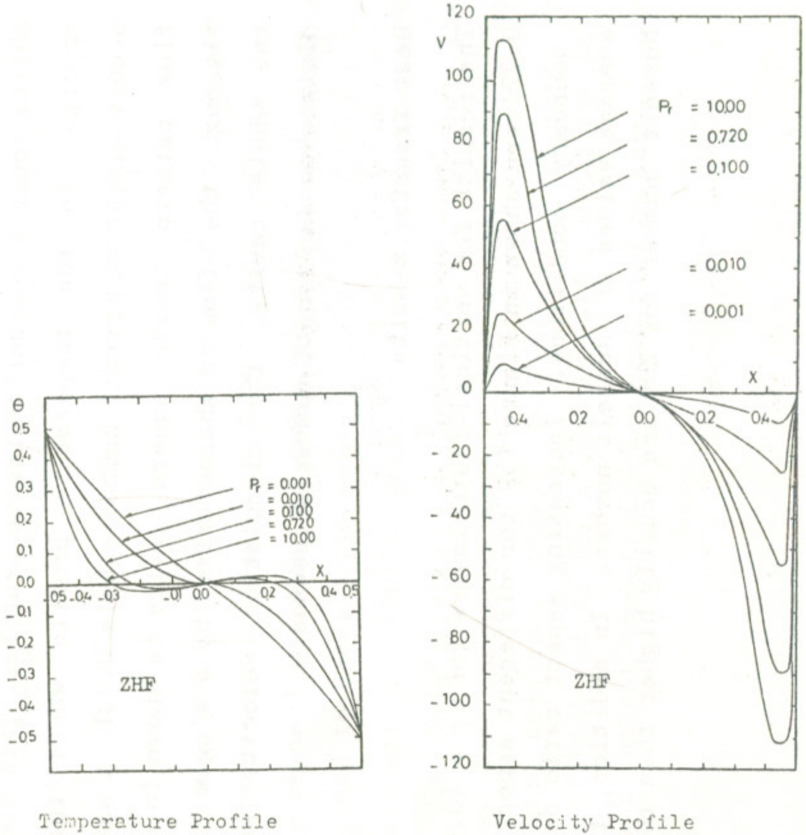
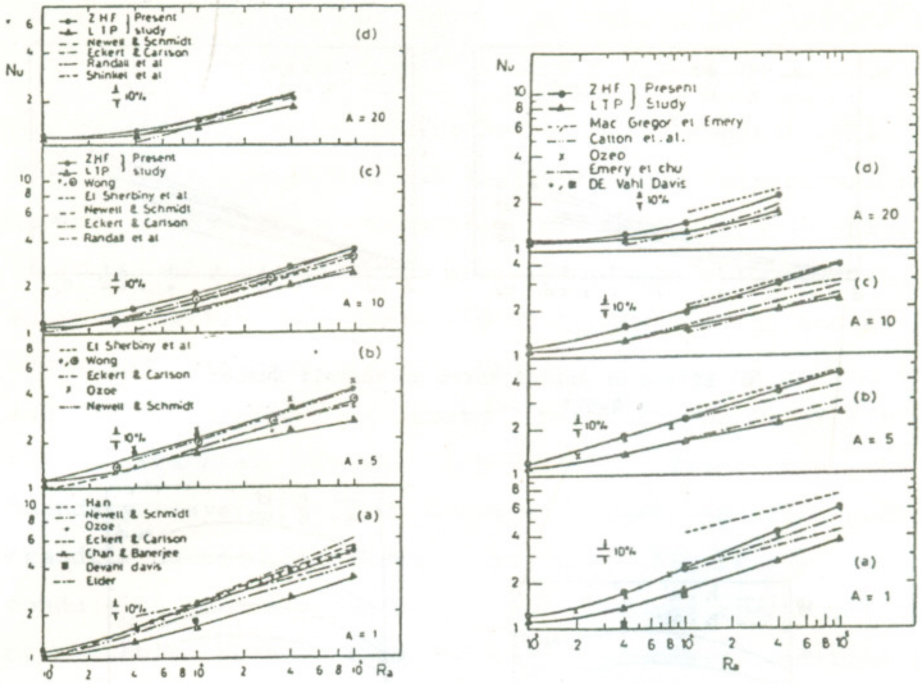


Fig. (4) Velocity and Temperature Profiles For Different Prandtle Numbers (A=1 , ϕ=90° , and Ra=10^5)



Pr = 0.72 Pr = 10

Fig. (5) Average Nusselt Number For Vertical Layers

The present results show good agreement with the experimental and numerical published results. Comparison between the numerical results with Pr=0.72 and Pr=10 is shown in Figure (6) for different values of aspect ratios and boundary conditions.

In Figure (7) comparison given for different values of tilt angle from 90° (vertical) to 180° (horizontal). The differences of results between Pr=0.72 and Pr=10 are more pronounced at high Rayleigh numbers and low aspect ratios. A continuous change in Nusselt number was found as tilt angle

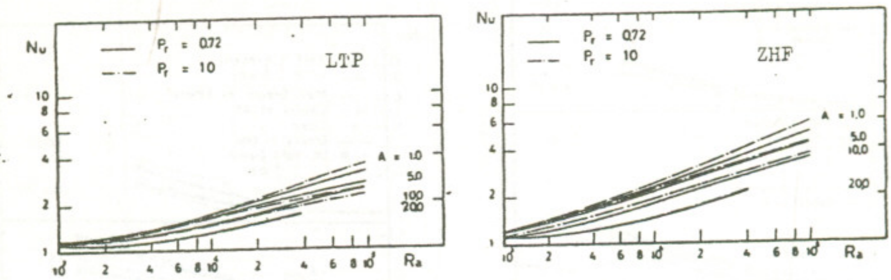


Fig.(6) Effect of Aspect Ratio on Nusselt Number
(ϕ=90° , A=1)

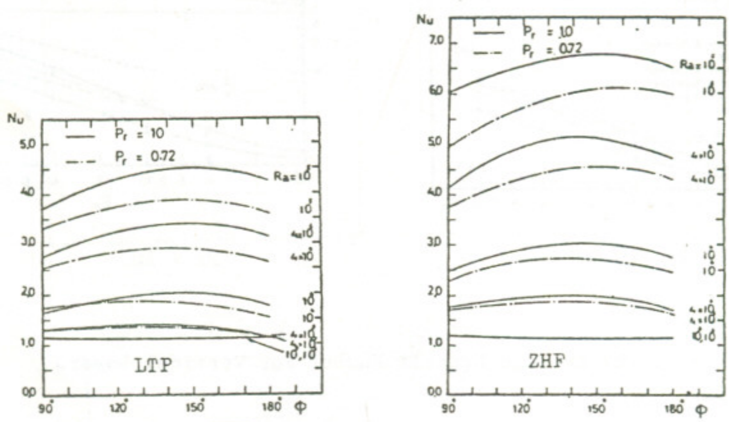


Fig.(7) Effect of Tilt Angle on Nusselt Number
(ϕ = 90° , A = 1)

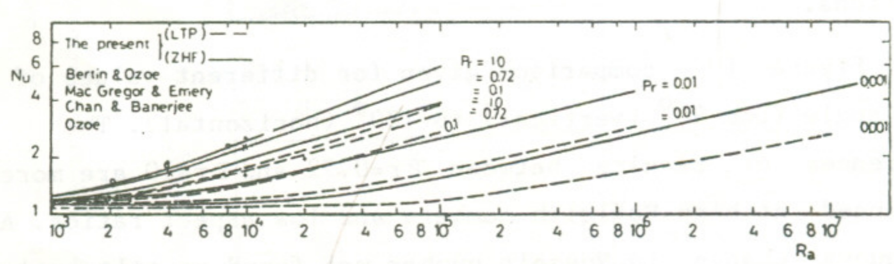


Fig.(8) Nu - Ra Relationship (ϕ=90°,A=1) increases with maximum values lie between 120° and 165°. For low Ra the maximum Nusselt number occurs at $\varphi = 90^{\circ}$.

The effect of Prandtl number and boundary conditions on Nu-Ra relationship is also shown in Figure (8). Decreasing Prandtl number leads to a reduction of Nu for the same Ra. Some limited published results are also given for comparison, for example Bertin and Ozoe (1986) for Pr = 0.01, and Ra 2500, Chan and Benergee (1979), Ozoe (1976) for Pr = 0.72 and 10 and Mac Georegor and Emery (1969). The oncet of different heat transfer regimes (conduction, transition, laminar boundary layer) shifts towards higher Rayleigh number as Prandtl number decreases, see also Figure (9). Wall heat conduction boundary condition (LTP) reduces always the values of Nusselt number than adiabatic boundary condition (ZHF) which agrees with the results of Kim and Wiskanta (1985).

Correlation of Nu-Ra Relationship

The complexity of the dependence of Nu on Ra, A, Pr and rules-out obtaining a single equation that correlates all results. Therefore, different equations that covers mainly the influence of boundary conditions and Prandtl number for vertical enclosures
($\varphi = 90^{\circ}$) are given below:

(a) Pr = 0.72 or 10

(i) ZHF, $(10^{3} \leq Ra \leq 10^{5})$, $(1 \leq A \leq 20)$

$$Nu=[(1.179)^{20}+(0.133\, Ra^{0.32}\, A^{-0.169}\, Pr^{0.028})^{20}]^{0.05}$$

(ii) LTP, $(10^3 \le Ra \le 10^5)$, $(1 \le A \le 20)$

$$Nu=[(1.145)^{20}+(0.214\,Ra^{0.238}\,A^{-0.123}\,Pr^{0.001})^{20}]^{0.05}$$

(b) $10 \ge Pr \ge 10^{-3}$, $A = 1$

(ZHF) $Nu = 0.373\ Ra^{0.202}\ Pr^{0.089}$

(LTP) $Nu = 0.323\ Ra^{0.187}\ Pr^{0.109}$

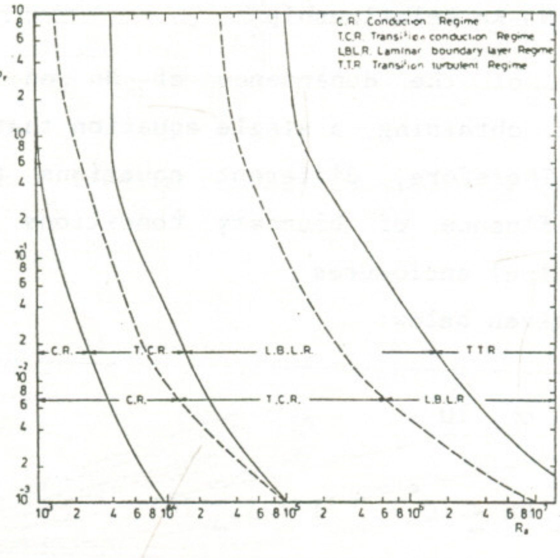


Fig. (9) Effect of Pr on Shifting Convection Regimes

($\phi=90^{\circ}$, A=1)

CONCLUSION

The effect of Prandtl number and boundary conditions on natural convection heat transfer inside rectangular enclosures have been studied for different aspect ratios and tilt angles. The results indicated that the maximum heat transfer rates occur at tilt angles between 120^{o} to 160^{o} depending upon the value of Rayleigh numbers. The influence of Prandtl number is more pronounced at low aspect ratio and high Rayleigh numbers. The computed values of Nusselt number for ZHF are usually higher than that for LTP. A correlation equations for Nu-Ra relationship are also given.

REFERENCES

[1] Arnold, J.N., Catton I., and Edwards D.K., 1976 "Experimental Investigation of Natural convection in Inclined Rectangular Regions of Differing Aspect Ratios", J. Heat Transfer, Vol. 98, pp. 67-71.

[2] Batchelor G.K., 1954 "Heat Transfer by Free Convection Across a Closed Cavity Between Vertical Boundaries at Different Temperature," Quart. of Applied Mathematics, Vol. XII, PP. 209-233.

[3] Bertin H. and Ozoe H, 1986 "Numerical Study of Two-Dimensional Natural Convection: A Horizontal Fluid Layer Heated from Below, By Finite Element Method: Influence of Prandtl Number," J. Heat Mass Transfer, Vol. 29, PP. 429-449.

[4] Chan A.M.C. and Banerjee S., 1979 "Three- Dimensional Numerical Analysis of Transient Natural Convection in Rectangular Enclosure," J. Heat Transfer, Vol. 101, PP. 114-119.

[5] De Vahl Davis G., 1968 "Laminar Natural Convection in an Enclosed Rectangular Cavity", Int. J. Heat Mass Transfer, Vol. 11, PP. 1975-1693.

[6] Eckert E.R.G. and Carlson W.D., 1961 "Natural Convection in an Air Layer Enclosed Between Two Vertical Plates with Different Temperatures", Int. J. Heat Mass Transfer, Vol. 2, PP. 106-120.

[7a] Elsherbiny S.M., Raithby G.D. and Hollands K.G.T. 1982, " Heat Transfer by Natural Convection Across Vertical and Inclined Air Layers", J. Heat Transfer, Vol. 104, PP. 96-102.

[7b] Elsherbiny S.M., Fath H.E.S. and Refai G. 1988 "Numerical Study of Natural Convection in Vertical and Inclined Air Layers," J. of Modelling and Simulation Techniques, Vol. 19, No. 8, PP. 33-40.

[8] Emery A. and Chu N.C., 1965 "Heat Transfer Across Veryical Layers, J. Heat Transfer, Vol. 87, PP. 110-116.

[9] Gill A.E. 1966, "The Boundary Layer Regime for Convection in a Rectangular Cavity", J. Fluid Mechanics, Vol. 26, PP. 515 - 536.

[10] Hirt C.W. , Nichols B.D. and Romero N.C. 1975, "SOLA-A Numerical Solution Algorim for Transient Flows," LA-5852, Los Alamos.

[11] Kim D.M., and Viskanta R. 1985, "Effect of Wall Heat Conduction on Natural Convection Heat