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GEOMETRICAL CONFIGURATION OF STRAIGHT EMBANKMENTS IN TRANSITION ZONES

Magued N. Abdel Malek*, M.Sc.; Ph.D.,
Youssef F. Girges**, M.Sc.

* Lecturer, Faculty of Engineering, Alex. University.
** Assistant Lecturer, Faculty of Engineering, Alex. Univ.

ABSTRACT

The geometrical construction and properties of transition zones in embankments are seldom discussed in the literature.

The main objective of this paper is to find the suitable geometrical construction giving a safe and stable embankment within a transition zone. Three different cases are considered in the study, in each, the side surfaces, their properties and areas were determined. Also, the embankment volume and the minimum transition length were derived. Finally dimensionless relationships were presented for engineering practical use.

rformed gradually from h_1 to h_2 along the inclined top rface of the embankment (see Figure 1). "h" changes nearly according to the relationship

$$h = b\ell + f \tag{1}$$

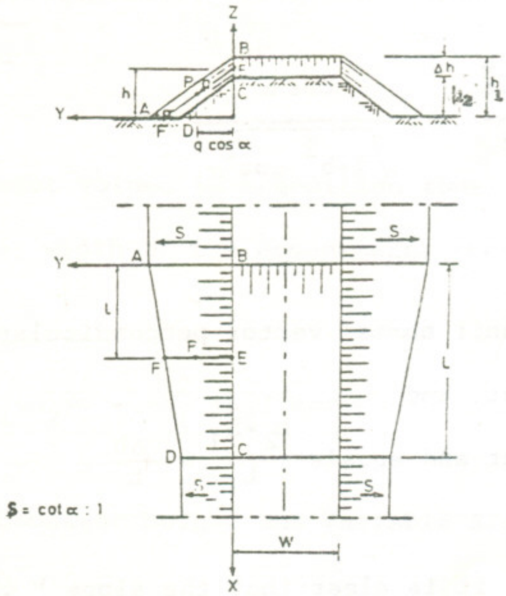


Fig. 1 Embankment with variable height and constant side slope.

As the slope of the side surfaces is the same before and after the transition zone, the side surface *ABCD is a plane* defined by the two parallel straight lines AB and DC. It is clear that any frontal section intersects the side surface in a straight line EF parallel to AB and DC.

2.1.1 Maximum Slope

The vector equation which represents the plane ABCD in the two parameters ℓ,q is :

$$\vec{r}(\ell,q) = (\ell)\vec{i} + (q \cos \alpha)\,\vec{j} + (h - q \sin \alpha)\,\vec{k} \tag{2}$$

where ℓ : is the horizontal distance measured in the direction of the X-axis.

q is the distance measured on the inclined line EF from the top of the embankment; and

α is the slope angle of the side surface.

Therefore, the slope " ϕ" of the plane ABCD is given by ;

$$\cos \phi = \frac{\vec{N}.\vec{K}}{1.1} = \frac{\cos \alpha}{\sqrt{1+b^2 \cos^2 \alpha}} \qquad (3)$$

where;

$\vec{N}$ is the unit normal vector perpendicular to the plane,

α is constant; and

b is constant and equals $\frac{h_2 - h_1}{L} = \frac{\Delta h}{L}$

From Eq. (3) it is clear that the slope " ϕ" of the plane ABCD is greater than "α " .

2.1.2 Minimum Transition Length

The minimum transition length "L_{min} " is obtained from Eq. (3) as follows:

$$L_{min} = \frac{(h_1 - h_2) \cos \alpha \cdot \cos \phi}{\sqrt{\cos^2 \alpha - \cos^2 \phi}} \qquad (4)$$

2.1.3 Side surface area of transition zone

The surface area "A" of the plane ABCD is calculated as follows;

$$A = \frac{(h_1+h_2)}{2 \sin \alpha} \sqrt{L^2 + (h_1-h_2)^2} \qquad (5)$$

2.1.4 Embankment volume in transition zone

If w is the width of the embankment, then the embankment volume "V" is

$$V = \tfrac{1}{2} \left[(h_1+h_2)w + (h_1^2 + h_2^2) \cot \alpha \right] . L \qquad (6)$$

2.2 Case of Constant Height and Variable Side Slope

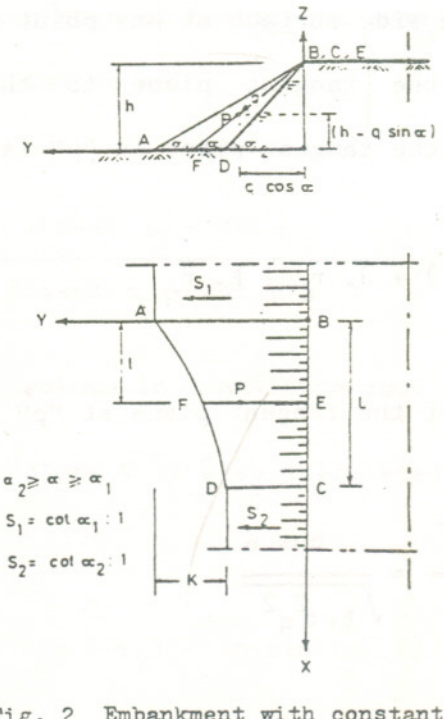


Fig. 2 Embankment with constant height and variable side slope

Figure (2) illustrates this case. The embankment height is kept constant while the slope of the side surfaces changes linearly from "α_1" to "α_2" according to the relationship

$$\alpha = c\,\ell + d \tag{7}$$

The side surface of the transition zone is limited by four lines AB,BC,CD and DA. The surface is bounded by a group of non-coplanar frontal straight lines which intersects BC and DA. A surface with such properties is defined as a ruled surface whose generator is the straight line EF, and is represented by the vector equation

$$\vec{r}(\ell,q) = (\ell)\vec{i} + (q\cos\alpha)\,\vec{j} + (h - q\sin\alpha)\,\vec{K} \tag{8}$$

2.2.1 Maximum slope of side surface

The slope of the side surface at any point "p" is defined as the slope of the tangent plane to this surface at "p" The equation of the tangent plane at "p" is

$$\vec{R}(J,K) = \vec{r}(\ell,q) + J.\,\vec{r}_\ell + K.\,\vec{r}_q \tag{9}$$

The slope "ϕ" of the tangent plane at "p" is given by

$$\cos\phi = \frac{\vec{N}\cdot\vec{K}}{1.1} = \frac{\cos\alpha}{\sqrt{1+c^2q^2}}\,. \tag{10}$$

From Eq. (10), the slope "ϕ" is equal to "α" at q= 0.0 i.e for all points on line BC. The slope "ϕ" increases by increasing the parameter "q". This means that point "D" is the point of maximum slope, where,

$$\cos \phi_{max} = \frac{\cos \alpha_2}{\sqrt{1+c^2 h^2 \operatorname{cosec}^2 \alpha_2}} . \qquad (11)$$

2.2.2 Minimum transition length

The minimum allowable transition length "L_{min}" is obtained from eq. (11);

$$L_{min} = \frac{(\alpha_2-\alpha_1)\, h \operatorname{cosec} \alpha_2 \cdot \cos \phi_{max}}{\sqrt{\cos^2 \alpha_2 - \cos^2 \phi_{max}}} \qquad (12)$$

2.2.3 Side surface area of transition zone

By simple integration we get ;

$$A = \frac{h.L}{(\alpha_2-\alpha_1)} . \ln \frac{(\operatorname{cosec} \alpha_2 - \cot \alpha_2)}{(\operatorname{cosec} \alpha_1 - \cot \alpha_1)} \qquad (13)$$

2.2.4 Embankment volume in transition zone

The embankment volume V is also calculated by integration, where we get ;

$$V = h.wL \frac{h^2 L}{(\alpha_2-\alpha_1)} [\ \ln \sin (\alpha_2) - \ln \sin (\alpha_1)] \qquad (14)$$

2.3 Case of Variable Height and Variable Side Slope

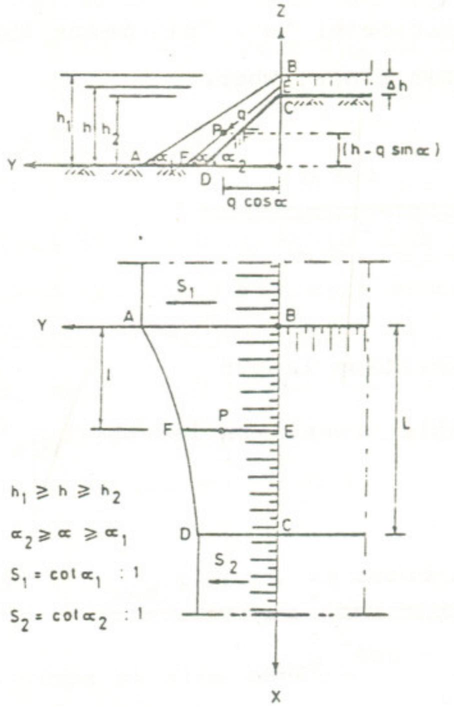


Fig. 3 Embankment with variable height and side slope

Figure (3) illustrated this case where both the height "h", and the side slope " α " are varied. The properties of the side surface are similar to those of case study (2). The side surface is a ruled surface represented by the vector equation;

$$\vec{r}(\ell,q) = (\ell)\vec{i} + (q \cos\alpha)\vec{j} + (h - q \sin\alpha)\vec{K} \cdot \quad (15)$$

2.3.1 Maximum slope of side surface

Similar to case (2), the slope " ϕ " of the side surface at any point p is equal to

$$\cos\psi = \frac{\vec{N}\cdot\vec{K}}{1.1} = \frac{\cos\alpha}{\sqrt{1+(cq+b\cos\alpha)^2}} \quad (16)$$

The slope " ψ " is always greater than the slope " α ". Also, it is obvious that the slope " ψ" increases with the increase of " α" and the parameter "q". This means that " ψ" is maximum at point D (see Figure 3) at which $\alpha = \alpha_2$ and $q = h_2/\sin\alpha_2$. Consequently

$$\cos\psi_{max} = \frac{\cos\alpha_2}{\sqrt{1+(ch_2\operatorname{cosec}\alpha_2+b\cos\alpha_2)^2}} \quad (17)$$

2.3.2 Minimum transition length

The minimum transition length "L_{min}" is obtained from the above Eq. 17 as follows:

$$L_{min} = \frac{[(\alpha_2-\alpha_1)h_2\operatorname{cosec}\alpha_2+(h_1-h_2)\cos\alpha_2]\cos\psi_{max}}{\sqrt{\cos^2\alpha_2-\cos^2\psi_{max}}} \quad (18)$$

2.3.3 Side surface area of transition zone

The surface area "A" is equal to

$$A = \int_0^L (b\ell+f)\operatorname{cosec}(c\ell+d)\,d\ell \quad (19)$$

However, it was found much more practical to calculate the surface area numerically using any of the well known methods such as Simpson's rule.

2.3.4 Embankment volume in transition zone

The volume "V" of the transition zone is calculated by carrying out the integration ;

$$V = \int_{0}^{L} [(b\ell+f)W+(b\ell+f)^2 . \cot (c\ell+d)] d\ell \qquad (20)$$

The evaluation of the above mentioned integral is not practical, and the volume is better calculated using numerical methods as for the side surface area.

3. Application and Analysis of Results

The relationship between the embankment length "ℓ" and the slope " ψ " of the side surface is represented for both study case (2) and (3) in Figure (4-a) & (4-b) respectively.

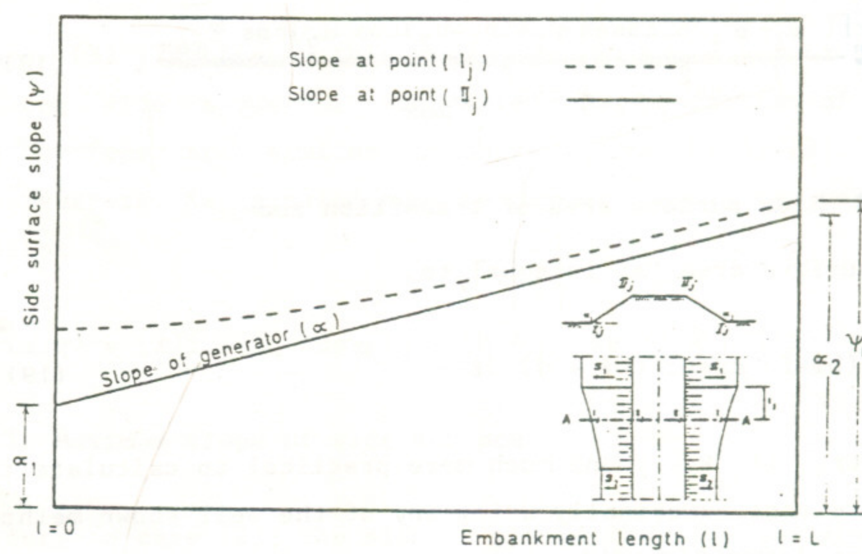


Fig. 4.a Embankment length - side surface slope relationship for case study (2).

Considering Fig. (4-a), case study 2, the slope of the surface at point "II_j" which has the parameter q=0, is always equal to "α_j". At point "I_j", which has the parameter $q = h/\sin \alpha_j$, the slope of the side surface is always greater than "α_j". The maximum slope occurs at "$\ell = L$ and $\alpha_j = \alpha_2$".

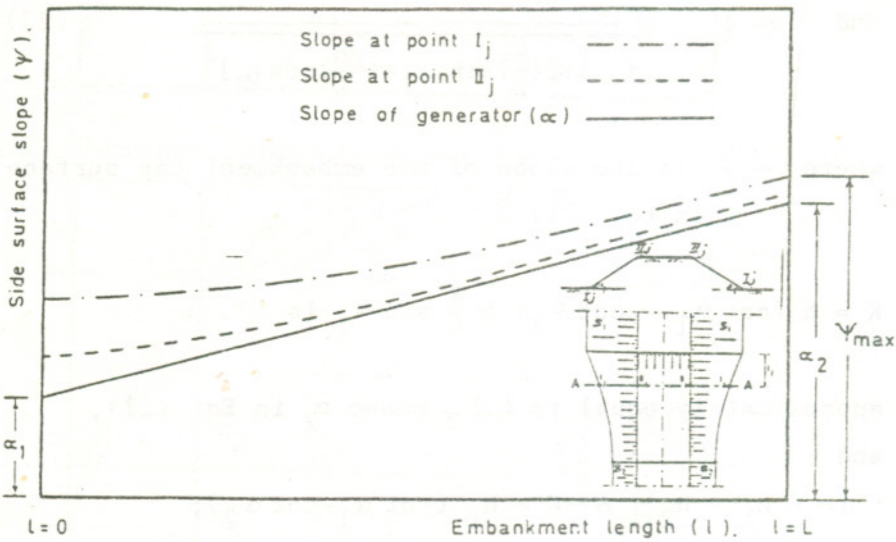


Fig. 4.b Embankment length - side surface slope relationship for case study (3)

In Fig. (4-b), case study 3, the slope of the surface at point "II_j", which has the parameter $q = 0$, is always greater than "α_j". At point "I_j" which has the parameter $q = h_j / \sin\alpha_j$, the surface slope is always the maximum for this section. he maximum slope "ψ_{max}" occurs at "$\ell = L$" and "$\alpha_j = \alpha_2$".

In order to study the influence of the different parameters on the side slope within the transition zone, equations (3), (11) and (17) are simplified, respectively, to the following forms:

$$\cos\psi = \frac{\cos\alpha}{\sqrt{1+\left(\frac{\Delta h}{L}\right)^2\cos^2\alpha}} \tag{21}$$

$$\cos\psi_{max} = \frac{\cos\alpha_2}{\sqrt{1+\left(\frac{K}{L}\right)^2\sin^2\alpha_1}} \tag{22}$$

and
$$\cos\psi_{max} = \frac{\cos\alpha_2}{\sqrt{1+\left[\left(\frac{K}{L}\right)\sin\alpha_1+\left(\frac{\Delta h}{L}\right)\cos\alpha_2\right]^2}} \tag{23}$$

where $\frac{\Delta h}{L}$ is the slope of the embankment top surface in Eq. (21) ;

$K = h\ (\cot\alpha_1 - \cot\alpha_2)$ & $\frac{K}{L}\sin\alpha_1$ is

approximately equal to $c.h_2$ cosec α_2 in Eq. (22),

and ;

$\Delta h = h_1 - h_2$ & $K = h_2\ (\cot\alpha_1 - \cot\alpha_2)$;

& $\frac{K}{L}\sin\alpha_1$ is approximately equal to

ch_2 cosec α_2 is Eq. (23).

Three computer programmes were used to solve the above mentioned three equations for wide range of the following parameters:

α & $\frac{\Delta h}{L}$ in Eq. (21) ;

α_1 & α_2 and $\frac{K}{L}$ in Eq. (22) and

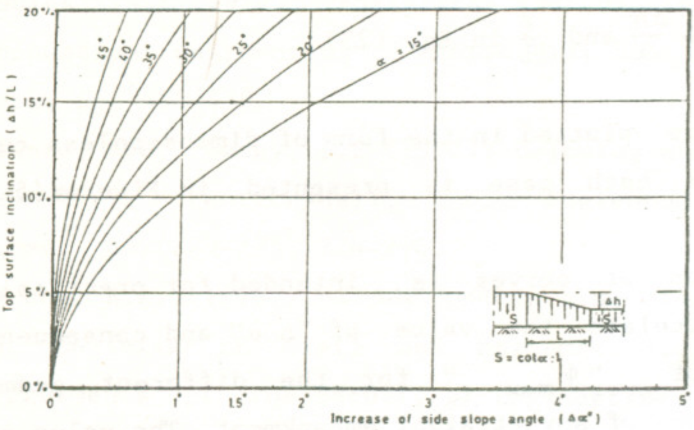


Fig.5.a Relationship between (Δh / L) and (Δα)

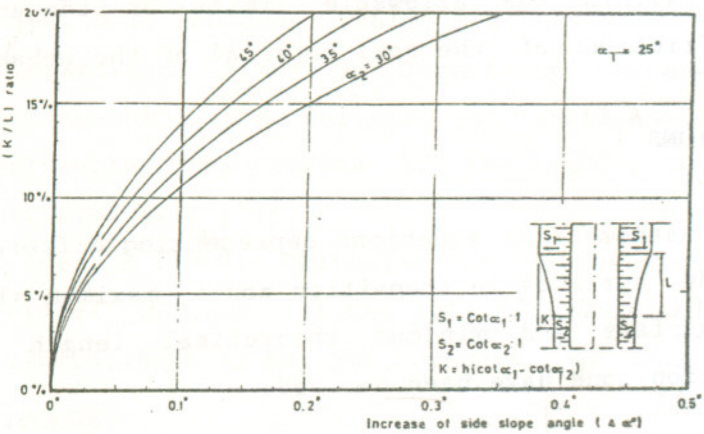
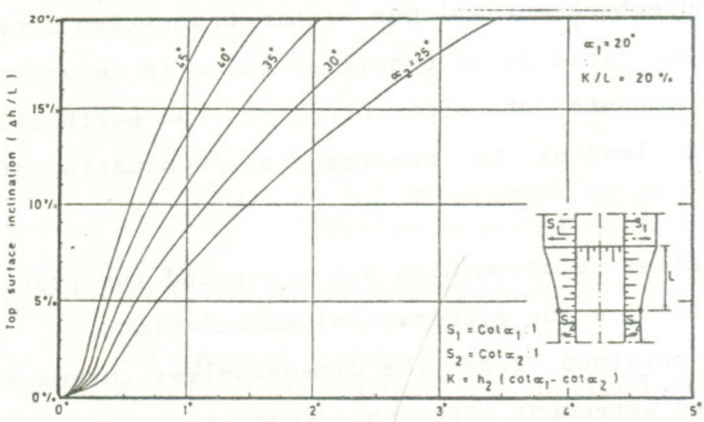


Fig.5.b Relationship between (Δ α°) and (K / L) for ($\alpha_1 = 25°$)



g.5.c Relationship between (Δα°) and ($\frac{\Delta h}{L}$) for ($\alpha_1 = 20°$) and (K / L = 20%).

α_1 & α_2 & $\frac{\Delta h}{L}$ and $\frac{K}{L}$ in Eq. (23)

Results are plotted in the form of dimensionless curves, a sample for each case is presented in Figures (5 a,b,c).

This group of curves is intended for practical use to simply calculate the value of "$\Delta \alpha$" and consequently the volume of "ϕ_{max}" for the different geometrical constructions of any straight embankment. The value of "ϕ_{max}" should be within the allowable limits of the angle of internal friction of the soil material of the embankment.

4. CONCLUSIONS

1. Useful mathematical equations representing different types of side surfaces in transition zones, maximum slope of each surface and minimum theoretical length of the transition zone have been derived.
2. Workable equations for evaluation of the side surface areas and embankment volumes within transition zones are introduced. This is of practical value in determination of the amounts of side pitching and earth filling; therefore leading to precise cost evaluation of such jobs.
3. Dimensionless relationships are presented for practical use to minimize the mathematical calculations.
   Results obtained from the dimensionless curves showed very close agreement with those from the exact solution.

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NOMENCLATURE

A	=	Side surface area of transition zone
b	=	$\Delta h/L = (h_1 - h_2)/L$
c	=	$\Delta\alpha /L = (\alpha_2 - \alpha_1)/L$
d	=	α_1
f	=	h_2
h	=	Embankment height
Δh	=	The change in the embankment height
ℓ	=	Horizontal distance measured in direction of X-axis
L	=	Total length of the transition zone
L_{min}	=	Minimum transition length
$\vec{N}$	=	Unit normal vector
q	=	Distance measured on the inclined line EF from the top of embankment
V	=	Volume of transition zone
w	=	width of embankment
α	=	The slope angle of transition surface
$\Delta\alpha$	=	The change in the slope angle of the side surface
ϕ	=	Slope of transition surface.

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c	=	$\Delta\alpha /L = (\alpha_2 - \alpha_1)/L$
d	=	α_1
f	=	h_2
h	=	Embankment height
Δh	=	The change in the embankment height
ℓ	=	Horizontal distance measured in direction of X-axis
L	=	Total length of the transition zone
L_{min}	=	Minimum transition length
$\vec{N}$	=	Unit normal vector
q	=	Distance measured on the inclined line EF from the top of embankment
V	=	Volume of transition zone
w	=	width of embankment
α	=	The slope angle of transition surface
$\Delta\alpha$	=	The change in the slope angle of the side surface
ϕ	=	Slope of transition surface.