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A NUMERICAL APPROACH FOR SOLUTION OF NATURAL CONVECTION HEAT TRANSFER PROBLEM BETWEEN TWO PARALLEL PLATES

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ABSTRACT

A numerical model is proposed to solve the natural convection problem between two parallel plates. Computer results showed a good agreement as compared with the available reported results for air. It is also shown that the model is capable of carrying out solutions at transient as well as steady state conditions.

NOMENCLATURE

a	distance between two plates, m.
C_p	specific heat at constant pressure, kJ/kg^{o}K
C_{ps}	Specific heat for steel, kj/kg^{o}k.
g	graity acceleration, m/s^2
Gr	local Grashof number ($=\frac{g\beta(T-T_{b2})x^3}{\nu^2}$)
h	local heat transfer coefficient, w/m^2.^{o}C
k	thermal conductivity, w/m.^{o}k
L	length in x direction, m.
Nu	Nusselt number [= hx/k]
Pr	Prandtl number [$=\mu C_p/k$]
p	pressure, N/m^2.
q"	heat flux, kw/m^2
Ra	Rayleigh number [$= g\beta\Delta T\,\ell^3/(\alpha\nu)$]
Ra*	modified Rayleigh number [$= g\beta q''\ell^4/(\alpha\nu k)$]
t	time, sec.
t_1	plate thickness, m.
T	temperature, ^{o}C
u	velocity component in x direction, m/s.
v	velocity component in y direction, m/s.
W	wideness of the two parallel plate, m .
x	direction along the length of the plate, m.
y	direction perpendicular to the plate, m.

Greek-Symbols

α	thermal diffusivity, m^2/s.
β	the coefficient of volume expansion, 1/^{o}k
ρ	density, kg/m^3
θ	channel tilt angle, deg.

results showed a modification on Aung's equations when different values for the constant depending on Pr and Ra are used.

Nowadays, utilizing the solar energy is the interest of many investigators. The traditional way for utilization is usually performed using flat plate collectors. To increase the collector efficiency it is belived that, two parallel plate collector [5] provides the maximum efficiency, since all the surface area of the flow passage is subjected to the sun radiation.

To the knowledge of the author, no reported work so far took into account both variations of the wall temperature and the heat flux as well as the effect of tilting angle for the two parallel plates. This is the case for two parallel flat plates solar collector, since the sun radiation and the plate temperature vary from time to time.

The aim of this paper is to develop a general solution model for the case where variation in heat flux, wall temperature and the tilting angle are present. This model is applicable for transient as well as steady state solution for either fixed or variable wall heat flux and/or wall temperature.

2. THEORETICAL ANALYSIS

As mentioned previously, this research deels with the solution of natural convection for an open cavity consists of two parallel plates. A configuration of the problem is shown in Fig. 1. The momentum equation for the flow in x direction is given as [6].

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+X \quad (1)$$

Where X is the body force. In free convection, the body force can not be neglected because it sustains the fluid motion and hence; it certainly must be included. Upon selecting the x-coordinate axis as parallel to surface and y-coordinate axis as normal to it, the body force per unit mass will be:

$$X = g\beta\,(T - T_{b2})\,\sin\theta \qquad (2)$$

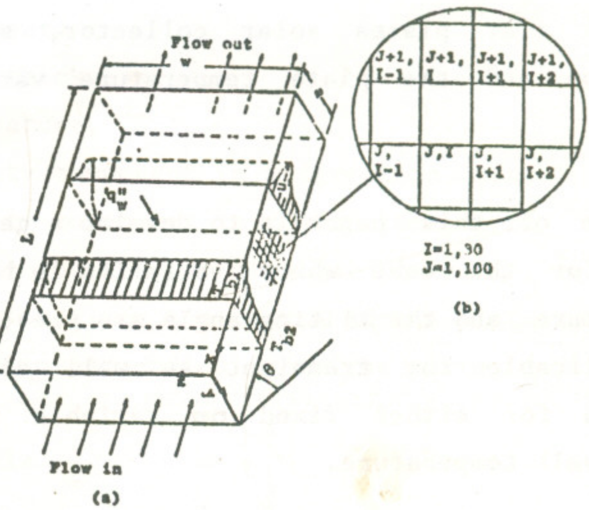

Fig. 1. The Problem Configuration, a) Coordinates and Flow Direction, b) A Miicroscopic Illustration of a Speciment Showing the Intervals Used in the Finite Element Method.

The following assumptions are applied:

1. In applying equation (1), a repeated steady state solution is considered for infinitensimal small descrete intervals of time. In each interval, using the energy balance equation (10), the temperature is changed by ΔT(as shown in Fig. 2) and in turn the velocity changes.
2. The width (W) is very large compared to the listance between the two plates (a), hence, the variation of velocity in z direction is neglected ($\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial z^2} = 0$)
3. The pressure gradient for a small increment in x direction may be neglected ($\frac{\partial \rho}{\partial x} = 0$).
4. As long as the distance (a) (in y direction) is very small compared to the height (L), the component of velocity (u) must change very rapidely from zero at both y=0 and y=a to u_{max} some where in between. Thus $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial y^2}$ are expected to be of significant magnitude. Therefore, the viscous dissipation term may be neglected in x direction as compared with that in y direction.

Applying the previous assumptions to equation (1) yield:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta \ (T - T_{b2}) \sin \theta \qquad (3)$$

Moreover applying these assumptions to the energy equation result:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The boundary conditions for equations (3 and 4) are

$$\left.\begin{array}{l} \text{at } y = 0 \ ; \quad T = T_{b1} \ , \quad \text{and} \quad u = 0 \\ \quad\ \ y = a \ : \quad T = T_{b2} \ , \quad \text{and} \quad u = 0 \end{array}\right\} \qquad (5)$$

Define similarity parameters as:

$$\eta = \frac{y}{x}\left[\frac{g\,\beta\,(T-T_{b2})x^3}{4\,\nu^2}\right]^{\frac{1}{4}} = \frac{y}{x}\left[\frac{Gr(x)}{4}\right]^{\frac{1}{4}}$$

$$\psi = 4\,\nu\left[\frac{Gr(x)}{4}\right]^{\frac{1}{4}} F(\eta)$$

$$\phi = \frac{T - T_{b2}}{T_{b1}-T_{b2}} \qquad (6)$$

$$d(x) = T_{b1} - T_{b2} = Nx^n$$

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = -\frac{\partial\psi}{\partial x}$$

Where N is some constant and n is index. It is easy to show that n = 0 for constant wall temperature and n=0.2 for constant wall heat flux. Applying equation (6) into equations (3 and 4) and neglecting very small magnitude terms F'F" and 'F' one may obtain:

$$F''' + \sin\theta \cdot \varphi + [(n+3)FF'' - 2\,(n+1)F'^2] = 0$$

viscous term + buoyancy term + inertia term = 0

$$\frac{\varphi''}{Pr} + [(n+3)\,\varphi'F - 4nF'\,\varphi] = 0 \quad (8)$$

conduction term + convection term = 0

The boundary conditions for equations (7 and 8) are

$$\text{at } y = 0 : \eta = 0 , \quad F(0) = 0 , \quad \varphi = 1$$
$$(9)$$
$$y = a : \eta = \quad , \quad F(\ _i) = 0 , \quad \varphi = 0$$

Since equations (7 and 8) are ordinary nonlinear differential equations, then their exact solution form is quite difficult.

In this paper the solution had been performed using a finite difference explicit method with an axial interval $\Delta x=L/100$ and perpendicular interval $\Delta y=a/30$. Fig. 1 shows a sample of the nodes used for finite difference method.

Applying the finite difference methods to equations (7 and 8) with boundary conditions given in equation (9), 28 equations for each F and are obtained which should be solved simultaneously.

2.1 Energy Balance

Energy balance for the control volume on the boundary of a segment in x direction is given as:

$$q_w = q_1 + q_2 + q_3 + q_4 + q_5 \quad (10)$$

where, $q_w = q''.\ \Delta x.\ W.\ \Delta t$

= heat added to the upper wall surface . (kj).

$q_1 = \Delta x.\ W.\ t_1.\ \rho_s,\ C_{ps}\ (T'_{b1} - T_{b1})$

= internal energy increase in upper plate. (kj).

$q_2 =\ x.\ a.\ W(\rho' C_p T'_{av} - \rho C_p T_{av})$

= internal energy increase for fluid inside the control volume. (kj).

$q_3 = \Delta x.\ W.\ t_1.\ \rho_s C_{ps} (T'_{b2} - T_{b2})$

internal energy increase of the bottom plate. (kj).

$q_4 = \bar{u}.\ W.\ a.\ \rho.\ T_{av}$

= energy associated with the flow entering the control volume. (kj).

$q_5 = \bar{u}'.\ W.\ W.a.\ \rho'\ T'_{av}$

= energy associated with the flow leaving the control volume (kj)

Notice that the (') implies the same variable as that calculated at the end of the segment.

3. SOLUTION TECHNIQUE

Applying the finite difference method to equations (7 and 8) at a certain height x_J, yields 28 equations for each F and φ .A first guess for φ's is assumed, and a corresponding values of F's are obtained. These values of F's are used to obtain improved values for φ's, which in turn are used to modify F's values again. This procedure is repeated untill the difference between successive values for F's and φ's approaches a prespecified values of tolerance. Once the final solution for F's and φ's is reached, the energy balance equation (10) is applied to a control volume bounded by x_{J-1} and x_J, with initial temperature of upper plate (T_{b1}), equal to that at x_{J-1}. An increase or decrease of the estimated value of

T_{b1} by a small increment ($0.005^{o}C$) is carried out untill the energy balance is reached. The exit parameters from station J will be considered as initial guess for station J+1. This pricess is repeated up to the last station.

The above procedure is repeated for a new time interval, for which the initial guesses are taken to be the final solution from the previous one. The time change stops when steady state solution is researched.

4. RESULTS AND DISCUTION

From a practical point of view a simulation model of two parallel plates with W=1 m, L=1.8 m, plate thickness 1.25 mm, and a=1 cm is considered. A computer program is developed for the solution technique discussed above and samples of results obtained are shown in Tables 1-4, and figures 2-8.

TABLE 1 Samples of Results Evaluated at a=1 cm, q''=.2 kw/m², and θ=52°.

Time (sec.)	150	450	900	1500	2250	3150	4200	5400	5550
$T_{av.}$ (C°)	18.603	19.28	19.9	20.40	20.98	21.41	21.8	22.17	22.18
T_{b1} (C°)	19.15	20.44	21.61	22.69	23.59	24.34	25.	25.6	25.605
T_{b2} (C°)	18.03	18.07	18.14	18.21	18.31	18.41	18.52	18.64	18.65
$Q \times 10^{-6}$ (m^3/s)	.0615	.1365	.2087	.279	.337	.388	.434	.477	.4779
U (cm/s)	.0006	.0014	.0021	.0028	.0034	.0039	.0043	.0045	.0045
Nu	884.5	502.	340.9	264.1	221.	196.7	179.9	167.	166.9

Table 1, shows the variation of T_{av}, T_{b1}, T_{b2}, Q, u, and Nu at the exit of the two parallel plates, with respect to time. These results were obtained for $q''=0.2$ kw/m^2 and =52 (optimum tilting angle for latitude 32° [8]). The results tabulated in Table 1, are plotted in Fig. 2. From this figure it can be seen that as the time increase, the rate of change of temperature, $\bar{u}$, Q and Nu decrease. For example, between time 1500, 2250 s, $\partial T/\partial t = 0.0012$, and between times 2250, 3150 s, $\partial T/\partial t=0.0008$.

In our model the losses to the surrounding are neglected, due to the fact that the losses depend upon the type of application. This is why a steady state solution could not be reached. However from the results, one can figure out that above the time 3150 sec., the rate of increase in

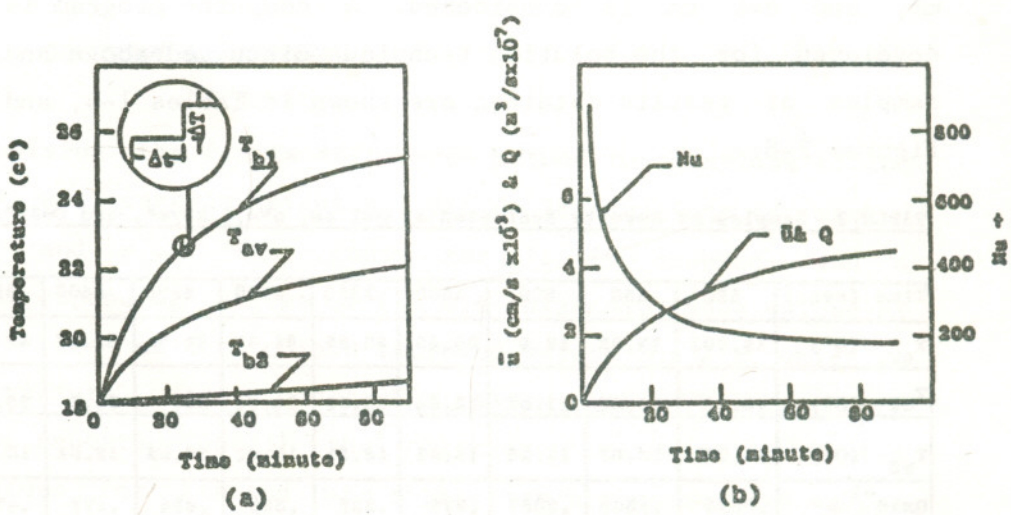

Fig. 2. Variation of exit parameters for two parallel plates with respect to time; a) variation of T_{b1}, T_{b2} and T_{av} b) variation of $\bar{u}$, Q and Nu.

temperature with respect to time is almost negibile (less than 0.0006) and steady state is considered after 3150 sec. Fig. 3 shows the variation of T_{bl} with respect to location at different times. From which it is clear that the enterance effect dies out quickly. In other words, a fully developed flow may be provided within a distance of less than 20 % of the full height.

Fig. 4 shows the velocity distribution as well as the temperature distribution of the water between the two parallel plates at different locations. It can be seen from Fig. 4-a, that the maximum velocity occures closer to the hotter surface. This is due to the fact that the density gradient is higher near the upper plate, which in turn provide higher driving force there.
In a trail to study the effect of variable parameters such as heat flux (q"), distance between two plate (a), and the tilting angle (θ), a series of computer runs were carried out. A sample of steady state results at channel exit are tabulated in Tables 2,3 and 4.

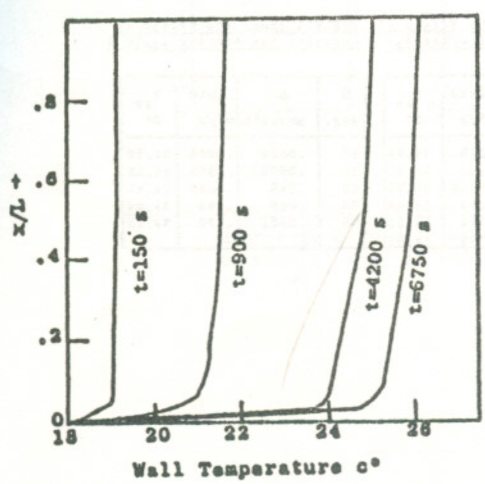

Fig. 3. Variation of upper plate temperature with respect to x/L at different times.

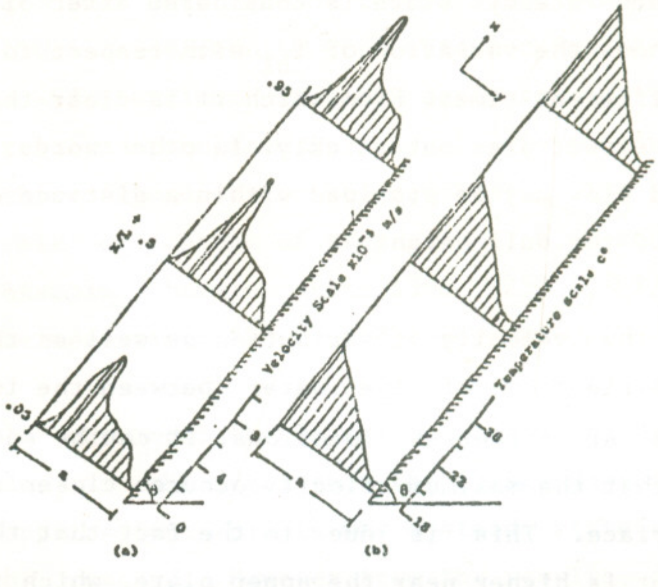

Fig. 4. The variation of: a) velocity distribution and b) temperature distribution between two parallel plates w.r.t. x/L.

TABLE 2 Shows the Effect of Changing the Heat Flux q".

q" kw/m²	h kw/(m²K)	Qx10^{-6} m³/s	T_{av} C°
0.2	0.068	.385	21.41
0.5	0.0234	.697	24.46
1.0	0.1114	1.808	28.87

TABLE 3 Shows the Effect of Changing the Thickness s.

s cm	h kw/(m²k)	Qx10^{-6} m³/s	T_{av} C°
0.5	.0347	.115	24.69
1.0	.068	.349	21.41
1.5	.1335	.5028	20.25
2.0	.1451	1.229	19.48
2.5	.1907	1.953	19.14
3.0	.2525	2.541	19.01

TABLE 4 Shows the Effect of Changing the tilting angle θ

θ deg.	h kw/(m²k)	Qx10^{-6} m³/s	T_{av} C°
10	.0612	.0973	21.76
30	.0682	.2595	21.56
52	.068	.3686	21.41
70	.069	.453	21.02
90	.0701	.476	21.02

Fig. 5, shows the variation of h, Q and T_{av} for both transient and steady state w.r.t. q" at the channel exit for a certain condition (θ =52 , a= 1 cm). From which it can be seen that, as q" increase h, Q and T_{av} increase.

Fig. 6, shows the effect of tilting angle (θ) variation on h, Q and T_{av} at the channel exit for a specified condition (q"=0.2 kw/m^2, a = 1 cm). From Fig. 6-a, it is clear that the variation of h w.r.t. is negligible. On the other hand, the variation of Q w.r.t. in Fig. 6-b, is noticible. This high variation in Q is due to the buoyancy

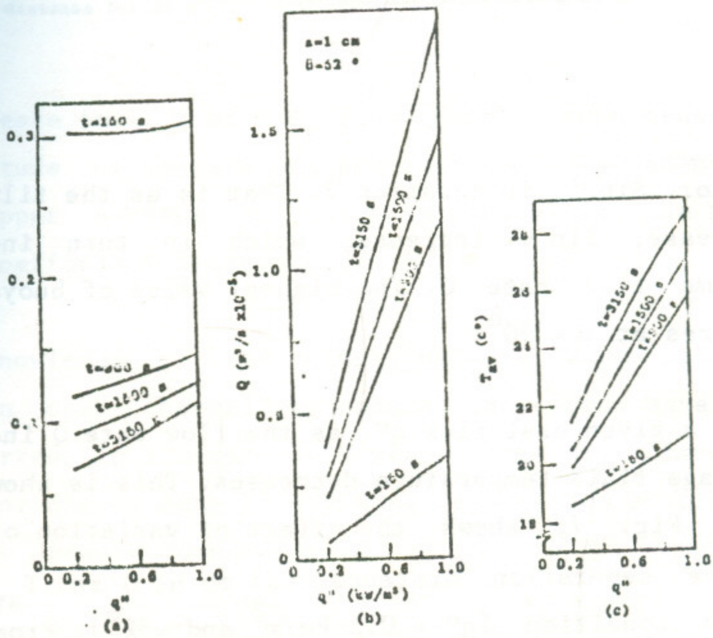

Fig. 5. Variation of: a) heat transfer coefficent h, b) volume discharge Q, and c) avarge temperature T_{av} with respect to heat flux q" at a=1 cm and θ=52°.

Fig. 5, shows the variation of h, Q and T_{av} for both transient and steady state w.r.t. q" at the channel exit for a certain condition (θ =52 , a= 1 cm). From which it can be seen that, as q" increase h, Q and T_{av} increase.

Fig. 6, shows the effect of tilting angle (θ) variation on h, Q and T_{av} at the channel exit for a specified condition (q"=0.2 kw/m^2, a = 1 cm). From Fig. 6-a, it is clear that the variation of h w.r.t. is negligible. On the other hand, the variation of Q w.r.t. in Fig. 6-b, is noticible. This high variation in Q is due to the buoyancy

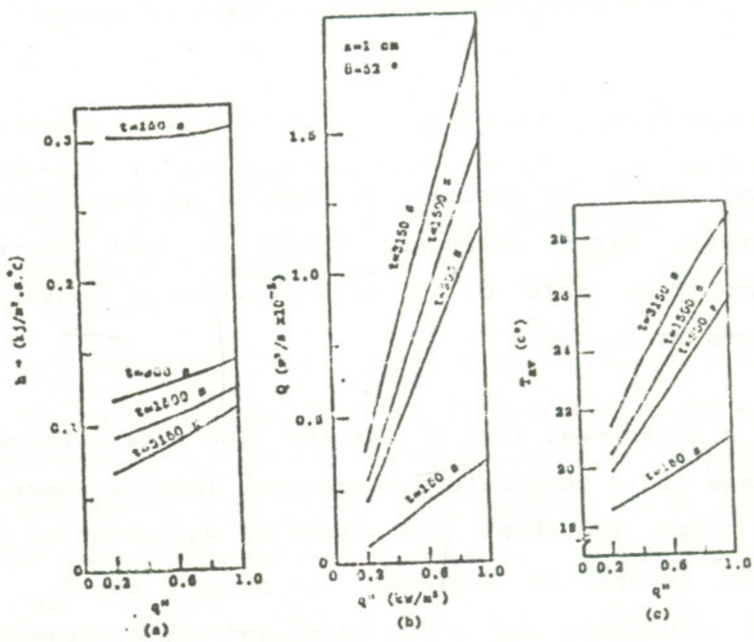

Fig. 5. Variation of: a) heat transfer coefficent h, b) volume discharge Q, and c) avarge temperature T_{av} with respect to heat flux q" at a=1 cm and θ=52°.

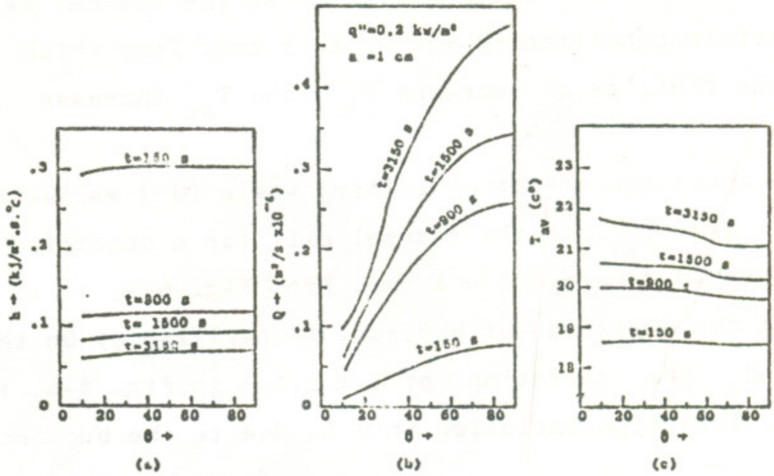

Fig. 6. Variations of: a) h, b) Q and T_{av} w.r.t. the tilting angle θ at q"=0.2 kw/m² and a=1 cm.

factor, Sin θ, in equation 7. That is as the tilting angle increase, Sin θ increase, which in turn increase the volume flow rate Q. The highest value of buoyancy force occures at $\theta = 90^{o}$.

For a given heat flux q", as the flow rate Q increase the average fluid temperature decreases. This is shown in Fig. 6-c. Fig. 7, shows the effect of variation of parallel plates separation distance (a) on h, Q and T_{av}. For a given condition (q" = 0.2 kw/m^2 and =52^{o}). From this it is clear that as a increases, both h and Q increase, while T_{av} decreases,. The idea behined increasing of Q and decreasing of T_{av}, shown in Fig. 7-b and 7-c is that: increasing a provides a higher flow rate which leads

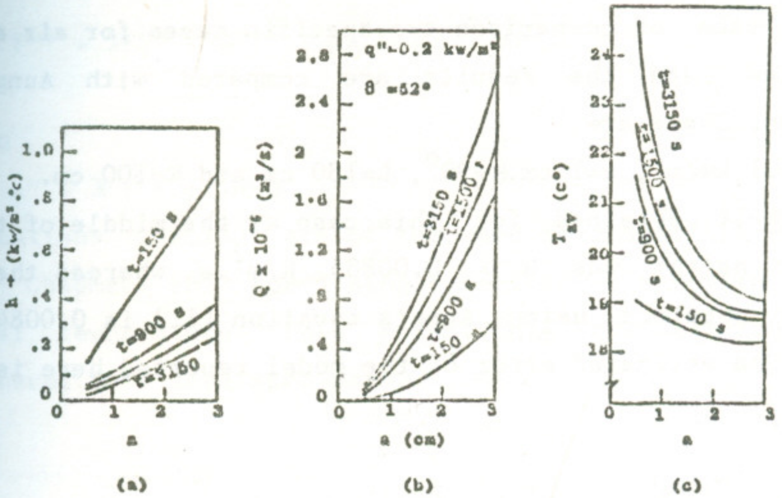

Fig. 7. Variations of: a) h, b) Q and c) T_{av} w.r.t. plate separation distance (a) at q"=0.2 kw/m² and θ= 52°.

to a decrease in T_{av}, of the fluid. This in turn causes the magnitude of temperature gradient ($\partial T/\partial y$) adjacent to the upper surface to increase, therefore, the heat transfer coefficient h increase.

To the knowledge of the author, data for water passing through an open channel by natural convection have not been reported in literature. However there are some data measured for air by Aung [3,4]. Aung reported an emperical formula for Nusselt number as:

$$Nu = \frac{ha}{k} = C(Ra^*)^{\frac{1}{3}} \quad ; \quad 10^2 \le Ra^* \le 10^4$$

where $Ra^* = \dfrac{g\beta\, \bar{q}'' a^4}{\nu\alpha\, k}$ and $\bar{q}''$ is the average heat flux applied to the channel walls.

For a sake of comparison two specific cases for air are simulated and the results are compared with Aung's equation. These are:

i) $q''=0.1$ kw/m^2, a=1 cm,$\theta=90^o$, L=180 cm and W=100 cm.
The result obtained for this case at the middle of the channel height was h = 0.00885 k/m^2.s whereas the results obtained using Aung's equation (11) is 0.008411 k/m^2. The estimated error of the model reported here is + 5.2 %.

i) $q''=0.1$ kw/m^2, a=3 cm, =90^o,L =180 cm and W=100 cm.
The result obtained this case was h=0.00866 kw/m^2. C, whereas that obtained from Aung's equation is 0.00675. Agrain an error of + 22% was estimated. It is to be noticed that the error estimated above is due to the fact that in Aung's work, heat from both sides of the channel was applied whereas in the present work the heat is applied only on the upper plat. This in turn affects the value of Ra*.

When the model applied for air, it is found that, the air can stand higher heat flux in natural convection due to low heat capacity of air. When moderate heat flux is applied a rapid steady state solution is obtained, e.g, for first case mentioned above the steady state is reached in 10 minutes. Returning back to equations (7,8), with n=0, and Sin θ=1, then

$$F''' + 3FF'' - 2F'^2 + \varphi = 0 \tag{12}$$

and

$$\varphi'' + 3Pr\,F\,\varphi' = 0 \tag{13}$$

Equations (12,13) are of the same form as those obtained by Chapman [8] for a single vertical plate with consist wall temperature. This establishes the validity of the general form developed here.

5. CONCLUSION

In this paper a numerical method for solution of natural convection between two parallel plates problem has been introduced and verified by computer simulation.

From the above results and discussion it can be conclude that:

1. The model developed in this paper provids agreeable results with those reported in the literature for air.
2. The steady state solution for water passing through two parallel plates may be reached within a time of less than one houre.
3. The fully developed flow may be reached within a distance of less than 20 % of the full height.
4. The proposed model is proved to be capable of carrying out transient as well as steady state solutions for both specified wall temperature and/or specified wall heat flux.

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