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THE ACTUAL DAMPING BEHAVIOR OF MATERIALS

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ABSTRACT

The actual internal damping of materials is characterized by the dissipation energy represented by the hystresis loop.

All types of materials are classified into three categories according to their hystresis dependance on stress level, stress rate, and number of cycles.

An example to use these dissipation terms in Lagrange's equation of motion for a longitudinal vibration of a circular cross section steel rod is given. A closed from solution to the nonlinear problem is obtained by using perturbation technique.

INTRODUCTION

The vibration and noise control is a very important problem in recent high speed machines. Such control highlight the importance of using a material which has high damping capacity as well as high strength. In additior the well known relations between the damping capacity of a material and its fatigue which state that a material of high damping capacity resists fatigue rupture more than that which has lower damping capacity owing to its notch sensitivity, and that the higher damping capacity material supports better the sudden overloading due to resonance phenomenon.

In spite of the lack of correspondence between the internal damping and viscosity (minimum for the classical materials) [1,2], many authors characterized the damping capacity (which is the variation of work done per cycle $\Delta W/W$) and different coefficients related to the material viscosity [3,4] as the logarithmic decrement $\delta = \frac{1}{2}\frac{\Delta W}{W}$, the phase angle between stress and strain φ, and the quality factor $Q^{-1} = \tan\varphi = \delta/\pi$.

Owing to the wide variation of damping behavior of materials, many investigations for modeling the mechanical behavior of soilds has been made. These models changes from material to another depending upon the observations of macroscopic [5], or microscopic [6] experimental results.

In the present study we will attempt to find a general description of the material internal damping (depending upon the dissipated energy per cycle) for all types of materials and present an example to use the expressions in the vibration analyses.

STUDY OF THE ENERGY DISSIPATION

The behavior of dissipated energy per cycle which is represented by the hystresis loop is different for different materials.

To obtain a theoretical expression for this dissipated energy, the relation between the hystresis loop and different parameters affecting its area such that; stress level, stress rate, and number of cycles or frequency must be discussed.

1. Effect of Stress Level

The relation between the energy dissipation ΔW and stress level has been discussed by several authors [7,8,9,10,11, 12]. It was found that a correlation of the form $\Delta W = K (\sigma)^n$ (where k is a constant and n takes three different values according to stress range) may satisfy the experimental results for cast iron [9] and some types of steel [10].

In fact a correlation of this type $\Delta W = f(\sigma)$ could be obtained directly using the area of the hystresis loop. Considering the case of unidirectional stress , Fig. 1, the stress-strain relation which describes the boundaries of the hystresis loop could be simply approximated to the well known dynamic stress-strain relation [13];

$$\frac{\varepsilon}{2} = \frac{\sigma}{2E} + \varepsilon'_f \left(\frac{\sigma}{2\sigma'_f}\right)^{\frac{1}{n'}}$$

The advantage of this approximation is to make the relation valid for all types of materials.

where ; ε = strain

σ = stress

E = Young's modulus

ε'_f = fatigue ductility coefficient

$\cong$ monotonic fracture ductility, ε_f

σ'_f = fatigue strength coefficient

$\cong$ monotonic true fracture strength, σ_f

n' = cyclic strain hardening exponent.

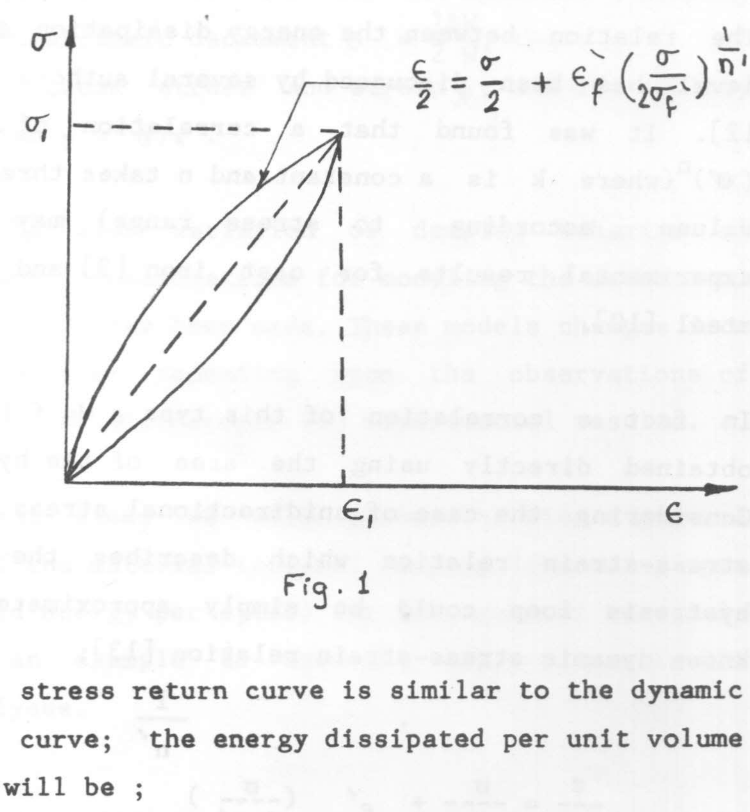

Fig. 1

Assuming the stress return curve is similar to the dynamic stress-strain curve; the energy dissipated per unit volume and per cycle will be ;

$$\Delta W = 2[\int_o^{\varepsilon} \sigma \, d\varepsilon - \tfrac{1}{2}\sigma_1 \; \varepsilon_1]$$

$$= \frac{2\,\varepsilon'_f}{(1+\frac{1}{n'})(\sigma'_f)^{\frac{1}{n'}}}\,(\sigma)^{1+\frac{1}{n'}}$$

or ;

$$\Delta W = B\,(\sigma)^{1+\frac{1}{n'}} \qquad (1)$$

where ;

$$B = \frac{2\,\varepsilon'_f}{(1+\frac{1}{n'})(\sigma'_f)^{\frac{1}{n'}}}$$

2. EFFECT OF NUMBER OF CYCLES

The hystresis loop behaves differently for different materials with respect to the number of cycles.

Some metals (for example mild steel) exhibit continually a hystresis loop of finite width even when subjected to stress below the fatigue limit.

Soft metals exhibit initially a wide hystresis loop which narrows rapidly (usually within first 5 % of expected life) before it maintains a constant width. For high stress level, the loop increases gradually, its width increases rapidly just prior to fracture [14,15].

In case of hard materials, the initially wide loop and its subsequent narrowing is absent, for such materials the loop increases progressively as a function of the number of cycles [16,17].

From the previous discussion, one can classify the materials according to the changing of their damping characteristics with respect to time into two categories;

a. Those which exhibit a hysreresis loop which stablizes after a short time and remain constant as least 80 % of its fatigue life. Thus, the expression (1) for dissipated energy per cycle is valid over this range of life.

b. Those which exhibit a cycle which increased continually as a function of number of cycles. In such materials (case b), the dissipated energy may be written in the form;

$$\Delta W = f (\sigma) g (N)$$

$$\Delta W = B\sigma^{1 + \frac{1}{n'}} \left(\frac{CN^{m} - 1}{C + 1} \right)$$

With C and m are constants depending on material. Equation (2) reduce to the expression (1) when the exponent m tends to zero.

Knowing that N = ft where f denotes frequency ; expression (2) is ;

$$\Delta W = \left(B\sigma^{\left(1 + \frac{1}{n'}\right)} \right) \left(\frac{C(ft)^{m} - 1}{C - 1} \right)$$

Which clarify the time dependent of the dissipation function for a given frequency.

3. EFFECT OF STRESS RATE

For the viscoelastic materials, the effect of stress rate can not be neglected. This effect is clearly in the direction of decreasing the hysteresis energy [18].

TANAKA et al, [19,20,21,22] made several experiments for the impact fatigue. In spite of the fact that their goal was to obtain fatigue characteristics under high strain rate conditions, one can notice the change of the hystersis loop area due to stress rate at the same level.

Generally the materials could be classified according to the stress rate effect on its damping characteristics into two groups;

i. Those which exhibt quasi-static loops, practically unaffected by the stress rate (most types of classical materials at room temperature). Thus, expressions (1) and (2) are valid in such materials.

ii. Those which exhibit dynamic hysteresis loop affected by stress rate (viscoelastics materials)

The dissipated energy per cycle in such materials is written in the form:

$$\Delta W = f(\sigma)\, g(N)\, h(\dot{\sigma})$$

$$\Delta W = B\left[\sigma^{1+\frac{1}{n'}}\right]\left[\frac{C(ft)^{n}-1}{c-1}\right]\left[\frac{1-D\,\dot{\sigma}^{n}}{1-D}\right] \quad (3)$$

Expression (3) is a general one which reduces to expression (2) when exponent n tends to zero.

APPLICATION

Vibration of Continuous Systems Considering The Internal Damping Effect

As a simple application, consider the longitudinal vibration of a uniform circular cross-sectional steel bar fixed atboth ends ,Fig.2.

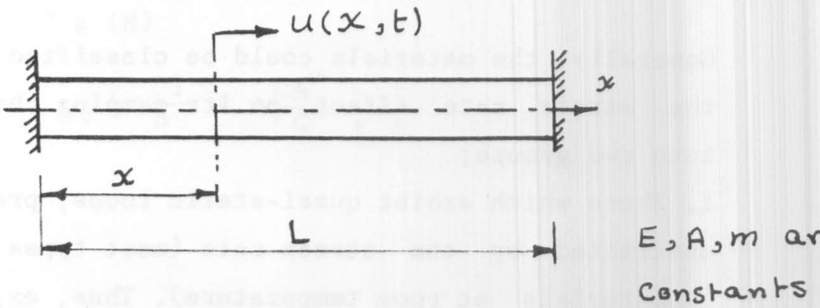

Fig.2

The kinetic and potential energy of such system could be written as [23];

$$\text{kinetic energy} = T = \frac{1}{2}\int_0^L m\left(\frac{\partial u(x,t)}{\partial t}\right)^2 dx$$

$$\text{potential energy} = V = \frac{1}{2}\int_0^L EA\left(\frac{\partial u(x,t)}{\partial x}\right)^2 dx$$

where A = cross-sectional area

m = mass per unit length

The shaft material (steel) obeys the form of dissipation energy function characterized by expression 1, the dissipation energy per cycle will be ;

$$W_d = \int_o^L A(B\sigma^{1+\frac{1}{n'}})dx$$

From experimental data, the exponent n' varies from 0.1 to 0.3. For the following analysis a value of n' = 0.25 will be assumed.

Thus, $W_d = \int_o^L A\,(B\,\sigma^5)\,dx$

Remembering that ;

$u(x,t) = u(x)\,e^{iwt}$ (Separable in space and time)

$$\frac{\partial u(x,t)}{\partial t} = \dot{u}(x,t) = u_{(x)}\,iwe^{iwt}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \ddot{u}(x,t) = -w^2 u_{(x)} e^{iwt}$$

$$\frac{\partial u(x,t)}{\partial X} = u'(x,t) = u'_{(x)} e^{iwt}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = u''(x,t) = u''_{(x)} e^{iwt}$$

The true stress $\sigma = \sigma_e(1+e)$

where, σ_e = engineering stress

e = engineering strain

Thus ; $\sigma = Eu'_{(x,t)}(1+u'_{(x,t)})$.

Substitution of the previous formula, taking the first approximation for stress expression gives;

$$W_d = \int_o^L ABE^5\, u'^5(x)e^{5iwt}]dx$$

knowing that N = ft where ;

f = frequency , t = time

the rate of energy dissipation for a given frequency (noting that for the type of material studied here the effect of frequency in changing the hysteresis area is neglected.) will be ; $W' = \frac{1}{f} W_d$

Substitution in lagrange's equation with a dissipation term gives ; [24].

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} + \frac{\partial}{\partial q}(W') = 0$$

$$-w^2 mu(x) + EA\ u''(x) + \frac{5ABE^5}{f}[u'(x)]^3 u''(x) = 0$$

Rewriting this equation we get the differential equation;

$$\frac{\partial^2 u(x)}{\partial x^2} - \frac{w^2 m}{EA} u(x) + \frac{5BE^4}{f}\left(\frac{\partial u}{\partial x}\right)^3 \left(\frac{\partial^2 u}{\partial x^2}\right) = 0 \quad (4)$$

Solution of The Equation of Motion

Equation (4) contains a nonlinear term. Knowing that the damping energy in a classical material is very small compared with the elastic energy, one can expect a harmonic solution slightly perturbed by small nonlinear terms.

Thus the perturbation method [25] could be used to solve the equation. According to this method limited to the second approximation, we attempt to find a formal solution of equation (4) in the form;

$$u(x,t) = u_o(x,t) + \alpha u_1(x,t) + \gamma u_2(x,t)$$

Putting: $$\beta^2 = \frac{w^2 m}{EA}$$

$$, M = \frac{5 B E^4}{f}$$

it is a straightforward application to get ;

$$\frac{\partial^2 u_o(x)}{\partial x_o^2} - \beta^2 u_o(x) = 0 \tag{5-a}$$

$$\frac{\partial^2 u_1(x)}{\partial x_1^2} - \beta^2 u_1(x) = M \left(\frac{u_o(x)}{\partial x_o}\right)^3 \left(\frac{{}^2 u_o(x)}{\partial x_o^2}\right) \tag{5-b}$$

$$\frac{\partial^2 u_2(x)}{\partial x^2} - \beta^2 u_2(x) = 3Mu_{1x} \left(\frac{u_o(x)}{\partial x_o}\right)^2 \left(\frac{\partial^2 u_o(x)}{\partial x_o^2}\right) + $$

$$+ M\,[u_1(x)] \left(\frac{\partial u_o(x)}{\partial x_o}\right)^3 \tag{5-c}$$

The generating solution, which is the solution of the zero order approximation is given by :

$$u_o(x) = \lambda_1 \sin \beta x + \lambda_2 \cos \beta x$$

Using our problem boundary conditions which are;

$$u_o(0) = u_o(L) = 0$$

The solution will take the form ;

$$u_o(x) = \lambda_r \sin \frac{r \pi x}{L} \quad (r = 1,2,3,...)$$

and λ_r are arbitrary constants.

Taking $\lambda_r = 1$ and r=1 for the first mode, one get ;

$$u_o(x) = \sin \frac{\pi x}{L}$$

The first order approximation (5-b) will be ;

$$\frac{\partial u_1(x)}{\partial x_1} - \beta^2 u_1(x) = -\frac{M \pi^5}{L^5} \left(\cos \frac{\pi x}{L}\right)^3 \sin \frac{\pi x}{L} \qquad (6)$$

Equation (6) is a nonhomogeneous equation. Using the method of undetermined coefficients [26], a particular solution may be assumed in the form,

$$u_{1p}(x) = H \cos^3 \frac{\pi x}{L} \sin \frac{\pi x}{L} + F \cos \frac{\pi x}{L} \sin \frac{x}{L}$$

Derivatives of such solution give;

$$\frac{\partial u_{1p}(x)}{\partial x} = H \frac{\pi}{L} \cos^4 \frac{\pi x}{L} + \frac{3H\pi}{L} \cos^2 \frac{\pi x}{L} \sin^2 \frac{\pi x}{L}$$

$$- F \frac{\pi}{L} \sin^2 \frac{\pi x}{L} + F \frac{\pi}{L} \cos^2 \frac{\pi x}{L}$$

$$\frac{\partial^2 u_{1p}(x)}{\partial x^2} = - \frac{4H \pi^2}{L^2} \cos^3 \frac{\pi x}{L} \sin \frac{\pi x}{L}$$

$$+ \frac{6H \pi^2}{L} \sin \frac{\pi x}{L} \cos \frac{\pi x}{L}$$

$$- \frac{2F \pi^2}{L^2} \sin \frac{\pi x}{L} \cos \frac{\pi x}{L}$$

$$- \frac{2F^{2}}{L^{2}} \sin \frac{\pi x}{L} \cos \frac{\pi x}{L}$$

Substition in Equation (6) gives;

$$H = \frac{M \pi^{5}}{L^{3}(4\pi^{2} + \beta^{2} L^{2})} , \quad F = \frac{6 H \pi^{2}}{4 \pi^{2} + L^{2}}$$

The solution of the first order approximation equation will be ;

$$u_1(x) = u_o(x) + u_{1p}(x)$$

$$u_1(x) = \sin \frac{\pi x}{L} + H \cos^{3} \frac{\pi x}{L} \sin \frac{\pi x}{L}$$

$$+ F \cos \frac{\pi x}{L} \sin \frac{\pi x}{L}$$

$$= \sin \frac{\pi x}{L} (1 + F \cos \frac{\pi x}{L} + H \cos^{3} \frac{\pi x}{L})$$

Using the solution $u_o(x)$ and $u_1(x)$ in (5-c) to get the second order approximation equation one gets,

$$\frac{\delta^{2} u_2(x)}{\delta x^{2}} - \beta^{2} u_2(x) = \frac{M \pi^{3}}{L^{3}} (1 + F \cos \frac{\pi x}{L} + H \cos^{3} \frac{\pi x}{L}) *$$

$$\sin \frac{\pi x}{L} \cos^{2} \frac{\pi x}{L} (\cos \frac{\pi x}{L} - \frac{3 \pi}{L} \sin \frac{\pi x}{L}) \qquad (7)$$

Using the method of undetermined coefficient in a similar way as in the first order approximation solution; the particular solution of Eq. (7) will be in form ;

$$u_{2p}(x) = Q \sin\frac{\pi x}{L}\cos^3\frac{\pi x}{L} * (1 + \cos\frac{\pi x}{L} + \cos^3\frac{\pi x}{L}) + K \sin^2\frac{\pi x}{L}\cos^2\frac{\pi x}{L}(1 + \cos\frac{\pi x}{L} + \cos^3\frac{\pi x}{L})$$

Where the constants Q and K are given by:

$$Q = -\frac{M\pi^3}{L(10\pi^2 + \beta^2 L^2)}$$

$$K = \frac{3M\pi^4}{L^2(21\pi^2 + \beta^2 L^2)}$$

and the solution for $u_2(x)$ will be ,

$$u_2(x) = u_o(x) + u_{2p}(x)$$

$$u_2(x) = \sin\frac{\pi x}{L} + \sin\frac{\pi x}{L}\cos^2\frac{\pi x}{L}(1 + \cos\frac{\pi x}{L} + \cos^3\frac{\pi x}{L}) * (Q\cos\frac{\pi x}{L} + K\sin\frac{\pi x}{L})$$

Referring to the formal solution, the solution of the nonlinear equation will be;

$$u(x,t) = u_o(x)e^{iwt} + u_1(x)e^{2iwt} + u_2(x)e^{5iwt}$$

where and are arbitrary constants and could be taken to unity.

Thus the solution is :

$$u(x,t) = u(x)e^{iwt} + u_1(x)e^{i2wt} + u_2(x)\,e^{i5wt}$$

Discussion of Results

The representation of the damping energy by the area of the hysteresis loop means that their is no damping effect during the first half of the cycle (car their is no area enclosed in the hysteresis loop), i.e., the solution is a harmonic one with an amplitude $u_o(x)$ which is the amplitude of solution of the zero order approximation equation. In the second half of the cycle the amplitude $u_{(x)}$ is affectect by the two terms $u_1(x)$, $u_2(x)$ and the damping effect appears in a reduction of the amplitude in the second half of the cycle which gives a new amplitude at the end of the cycle. This final amplitude of the first cycle is the starting amplitude for the second cycle $u_o^1(x)$ which remains coonstant for the first half of the second cycle. For the second half of the second cycle this amplitude $u_o^1(x)$ decreases by the new amplitudes $u_1^1(x)$ and $u_2^1(x)$ and so on (note that the values $u_1^i(x)$ and $u_2^i(x)$ depend on the starting amplitude at each cycle $u_o^i(x)$ where i is the cycle's number). Fig. 3.

Thus the general solution is:

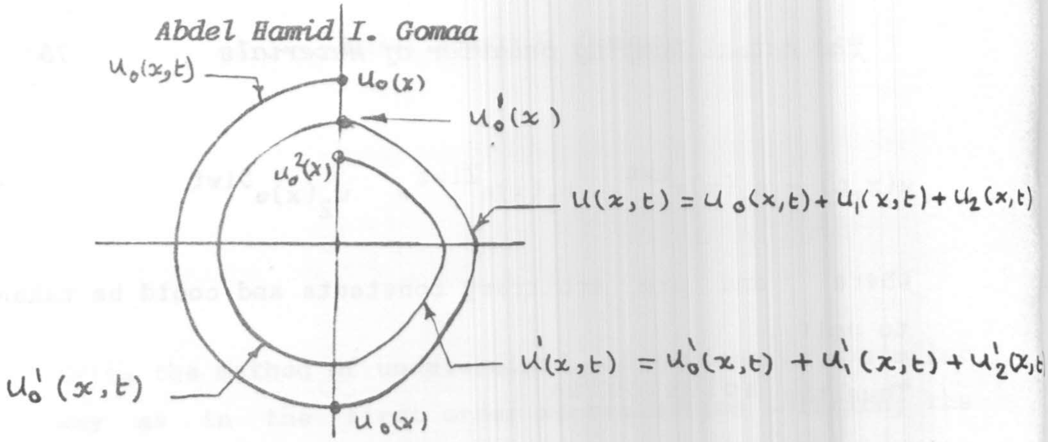

Fig. 3

$$u_{(x,t)} = u_{o(x,t)} \quad \text{for } o \le t \le \frac{\pi}{}$$

$$u_{(x,t)} = u_o^{(x,t)} + u_{1(x,t)} + u_{2(x,t)} \quad \text{for } \frac{\pi}{} \le t \le \frac{2\pi}{\omega}$$

CONCLUSION

The exact internal damping characteristics are determined using the actual behaviour of the hysteresis loop of each material.

Three different formulas characterizing the damping effect of different materials were suggested.

An application of the simplest formula to the longitudinal vibration of a circular cross-section steel rod showed a nonlinear lagrangian derived equation of motion.

A closed form solution using the perturbation technique was found.

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