Alex. Eng. J., Alex. Univ., Volum 27, pp. 21-34 (1988)

LATERAL VIBRATION OF BEAMS WITH INTERNAL DAMPING

Abdel Hamid I. Gomaa
Mechanical Engineering Dept. Faculty of Engineering
University of Alexandria

ABSTRACT

The actual damping behavior described by the dissipated energy expression is used. The lateral vibration of simply supported beam with internal damping effect is analyzed. A closed form solution for the nonlinear fourth order lagrange's derived equation using the perturbation technique and the method of variation of parameters is presented .

INTRODUCTION

The studies of the effect of internal damping in vibration of continuous systems have been oriented for modeling the behavior of the material with a viscoelastic model. Many studies have been presented for determination of the parameters of these models by comparing experiemntal data with theoretical predictions.

These studies, practically started by Chiu-Neubert, [1] need complicated calculations for determining the model parameters. Furthermore a choice of model type for a material must be made in advance of the analysis from few data [2].

Another problem of the application of these models (knowing that each material has its own model parameters) is the lack of precision in applying the macroscopic behavior owing to the probable difference between the microscopic behaviors for the same macroscopic features [3].

FORMULATION OF THE PROBLEM

The problem is a simply supported, circular cross-sectional, steel beam laterally oscillated by a harmonic oscillation u (x,t), Fig. 1.

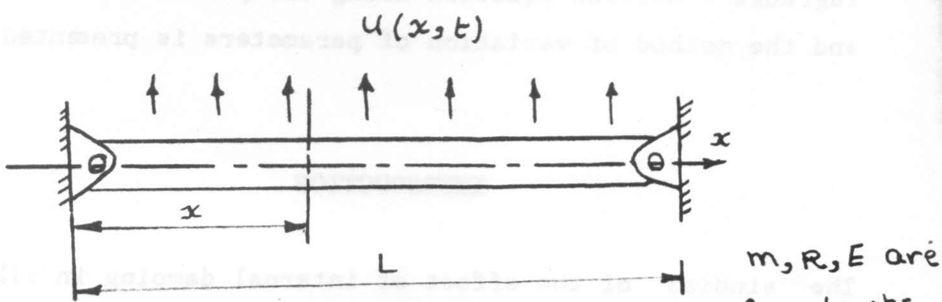

Fig. 1

The selection of beam material gives the possibility of using the first damping energy expression [4], which is

$$\Delta W = B\,(\sigma)^{1 + \frac{1}{n'}}$$

where; ΔW = the energy dissipated per unit volume and per cycle

σ = stress level

E = modulus of elasticity

n' = cyclic strain hardening exponent

$$B = \frac{2\,\epsilon' f}{(1+\frac{1}{n'})(\sigma' f)^{\frac{1}{n'}}}$$

$\epsilon' f$ = fatigue ductility coefficient

$\sigma' f$ = fatigue strength coefficient

Remembering that ; $\sigma = \frac{M}{I} y = E y \frac{\partial^2 u(x,t)}{\partial x^2}$

and knowing that in this type of dissipation the frequency effect could be neglected, thus the rate of energy dissipation in our problem could be written as ;

$$W' = \frac{1}{f}\left[2 \int_o^L \int_o^R \int_{-R}^{+R} B\, E^5 y^5 u''^5 (st)\, dxdydz\right.$$

Assuming that u (x,t) is separable in time and space, $u(x,t) = u(x)\, e^{iwt}$

Thus,

$$\Delta W' = \int \frac{1}{f} \int_o^L \left(\frac{2BE\, R^7}{3}\, u''^5(x)\, e^{5iwt}\right) dx \qquad (1)$$

where, f = frequency

R = radius of beam cross section

The kinetic and potential energies of such system are given by [5];

$$\text{kinetic energy } T = \frac{1}{2} \int_o^L m \left(\frac{u(x,t)}{t}\right)^2 dx \qquad (2)$$

$$\text{Potential energy} = V = \frac{1}{2} \int_o^L R^2 E \left(\frac{\partial^2 u(x,t)}{\partial x^2}\right) dx \qquad (3)$$

Substitution of expressions 1,2,3 in lagrange's equation with dissipation term [6] and rearranging of the terms gives;

$$\frac{\partial^4 u}{\partial x^4} - \beta^4 u(x) + C \left(\frac{\partial^2 u(x)}{\partial x^2}\right)^3 \left(\frac{\partial^4 u(x)}{\delta x^4}\right) + 3C \left(\frac{\delta^2 u(x)}{\partial x^2}\right)^2 \left(\frac{\partial^3 u(x)}{\partial x^3}\right)^2 = 0 \qquad (4)$$

Where

$$\beta^4 = \frac{mw^2}{\pi R^2 E}$$

$$C = \frac{40\ BE^4\ R^5}{3\ \pi}$$

SOLUTION OF THE EQUATION OF MOTION

The equation of motion, (4), has two nonlinear terms. The perturbation technique [7] could be used in solving such equation owing to the well known fact which states that the damping energy is very small comparing with the elastic energy.

According to this method, we attempt to find a solution of Eq. (4) in the form ;

$$u(x,t) = u_o(x,t) + Mu_1(x,t) + Nu_2(x,t)$$

Where M, N are constants.

The derivatives of such solution with respect to x give

$$\frac{\partial^r u(x,t)}{\partial x^r} = \frac{\partial^r u_o(x,t)}{\partial x^r} + M\frac{\partial^r u_1(x,t)}{\partial x^r} + N\frac{\partial^r u_2(x,t)}{\partial x^r}$$

where r = 1,2,3,4 denotes the order of derivative.
Substitution in Eq. 4 gives the following equations;

$$\frac{d^4u_o(x)}{dx^4} - \beta^4 u_o(x) = 0 \qquad (5.a)$$

$$\frac{d^4u_1(x)}{dx^4} - \beta^4 u_1(x) - C\left(\frac{d^2u_o(x)}{dx^2}\right)^3\left(\frac{d^4u_o(x)}{dx^4}\right) - 3c\left(\frac{d^2u_o x}{dx^2}\right)^2$$

$$\left(\frac{d^3u_o(x)}{dx^3}\right)^2 = 0 \qquad (5\text{-}b)$$

$$\frac{d^4u_2(x)}{dx^4} - \beta^4 u_2(x) - u_1(x)\, C\left[3\left(\frac{d^2u_o(x)}{dx^2}\right)^2\left(\frac{d^4u_o(x)}{dx^4}\right) + \left(\frac{d^2u_o(x)}{dx^2}\right)^3\right]$$

$$-6Cu_1(x)\left[\left(\frac{d^2u_o(x)}{dx^2}\right)\left(\frac{d^3u_o(x)}{dx^3}\right)^2 + \left(\frac{d^2u_o(x)}{dx^2}\right)^2\left(\frac{d^3u_o(x)}{dx^3}\right)\right]$$

$$(5\text{-}c)$$

The solution begins by solving the zero order approximation equation (5-a) to get the generating solution.

The linearized equation (5-a) has a characteristic equation in the form;

$$\lambda^4 - \beta^4 = 0$$

Thus,

$$\lambda_1 = \beta \ , \quad \lambda_2 = -\beta \ , \quad \lambda_3 = i\beta \ , \quad \lambda_4 = -i\beta$$

and the generating solution takes the form [8];

$$u_o(x) = k_1 e^{\beta x} + k_2 e^{-\beta x} + k_3 e^{i\beta x} + k_4 e^{-i\beta x}$$

where; K_1, k_2, k_3, and k_4 are constants could be obtained by using the boundary conditions,

$$\text{at } x = 0 \qquad u_o \Big| 0 = 0 \ , \qquad \frac{\partial^2 u(o)}{\partial x^2}\Big|_o = 0$$

$$\text{and } x = L \qquad u^o \Big| L = 0 \ , \qquad \frac{\partial^2 u(0)}{\partial x^2}\Big|_L = 0$$

Thus we have the set of equation ;

$$k_1 + k_2 + k_3 + k_4 = 0$$

$$k_1 e^{\beta L} + k_2 e^{-\beta L} + k_3 e^{i\beta L} + k_4 e^{-i\beta L} = 0$$

$$\beta^2 k_1 + \beta^2 k_2 - \beta^2 k_3 - \beta^2 k_4 = 0$$

$$\beta^2 k_1 e^{\beta L} + \beta^2 k_2 e^{-\beta L} - \beta^2 k_3 e^{i\beta L} - \beta^2 k_4 e^{-i\beta L} = 0$$

Solving these equations simultaneously gives [9] ;

$$k_1 = \frac{\Delta k_1}{\Delta}, \; k_2 \; \frac{\Delta k_2}{\Delta}, \; k_3 = \frac{\Delta k_3}{\Delta}, \; k_4 = \frac{\Delta k_4}{\Delta}$$

Where

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{\beta L} & e^{-\beta L} & e^{i\beta L} & e^{-i\beta L} \\ \beta^2 & \beta^2 & -\beta^2 & -\beta^2 \\ \beta^2 e^{\beta L} & \beta^2 e^{-\beta L} & -\beta^2 e^{-i\beta L} & \beta^2 e^{-i\beta L} \end{vmatrix}$$

$$\Delta k_1 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & e^{-\beta L} & e^{i\beta L} & e^{-i\beta L} \\ 0 & \beta^2 & -\beta^2 & -\beta^2 \\ 0 & \beta^2 e^{-\beta L} & -\beta^2 e^{-i\beta L} & -\beta^2 e^{-i\beta L} \end{vmatrix}$$

$$\Delta k_2 = \begin{vmatrix} 1 & 0 & 1 & 1 \\ e^{\beta L} & 0 & e^{i\beta L} & e^{-i\beta L} \\ \beta^2 & 0 & -\beta^2 & \beta^2 \\ \beta^2 e^{\beta L} & 0 & -\beta^2 e^{i\beta L} & -\beta^2 e^{-i\beta L} \end{vmatrix}$$

$$\Delta K_3 = \begin{vmatrix} 1 & 1 & 0 & 1 \\ e^{\beta L} & e^{-\beta L} & 0 & e^{-i\beta L} \\ \beta^2 & \beta^2 & 0 & \beta^2 \\ \beta^2 e^{\beta L} & \beta^2 e^{-\beta L} & 0 & -\beta^2 e^{-i\beta L} \end{vmatrix}$$

$$\Delta k_4 = \begin{vmatrix} 1 & 1 & 1 & 0 \\ e^{\beta L} & e^{-\beta L} & e^{i\beta L} & 0 \\ \beta^2 & \beta^2 & -\beta^2 & 0 \\ \beta^2 e^{\beta L} & \beta^2 e^{-\beta L} & -\beta^2 e^{i\beta L} & 0 \end{vmatrix}$$

To simplify the representation, the tensor notation may be used. The zero order approximation equation solution may take the form;

$$u_o(x) = \sum_{j=1}^{4} k_j e^{\lambda_j x} \tag{6}$$

The first order approximation equation is a nonhomogeneous one. To solve this equation the method of variation of parameters [8] could be used.

According to this method, the solution of Eq. (5-b) may be written as [9];

$$u_1(x) = \sum_{i=1}^{4} [H_{1i}(x) + Li]\, e^{\lambda i x}$$

Where

$$H_{1i} = \frac{1}{\prod_{\substack{j=1 \\ j\neq i}}^{4} (\lambda_i - \lambda_j)} \sum_{j=1}^{4} \frac{k_j^2 \lambda_j^5}{(2\lambda_j - \lambda_i)} e^{(2\lambda_j - \lambda_i)x}$$

$$+ \sum_{j=1}^{4} \sum_{n=j+1}^{4} \frac{k_j k_n \lambda_j^2 \lambda_n^2 (\lambda_j + \lambda_n)}{(\lambda_j + \lambda_n - \lambda_i)} e^{(\lambda_j + \lambda_n - \lambda_i)x} \tag{7}$$

and ; L_i $(i = 1,2,3,4)$ are constants obtained by the application of boundary conditions;

At $x = L$ the bending moment $\frac{d^2 u_o(x)}{dx^2} = 0$

Thus, $\frac{d^4 u_1(L)}{dx^4} - \beta^2 u_1(L) = 0$

From which u_1 (L) could be considered as solution for the homogeneaus equation (5-a).

Thus; $u_1(L) = u_o(L)$

In addition $e^{\lambda ix}$ form a basis of linearly independent vectors [10], thus ;

$Hi(L) + Li = ki$

Now, substitution in the general equation gives ;

$$u_1(x) = \sum_{i=1}^{4} H_{1i}(x)\, e^{\lambda ix} - \sum_{i=1}^{4} H_{1i}(L)\, e^{\lambda iL} + \sum_{i=1}^{4} k_i e^{ix}$$

$$= \sum_{i=1}^{4} [H_{1i}(x) - H_{1i}(L)]\, e^{\lambda ix} + u_o(x)$$

using expression (7), one gets,

$$u_1(x) = u_o(x) + \sum_{i=1}^{4} \frac{e^{\lambda ix}}{\prod_{\substack{j=1 \\ j\neq i}}^{4} (\lambda_i - \lambda_j)} \left[\sum_{j=1}^{4} \frac{k_j^2 \; k_j^5}{(2\lambda j - \lambda_i)} \right. *$$

$$(e^{(2\lambda_j - \lambda_i)x} - e^{(2\lambda_j - \lambda_i)L}) + \sum_{j=1}^{4} \sum_{n=j+1}^{4} \frac{k_j k_n \lambda_j^2 \lambda_n^2 (\lambda_j + \lambda_n)}{(\lambda_j + \lambda_n - \lambda_i)} *$$

$$\left. [e^{(\lambda_j + \lambda_n - \lambda_i)x} - e^{(\lambda_j + \lambda_n - \lambda_i)L}] \right. \quad (8)$$

similarly for the second approximation equation (5-c), the solution may be written in the form ;

$$u_2(x) = \sum_{k=1}^{4} [H_{2k}(x) + L_k]\, e^{\lambda_k x}$$

where;

$$H_{2k}(x) = \frac{1}{\prod_{\substack{j=1\\ j\neq k}}^{4} (\lambda_k - \lambda_j)} \sum_{j=1}^{4} \frac{k_j^3 \, \lambda_j^7}{(3\lambda_j - \lambda_k)} e^{(3\lambda_j - \lambda_k)x}$$

$$+ \sum_{j=1}^{4} \sum_{n=j+1}^{4} k_j k_n \lambda_j^2 \lambda_n^2 \frac{(k_j \lambda_n e^{\lambda j x} + k_n \lambda_j e^{\lambda n x})}{(\lambda_j + \lambda_n - \lambda_k)} e^{(\lambda_j + \lambda_n - \lambda_k)*}$$

$$+ 2 \sum_{j=1}^{4} \sum_{n=1}^{4} \sum_{m=j+1}^{4} \frac{k_j k_n k_m \lambda_j \lambda_n \lambda_m}{(\lambda_j + \lambda_n + \lambda_m - \lambda_k)} e^{(\lambda_j + \lambda_n + \lambda_m - \lambda_k)} + \text{const.}$$

Using the same procedure we get ;

$$u_2(x) = u_o(x) + \sum_{k+1}^{4} \frac{e^{\lambda k}}{\prod_{\substack{j=1\\ j\neq k}}^{4} (\lambda_k - \lambda_j)} \left[\sum_{j=1}^{4} \frac{k_j^3 \lambda_j^7}{(3\lambda_j - \lambda_k)} * \right.$$

$$\left. \left(e^{(3\lambda_j - \lambda_k)x} - e^{(3\lambda_j - \lambda_k)L} \right) \right]$$

$$+ \sum_{j=1}^{4} \sum_{n=j+1}^{4} \frac{k_j k_n \lambda_j^2 \lambda_n^2}{(\lambda_j + \lambda_n - \lambda_k)} * [(k_j \lambda n e^{jx} + k_n \lambda_j e^{\frac{x}{n}})$$

$$e^{(\lambda_j + \lambda_n - \lambda_k)x} - (k_j \lambda n e^{\lambda j L} + kn \lambda_j e^{\lambda n L} {}_j + \lambda_n - \lambda_k)L]$$

$$+ 2 \sum_{j=1}^{4} \sum_{n=1}^{4} \sum_{m=j+1}^{4} \frac{k_j k_n k_m \lambda_j^2 \lambda_n^2 \lambda_m^2}{(\lambda_j + \lambda_n + \lambda_m - \lambda_k)} [e^{(\lambda_j + \lambda_n + \lambda_m - \lambda_k}$$

$$-e^{(\lambda_j + \lambda_n + \lambda_m + \lambda_k) L}] \tag{9}$$

Substitution of Eqs. 6,8,9 in the formal solution gives the a solution in the form ;

$$u(x,t) = u_o(x)\, e^{iwt} + u_1(x)\, e^{2iwt} + u_2(x)\, e^{5iwt}.$$

Thus, the general solution will be in the form (4);

$$u_{(x,t)} = u_o(x,t) \qquad \text{for} \quad 0 \le t \le \frac{\pi}{\omega}$$

$$u_{(x,t)} = u_o(x,t) + u_1(x,t) + u_2(x,t) \quad \text{for} \quad \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega}$$

CONCLUSION

A fourth order nonlinear differential equation describes the lateral vibration of beams, taking into account the actual damping characteristics of beam's material could be derived using largrange's technique.

A closed form solution of such problem could be obtained by using the pertubation method and the method of variation of parameters.

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