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GRID ON ELASTIC FOUNDATION

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ABSTRACT

An accurate stiffness matrix for a grid on elastic foundation element is formulated considering torsion. Few number of elements are required to represent grid foundations. The stiffness formulation is used adequately to analyze monolithic plane grid foundations composed of grade beams. The application of the proposed formulation is implemented into a computer program.

INTRODUCTION

Beams resting on an elastic foundation are commonly used in structural engineering and closed form solutions of the differential equation have been proposed (1-4). Numerical methods such as the finite difference and the finite element method has been applied to solve this problem by using discrete spring connected to the nodes of the structure, thus approximating the solution (5-6). M. Eisnberg and D. Yankelevsky (7) derived an exact stiffness matrix for a finite beam on continuous wrinkler elastic foundation.

The problem of a plane grid on elastic foundation can be approximated by using discrete springs connected to the structure nodes (8). In this study a stiffness matrix for a gird element on wrinkler elastic foundation is proposed considering torsion. Thus a few number of elements can be used to yield the exact solution. This proposed formulation can be used to analyze monolithic plane grid foundation composed of grade beams.

Stiffness Matrix Formulation

The terms of plane grid element stiffness matrix are defined as the holding actions at the ends of the element due to unit rotation and translations as shown in Figure (1).

The differential equation for an infinite bar resting on elastic foundation

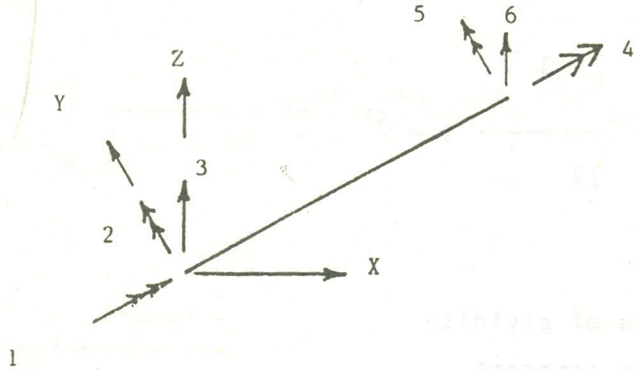
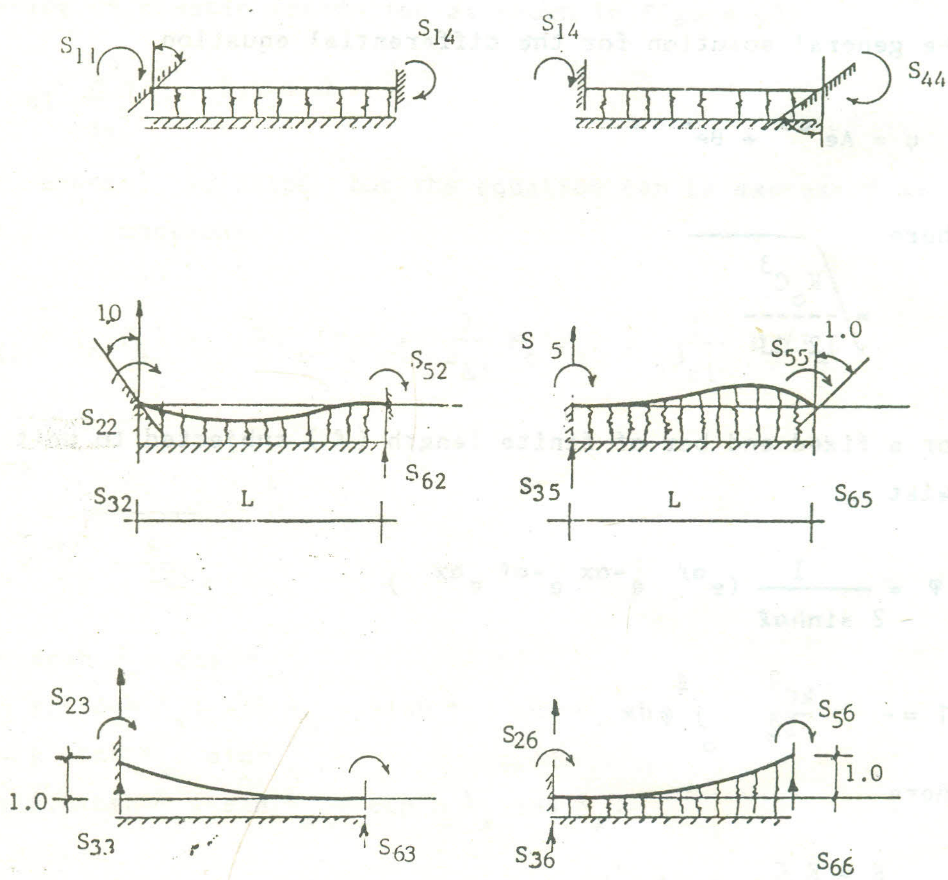


Fig. (1) Member Stiffnesses



$$GJ = \frac{d^2 \varphi}{dx^2} - \frac{K_o C^3}{12} \varphi = 0$$

where

G = Modulus of rigidity

J = Torsion constant

= Angle of twist

K_o = Modulus of foundation

G = Width of bar

The general solution for the differential equation

$$\varphi = Ae^{\alpha x} + Be^{-\alpha x}$$

where

$$= \sqrt{\frac{K_o C^3}{12\ GJ}}$$

for a fixed end bar of finite length (ℓ) subjected to unit twist

$$\varphi = \frac{1}{2\ \sinh \alpha\ell} (e^{\alpha\ell} \quad e^{-\alpha x} \quad e^{-\alpha\ell} \quad e^{\alpha x} \quad)$$

$$T = - \frac{kC^2}{12} \int_o^{\ell} \varphi\, dx$$

where

$$K = K_o C$$

$$T = \frac{KC^2}{12 \sinh \alpha l} \int_o \frac{1}{2 \sinh \alpha \ell} (e^{\alpha \ell} e^{-\alpha x} - e^{-\alpha \ell} e^{\alpha x}) \, dx$$

solving

$$S_{1,1} = \frac{KC^2}{12} \left(\frac{\cosh \alpha \ell - 1}{\sinh \alpha \ell} \right)$$

and $S_{4,4} = - S_{4,1} = - S_{1,4} = S_{1,1}$

The differential equation for the deflection curve of a bar resting on elastic foundation as shown in Figure (3)

$$EI \frac{d^3 y}{dx^4} + k.y = 0$$

The general solution for the equation can be expressed as the four functions

$$y_{(x)} = Y_o(F_1) - \frac{1}{}\theta_o(F_2) + \frac{1}{2_{EI}} M_o(F_3) + \frac{1}{\lambda^3 EI} Q_o(F_4)$$

where

$$= \sqrt[4]{\frac{K}{4EI}}$$

$E_1 = \cosh \lambda_x . \cos \lambda_x$

$F_2 = \frac{1}{2} (\cosh \lambda_x . \sin \lambda_x + \sinh \lambda_x . \cos \lambda_x)$

$F_3 = \frac{1}{2} \sinh \lambda_x . \sin \lambda_x$

$F_4 = \frac{1}{4} (\cosh \lambda_x . \sin \lambda_x - \sin h \lambda_x \cos \lambda_x)$

Then the solpes, moments and shears along the bar are expressed as:

$$Q_{(x)} = -4\lambda y_o(F_4) - \theta_o(F_1) + \frac{1}{\lambda EI} M_o(F_2) + \frac{1}{\lambda^2 EI} Q_o(F_3)$$

$$M_{(x)} = \frac{k}{\lambda^2} y_o(F_3) - \frac{k}{\lambda} \theta_o(F_4) - M_o(F_1) + \frac{1}{\lambda} Q_o(F_2)$$

$$Q_{(x)} = \frac{k}{\lambda} y_o(F_2) - \frac{k}{\lambda^2} \theta_o(F_3) - 4\lambda M_o(F_4) - Q_o(F_1)$$

The terms of the stiffness matrix can be derived from the above expressions due to the unit rotations and translations simultaneouly as follows.

$$S_{2,2} = \frac{k}{2\lambda^3 C} (\sin\lambda\ell \,.\, \cosh\lambda\ell - \sin\lambda\ell \,.\, \cos\lambda\ell)$$

where

$$C = \sinh^2\lambda\ell - \sin^2\lambda\ell)$$

$$S_{3,2} = -\frac{k}{2\lambda^2 C} (\sinh^2\lambda\ell + \sin^2\lambda\ell$$

$$S_{5,2} = \frac{k}{2\lambda^3 C} (\cosh\lambda\ell \,.\, \sin\lambda\ell - \sinh\lambda\ell \,.\, \cos\lambda\ell$$

$$S_{6,2} = \frac{k}{\lambda^2 C} (\sinh\lambda\ell \,.\, \sin\lambda\ell)$$

and $S_{3,3} \frac{k}{\lambda C} (\cosh\lambda\ell \,.\, \sinh\lambda\ell + \cos\lambda\ell \,.\, \sin\lambda\ell$

$$S_{5,3} = S_{3,2}$$

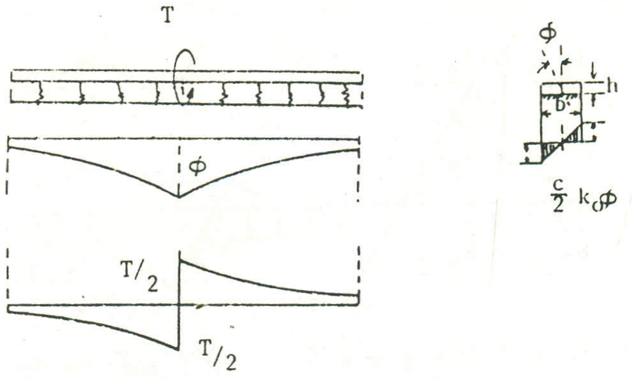


Fig. (2) Bar on Elastic Foundation

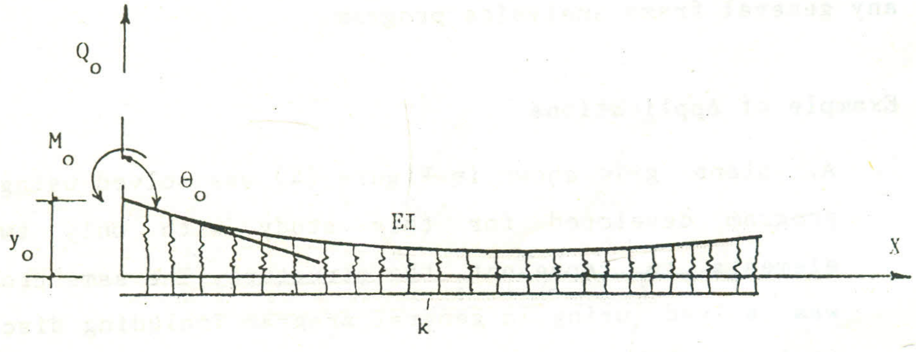


Fig. (3)

$$S_{5,3} = - S_{6,2}$$

$$S_{6,3} = - \frac{k}{\lambda C} \cdot (\sinh\lambda\ell . \cos\lambda\ell + \cosh\lambda\ell \sin\lambda\ell)$$

$S_{5,5} = S_{2,2}$ $\qquad$ $S_{6,6} = S_{3,3}$

$S_{2,5} = S_{5,2}$ $\qquad$ $S_{2,6} = S_{6,2}$

$S_{3,5} = S_{5,3}$ $\qquad$ $S_{3,6} = S_{6,3}$

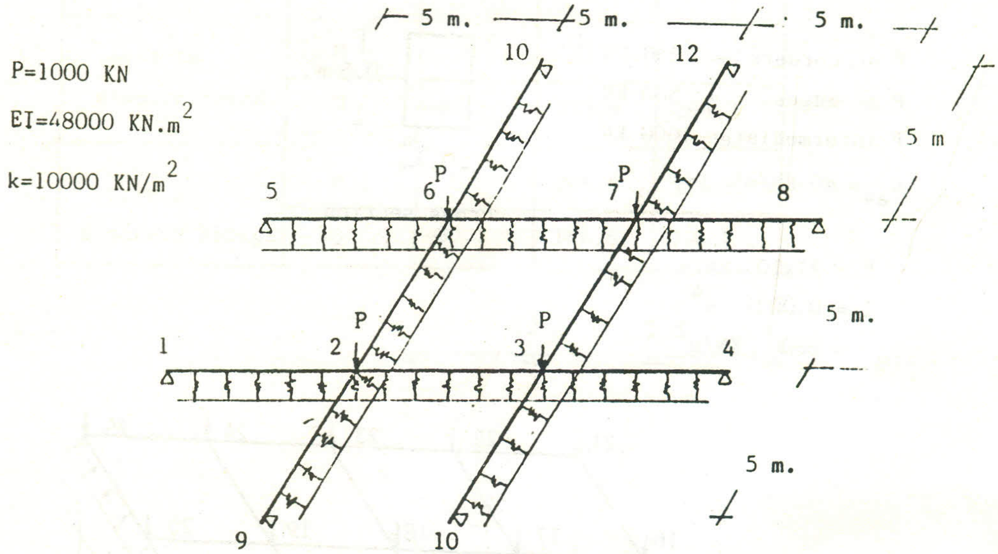
$S_{6,5} = -S_{3,2}$ $\qquad$ $S_{5,6} = S_{6,5}$

A general grid computer program was developed and written in Basic language to be used on any personal microcomputer. Only minor changes are required to incorprate the formulated stiffness matrix for grid element on elastic foundation in any general frame analysics program.

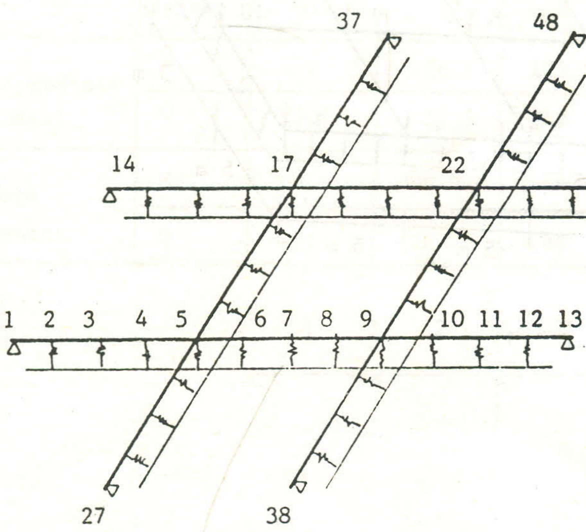
Example of Applications

1. A. plane grid shown in Figure (4) was solved using the program developed for this study with only twelve elements to represent the structure. The same problem was solved using a general program including discrete springs with fourty eight elements to represent the structure. The results of the solution are tabulated in table (1).

2. A plane grid shown in Figure (5) to represent a monolithic foundation composed of grade beams. The results of the analysis are tabulated in table (2). It is reasonable to recommend the moments at midspan of



Grid on Elastic Foundation



Grid on Discrete Elastic Springs

Fig. (4)

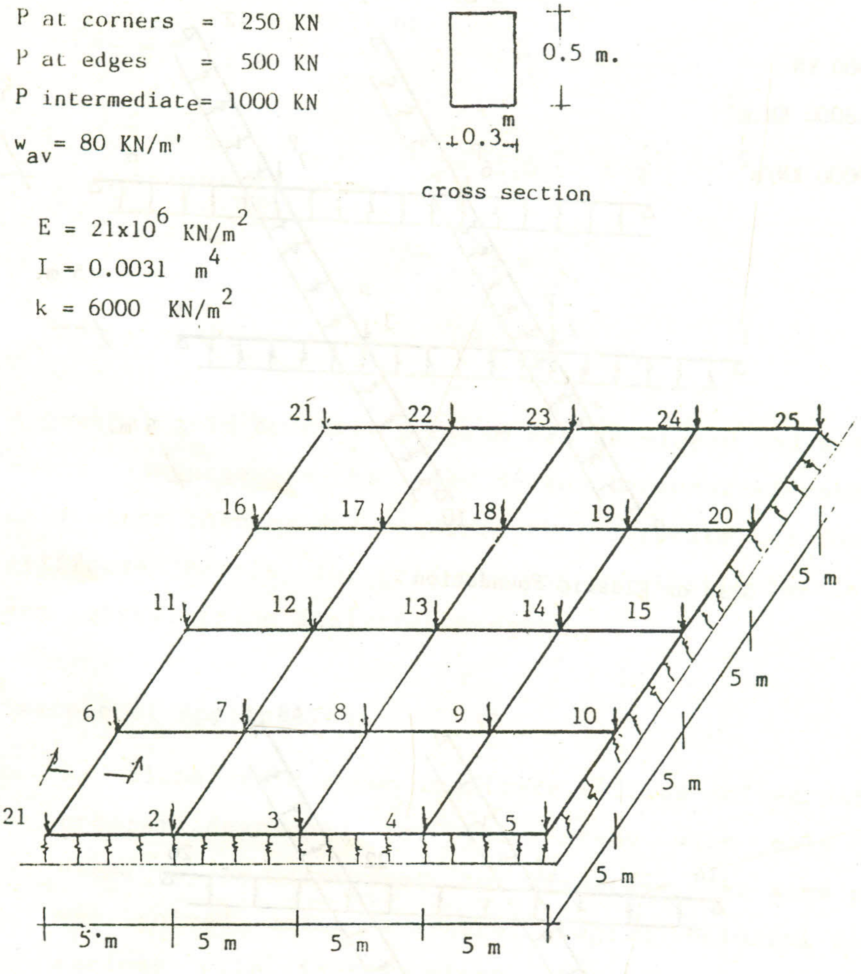

P at corners = 250 KN
P at edges = 500 KN
P intermediate= 1000 KN
w av = 80 KN/m'
E = 21x10^6 KN/m^2
I = 0.0031 m^4
k = 6000 KN/m^2
0.5 m.
0.3 m
cross section
21
22
23
24
25
16
17
18
19
20
11
12
13
14
15
6
7
8
9
10
21
2
3
4
5
5 m
5 m
5 m
5 m
5 m
5 m
5 m
5 m

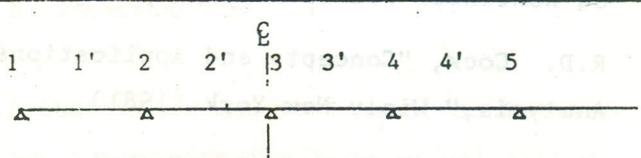
Fig. (5)

Table (1)

	joint	1	2	3
Grid on elastic found.	M	0	212	212
	Q	-33.1	233.8	-266.2
Grid on discrete sprigs	M	0	195.5	194.8
	Q	-30.3	164.6	-194.8

Table (2)

	joint	1	1'	2	2'	3
Intermediate Element	M	0	-58.3	175.5	-70.1	172.1
	Q	-105	12.2	242	0.1	-250
Edge Element	M	0	-56.6	140.4	-55.3	136
	Q	-125	18	194	0.4	-197.5



and at supports of for practical design of plane gride on elastic foundation.

Conclusion

An accurate stiffness matrix for a grid on a continuous Wrinkler foundation is formulated. This procedure can be implemented into any simple frame analysis computer program using the stiffness method of analysis. The examples showed that few elements are required to represent grid fourdations to yield exact solution for the problem.

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