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ELASTO PLASTIC ANALYSIS OF WEDGE ANCHORAGE

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ABSTRACT

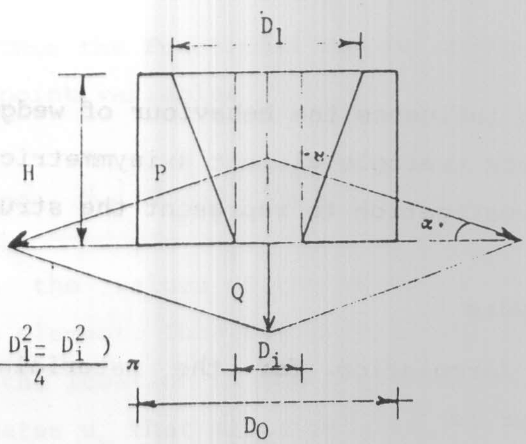
A nonlinear analysis of wedge anchorages, used in prestressed steel structures, is performed by the use of an imperoved finite element formulation. Development of the plastic region, the load displacement relationship and stresses and strains distribution during continued loading are studied. A limited parametric study is formed to investigate the major shape variables which influence the ultimate capacity of wedge anchcrage.

INTRODUCTION

Problem concering wedge anchorage used in prestressed steel structures and bearing type steel sockets may by approximated as a thick wall cylinder subjected to internal pressure as shown in Figure (1). Johnson (1) studied the optimum configuration for steel sockets. The finite element formulation was developed originally for structural analysis and it has been used for solving many complex structural problems. The finite element method has been extended to analyze nonlinear problems using the initial strain method (2) or the tangent modulus method (3). Marcal and King (4) introduced a partial stiffness concept and made an eleasto-plastic analysis of axisymmetric problems as well as problems of plane stress and strain. The most extensive formulation of plane elasto-plastic problems appears to be made by Felippa (5). Yamada et al (6) obtained an explicit expression of the incremental stress-strain matrix for the Prandtl-Rasssis equations. Carter and lee (7) investigated the analysis of cylinders and ring compression using the nonlinear finite element analysis. They studied the role of internal friction on the deformation behaviour of metals.

In this paper, an improved finite element representation was used to deal with the detailed study of plane strain axisymmetric wedge anchorage having finite dimensions.

Development of the plastic region, the load displacement relationship and stress-stran distributions during continued loading were studied with the variation of the major



Fig, (1)

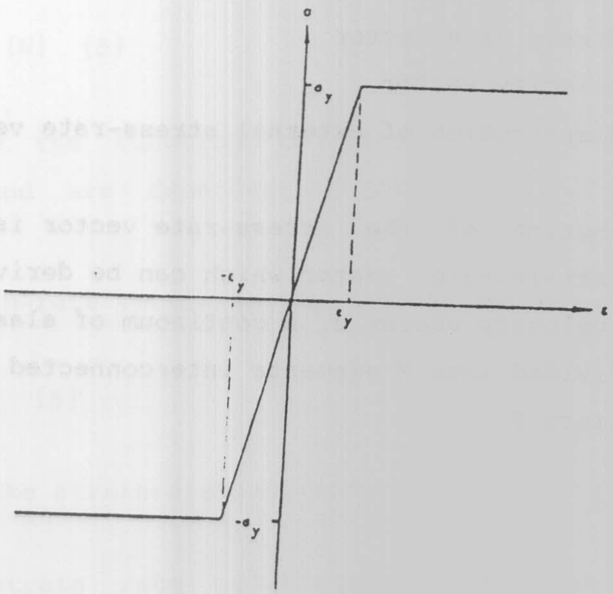


Fig. (2) Idealized stress-strain curve

variables which influence the behaviour of wedge anchorage. A constant strain traingle element axisymmetric element was used in this investigation to represent the structure.

Method and Analysis

A variational formulation for the materials of elastic-plastic behaviour is :

$$= \frac{1}{2} \int_V \sigma^T \varepsilon . \, dV - \int_S u^T F \, dS$$

where

σ = The stress rate vector

ε = The strain rate vector

u = The velocity vector

F = The distribution of external stress-rate vector.

The distribution of the stress-rate vector is associated with the strain-rate vector which can be derived from the continuous velocity vector u. A continoum of elastic-plastic body is divided into M elements interconnected at a number of rodal points P.

$$\Phi = \sum_m^M \varphi^m$$

where φ^m is the functional with respect to the m^{th} element. The discretization of the variational problem is performed on the elemental level approximating the functional Φ^m by a function φ^m of the m^{th} element. This approximation can be achieved by replacing the velocitrate u distribution by approximate velocity-rate for each element. Thus the functional can be approximated by the function of P-point variables

$$\Phi = \varphi\ (u_1, u_2 \ldots u_p) = \sum_m^M \varphi^m\ (u_k^m)$$

where u_k^m are the values of the velocity vectors at nodes in the element. Thus the initial variational problem is reduced to the location in the P-dimensional vector space of the coordinates u_k that minimize $\varphi\ (u_k)$.

The displacement rates at any point within the element can be expressed as function of the modal displacement increments

$$\{ u \} = [N]\ \{\delta\}$$

where [N] = the function of the position of the point considered and are dependent on the displacement function assumed.

The strain-rate vector in the element can be determined:

$$\{\varepsilon\} = [B]\ \{\delta\}$$

where [B] = the strain-displacement relation matrix

The stress-strain rate relationship can be expressed as

$$\{\sigma\} = [D]\ \{\varepsilon\}$$

where [D] = the material properties matrix of an elastic element

Thus

$$\varphi^{m} = \tfrac{1}{2}\int [D]\ [B]\{\delta\})^{T}\ [B]\ \{\delta\}dV - \int_{S} (N]\{\delta\}\ )^{T}\ dS\ \{F\}\ dS$$

Evaluation of surface and volume integrals results in the discrete representation of the functional at the element level

$$\varphi^{m} = \tfrac{1}{2}\{\delta\}^{T}\ [K]\{\delta\} - \{\delta\}^{T}\ \{F\}$$

where

[K] = element stiffness matrix

{ F } = nodal point force-rate vector

The element stiffness matrix [K] can be determined as

$$[K] = \int_{V} [B]^{T}\ [D]\ [B]\ dV$$

The global stifness matrix [K] and the equivalent nodal point force-rate vector F are the sume of those of the subregions.

The necessary condition for the function is to assume a stationary value for the unknown velocity-rate vector{ u} at the nodes of the region. Considering the mixed boundary conditions where the velocity vectors over the surface S are discribed. The following matrix equation of unknown nodal point vectors

$$\{F\} = [K] \{u'\}$$

where $\{F\}$ = the external nodal force-rate vector

$[K]$ = the global stiffness matrix

$\{u\}$ = the unknown displacement vector

In this study a trainglar axisymmetric element is used as shown in Figure (2) and the elemental stiffness matrix of this element can be expressed as

$$[K] = 2\pi \iint [B]^T [D] [B] \, r . \, dr . \, dz$$

The formulation of the stress-strain matrix [D] is based on Hooke's law and Prandtl-Reuss equations for the elastic-plastic state of an isotropic body.

$$\varepsilon^e_{ij} = \frac{\sigma_{ij}}{2G} + \delta_{ij} (1-2\nu) \frac{\sigma_m}{E}$$

$$\sigma_{ij} = 2G (\varepsilon^e_{ij} + \delta_{ij} \frac{\nu}{1-2} \varepsilon^e_{ii})$$

where σ^1_{ij} = deviatoric stress-rate tensor

ε_{ij} = elastic strain-rate tensor

σ_m = mean stress-rate

E = Modulus of Elasticity

G = Shear Modulus

ν = Poisson's ratio

δ_{ij} = Kroncker delta.

For the element in the plastic region, the plastic stress-strain matrix can be formed using the Prandtl-Reusse quations with von-Misses criterion during continued looding as

$$\varepsilon_{ij} = \sigma_{ij}\,\lambda + \frac{\sigma_{ij}}{2G}$$

$$\varepsilon_{ij} = \frac{1-2\nu}{E}\,\sigma_{ij}$$

$$\sigma_{ij}\quad\sigma_{ij} = \frac{2}{3}\,\sigma^{-2}$$

where $\lambda = \frac{2}{3}\left(\frac{\sigma}{\varepsilon H'}\right)$

ε_{ij} = deviatoric strain-rate tensor

σ_{ij} = deviatoric stress-rate tensor

H' = the slope of the effective stress and plastic strain curve

$$\sigma_{ij} = 2G\left(\varepsilon_{ij} + \delta_{ij}\,\frac{\nu}{1-2\nu}\,\varepsilon_{ij} - \sigma_{ij}\,\frac{\sigma_{kl}\,\varepsilon_{kl}}{S}\right)$$

where

$$S = \frac{2}{3}\,{}^{-2}\left[1 + \frac{H'}{3G}\right]$$

Numerical Solution Technique

A step by step incremental technique suggested by Yamada et al (6) for the solution of the nonlinear equation is used in this study. It is possible to trace the squential yielding of the elements in the correct order by using this incremental technique. The procedure used in this work can be summarized as follows.

1. Apply an initial loading and calculate the elastic displacements strains and stresses and the equivalent

stress $\bar{\sigma}^{e}$. Let $\bar{\sigma}^{e}$ the maximum of $\bar{\sigma}^{e}$.

2. Scale up all the elastic values in order to induce the element with the maximum equivalent stress to first yield. The scale factor $r = \frac{f_y}{\bar{\sigma}^{e}}$. Let $\{L_e\}$ the load at initial yield or the elastic limit.

3. Compute a new elastic-plastic element stiffness matrix for the element at yield $[K^p]$.

4. Modify the overall stiffness matrix $[K]$. Define an appropriate test increment $\{\Delta L^T\}$ of the load. Solve for the displacements and calculate the stress increment $\Delta\sigma_{ij}^T$ and strain increment $\Delta\varepsilon_{ij}^T$.

5. Calculate for every elastic element a yield scaling factor.

$$r = \frac{A + \sqrt{A^2 + \Delta(\Delta\sigma_{ij}^T)^2 (f_y^2 - \sigma^2)}}{2(\Delta\sigma_{ij}^T)^2}$$

$$A = (\Delta\sigma_{ij}^T) + 2\,\sigma \cdot \Delta\bar{\sigma}^{T} - (\Delta\bar{\sigma}^{T})^2$$

where σ is the present equivalent stress of the elastic element and $\Delta\bar{\sigma}^{T}$ the increment of included by the load increment $\{\Delta F^T\}$

6. Determine the minimum scaling factor r_{min}. Thus the load increment $r_{min}\ L^T$ is sufficient to induce the elastic element of r_{min} to yield and the element is treated as a post-yield element in all calculations.

7. Scale the nodal displacement, stresses and strains increments by r_{min} and add to the present values. Store the results.

8. Calculate and store the equivalent stresses for each

element. Calculate the increment $_p$ of the equivalent plastic strain for each post yield element.

$$\varepsilon_p = \frac{\sigma_{ij} \Delta\varepsilon_{ij}}{\sigma \;(1+H'/3G)}$$

and check that $\Delta\varepsilon^{-p}$ is positive for all post-yield plastic elements. If it is positive return to step (3) other wise stop computation. This closes the squence meaning that collapse load is reached.

In some instances, several elements instead of a single element can be allowed in the same loading increment in order to reduce the computation time. This can achieved by setting the number of elements to be yielded at a fixed number and the differences between r_{min} and the scaling factors of the element to be yielded should not be greater than a fixed tolerance factor.

Wedge Anchorage

A computer program written in FORTRAN IV is developed using the Finite Element technique for nonlinear material properties. The stress-strain relation ship is idealized as shown in Fig. (2). The properties of a high tensile steel having a yield strength of 360 KN/mm^2 and Poisson's ratio of 0.3 is used. A typical mesh of 77 nodes and 120 axisymmetric elements is used to represent the Wedge anchorage as shown in Fig. (3).

A limited parametric study is formed to study the effect of

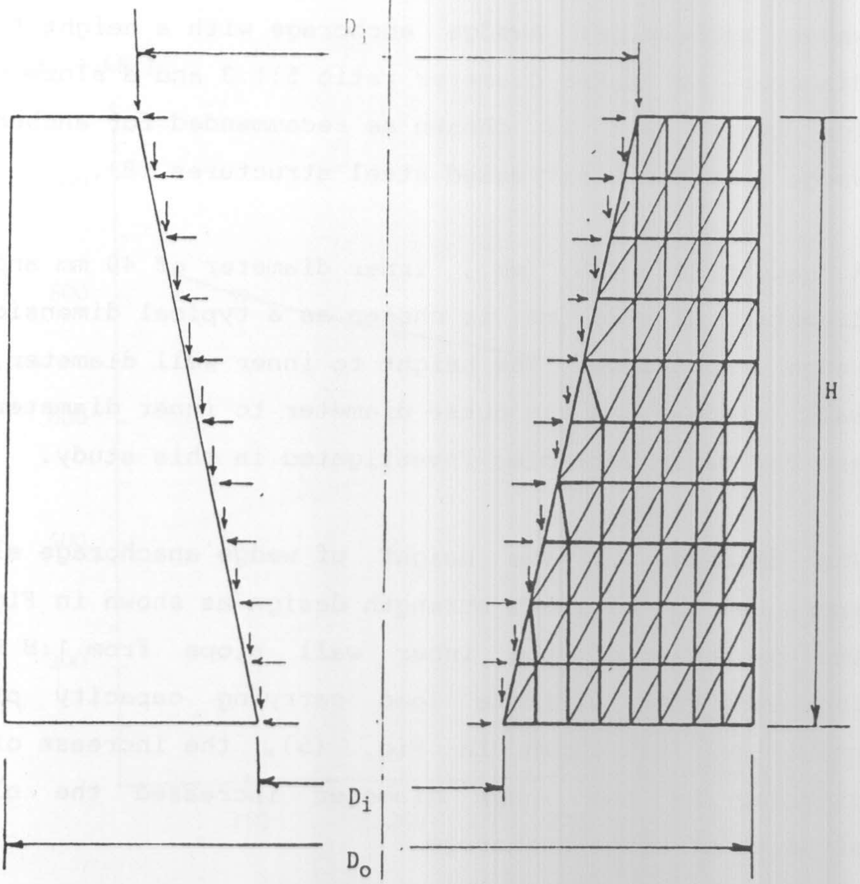


Fig (3) Finite Element Idealization

the major variables influence the behaviour and strength of wedge anchrorage: Awedge anchorage with a height to inner diameter to outer diameter ratio 5:1:3 and a slope of 1:10 for inner wall is chosen as recommended for anchors with wedge grip for prestressed steel structures (8).

A height of 200, mm., inner diameter of 40 mm and outer diameter of 120 mm. is chosen as a typical dimensions for wedge anchorages. The height to inner wall diameter, inner wall slopes and the outer diameter to inner diameter ratio are the major variables investigated in this study.

The decrease of the height of wedge anachorage slightly increased the ultimate strength design as shown in Fig. (4). The decrease of the inner wall slope from 1:8 to 1:12 increased the ultimate load carrying capacity of wedge anchorage, as shown in Fig. (5), the increase of outer diameter to the inner diameter increased the ultimate strength of wedge anchorage.

Conclusion

The procedure used here can perdict the ultimate load carrying capacity and behaviour of wedge anchorage, through the load history. A limited parameteric study is formed to investigate the major variables which influence, the ultimate carrying capacity of wedge anchorage.

The decrease of height is not significant in increasing the strength of wedge anchorage. The decrease of the slope of the inner wall increased the ultimate strength.The increase

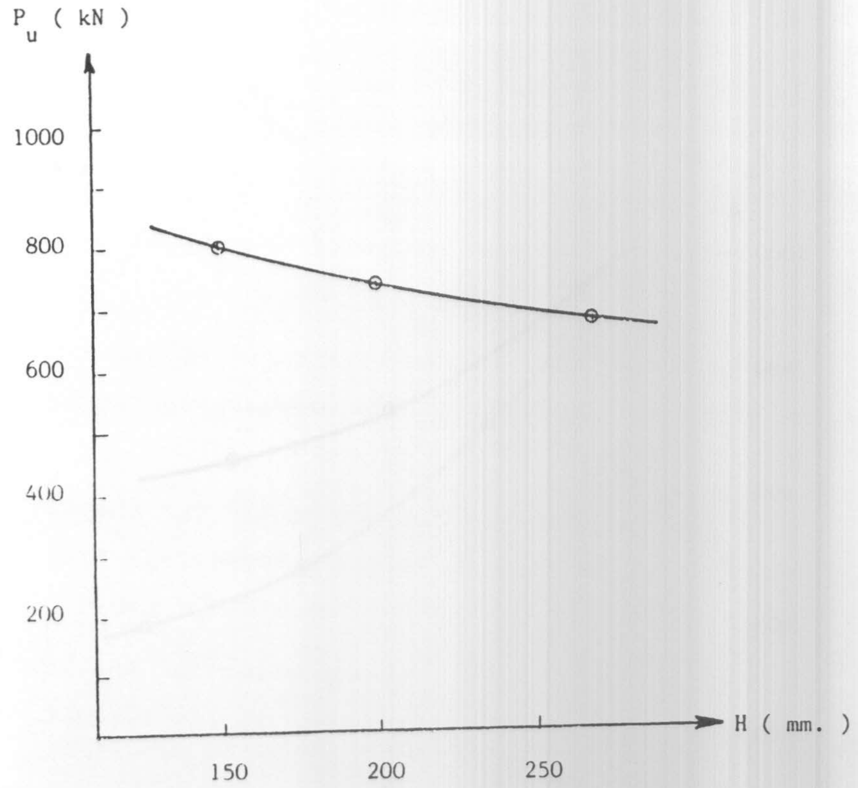


Fig. (4) Effect of Height Variation on Ultimate Stregth

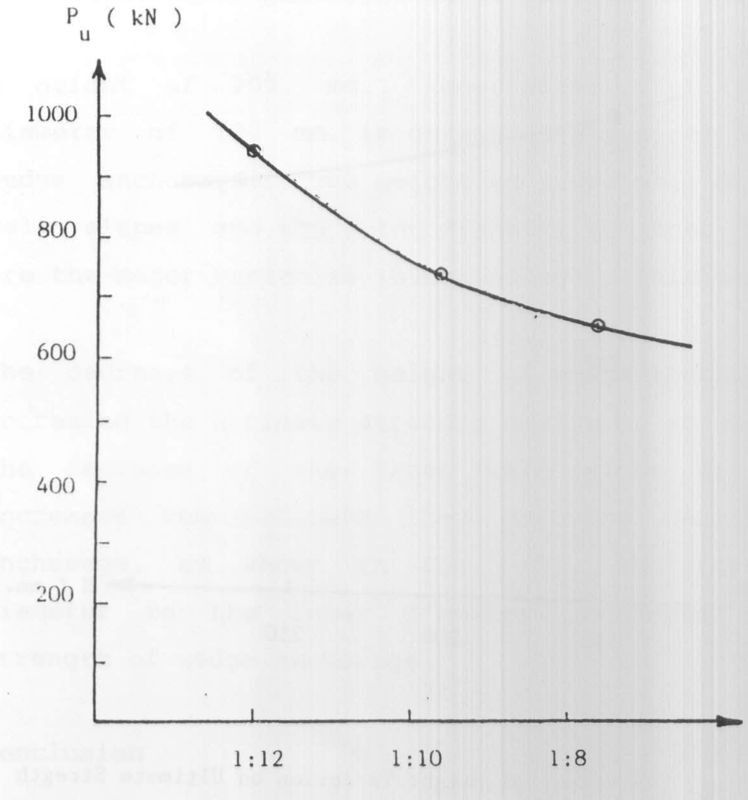


Fig.(5) Effect of Slope Variation on Ultimate Strength

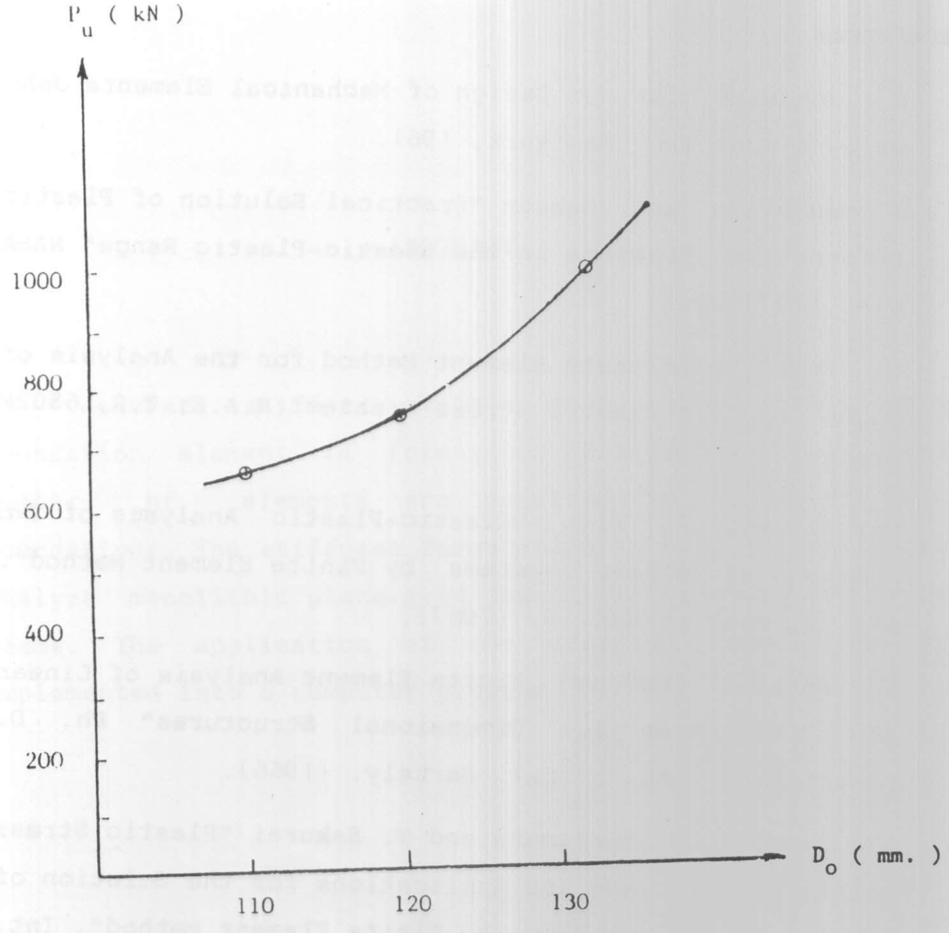


Fig.(6) Effect of Outer Diameter Variation on Ultimate Strength of wall thickness increased the ultimate carrying capacity of wedge anchorage. Therefore, having the height of the anchorage equale to the outer diameter, decreasing the slope of the inner wall and increasing the wall thickness is recommended for the design of wedge anchorages.

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